PHILOSOPHICAL

TRANSACTIONS

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MDCCCLI.

Adjudication of the Medals of the Royal Society for the year 1851 by the President and Council.

The Copley Medal to Professor Owen, for his important discoveries in Comparative Anatomy and Palæontology, contained in the Philosophical Transactions and numerous other works.

The Royal Medal in the department of Astronomy, to The Earl of Rosse, for his "Observations on the Nebulæ," published in the Philosophical Transactions for the year 1850.

The Royal Medal in the department of Physiology, to George Newport, Esq., for his paper "On the Impregnation of the Ovum in the Amphibia" (First Series), published in the Philosophical Transactions for the year 1851.

27726

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ERRATA.

Page 397, line 9 from the bottom, *for* garnet-octohedrons *read* garnet-dodecahedrons.

—— 398, line 13 from the bottom, *for* $(H_4 N)_2$ *read* $(H_4 N)$.

—— 500, line 16, *for* J *read* $\frac{dJ}{dr}$.

In Art. 6, page 519, change β to β_1 wherever it is a factor of quantities containing Y_2 or other terms in the development of y.

Page 522, line 13 from bottom, *for* c_1 *read* C_1.

—— 531, line 14 from bottom, *for* (23.) *read* (33.).

—— 539, line 2 from bottom, *for* j_2 *read* j.

—— 703, *for* Hewitt *read* Hewett; *for* Beautems Beaupres *read* Beautemps Beaupré; *for* Engineurs *read* Ingenieurs; *for* Française *read* Français.

—— 709 and 712, *for* Lemon *read* Leman.

PHILOSOPHICAL TRANSACTIONS.

XIII. *On the Results of Periodical Observations of the Positions and Distances of Nineteen of the Stars in Sir* JOHN HERSCHEL'S *Lists of Stars, favourably situated for the investigation of Parallax, contained in Part III. of the Philosophical Transactions for* 1826, *and Part I.* 1827*. *By Lord* WROTTESLEY, *F.R.S. &c.*

Received November 14, 1850,—Read January 16, 1851.

IN these communications, Sir JOHN HERSCHEL shows, that if the component members of a double star occupy a certain position with reference to the pole of the ecliptic, and one of them be supposed to be situated within a given distance from the earth, a change will be periodically produced in the angle of position, and in the distance of the two stars forming the double star, consequent on the motion of the earth in her orbit; that the maximum of the change in the angle of position will occur at two periods of the year distant from each other about six months; and he gives formulæ by which the epochs at which that maximum occurs may be computed, and by which the amount of parallax due to a given change in the angle of position may be also found for each star; also lists of stars favourably situated for the investigation of that element, with the times of the year at which they should be observed, and the amount of parallax, which an observed change of 30′ in the angle of position would indicate in the case of each star.

On the erection of my observatory in the autumn of 1842, comprising among its instruments an equatoreal of very considerable power, I determined to employ it in the observations of the double stars contained in these lists, with the view of ascertaining whether they exhibited such decided differences in their positions, when observed at the proper periods, as to give good grounds of hope that some definite conclusion might be arrived at, as to the existence of a parallax in the objects observed, capable of being measured, or at the least, confidently announced as subsisting in fact.

* I have to acknowledge my obligations to Mr. MAIN of the Royal Observatory, Greenwich, for many valuable suggestions during the preparation of this communication for the press.

In conducting the observations, however, difficulties arose that had not been anticipated: Sir JOHN HERSCHEL's lists contain sixty-nine stars; many of these, at the periods announced in the lists as proper for observation, if observed at all, must be observed at distances from the meridian too great, when the delicate nature of the inquiry is considered; many of them, having south or small northern declinations, are near the horizon when the proper period for their observation arrives. Another very serious impediment existed in the obligation imposed of obtaining half of the measures in the early morning hours; the observer, if employed in the observatory on the preceding night, was sometimes fatigued and unequal to the task; heavy fogs frequently came on at that time, and an enormous deposit of dew on the interior surface of the object-glass often seriously incommoded the observer; an evil which, until an opening was made in the tube near it, was irremediable, since the use of dew-tubes failed in preventing it.

To these and other causes it is owing, that after more than six years devoted to this course of observation, I am compelled to apologise for the meagre results which, for reasons about to be mentioned, I have still ventured to lay before the Royal Society in their present shape. Of sixty-nine stars I have only obtained observations of forty-eight, and of these forty-eight, twenty-nine have only been observed at one period of the year.

It is a most discouraging feature in this class of observations, that, however numerous and trustworthy the measures obtained at one period of the year may be, there may still be a failure to procure, at the expiration of six months from their date, measures worthy to be compared with them; and the function of the parallax, being the difference between the two results, the value of that element is of course affected by the whole amount of error with which either result is charged.

It has often happened that a star has been observed at one of the assigned periods, and that no corresponding observations have been procured at the expiration of six months, or the next succeeding period; thus, for example, calling the first epoch in the year at which the star is marked for observation, the *early*, and the second the *late* period, a star has been observed two or more times successively at the early period, when no corresponding observations could be procured at the following late period.

The question then naturally occurs, whether the communication of the measures actually obtained may not be deemed premature. I consider, however, that I have proceeded sufficiently far to demonstrate the impolicy of further perseverance, with the means at my command; the rather that instruments are now erected both at Liverpool and Oxford, which are pre-eminently suited to this class of observations, and therefore it would be only a waste of time and force, which may be more profitably employed in other ways, to devote any further attention to the inquiry.

I proceed, therefore, without further apology, to describe the means employed, the mode of employing them, and the results obtained.

The instrument employed was an equatoreally mounted telescope of 10 feet 9 inches focal length, with an object-glass of $7\frac{3}{4}$ inches clear aperture, of which the flint-glass is by GUINAND, and the crown glass English, the whole having been finished and perfected by DOLLOND.

This telescope is mounted in the manner usually adopted by the opticians of this country in a fixed observatory, in the immediate vicinity of my residence in Staffordshire; the polar axis is formed of four mahogany planks 14 feet 3 inches long and 10 inches square in the middle, the pivots of which are of hard bell-metal, and rest above and below on Y's attached to large and solid stone piers, which are supported by a foundation of brick-work, joined with cement, and formed into a solid mass of great extent by filling up the space enclosed by an outward circuit of brick with stones and mortar pounded together: this mass of brick-work and stone extends beyond the equatoreal room, and forms also the support of the piers of a 5-foot transit.

The Declination and the Hour Circle are each 3 feet in diameter; the two verniers of the former read off to 10″ in space, and that of the latter to seconds of time. To the telescope there is attached a parallel-wire micrometer with one equatoreal fixed, and two moveable wires; the screw heads are divided into 100 parts; the micrometer is provided with the usual position circle, graduated on silver, with its vernier which reads to 6′ in space. The value of one part of the micrometer first employed was 0″·15628, but from the 1st of January 1847 the value 0″·15641 was used, the first having been determined by Mr. BEAUMONT, the former possessor of the instrument, and the latter by myself. There are six eye-tubes that can be used with the micrometer, with powers varying from 85 to 820, but the power almost invariably used in the observations about to be described was 450; one of 320, and sometimes a lower power, was occasionally, but very rarely, employed, and when this occurs a notice to that effect will be found in the Table containing the results of observations subjoined. The telescope is provided with a clock-work motion; the performance of the object-glass, so far as its powers have been tested, leaves nothing to be desired, but the mounting of the telescope, notwithstanding the precautions above mentioned, does not appear to be equally steady with that of some other recently erected equatoreals. Whenever the night admitted of it, ten measures both in position and distance were obtained of each object observed; when in the sequel I speak of a *set* of observations, the term is to be understood as applying to all the observations of a single night, which almost always comprised that number of measures both of position and distance. To each individual measure, whether of position or distance, an arbitrary weight was assigned, and registered at the time of making it, the number 10 being supposed to express a result with which the observer was entirely satisfied, and smaller numbers to denote a less degree of confidence in the value obtained; in point of fact, however, no higher number than 8 seems to have been used, and the weights employed generally range from 4 to 7. The sum of the weights of the indi-

vidual measures, or the weight of the set, has been divided by 10 prior to its entry in the subjoined tables; while therefore the number 5 should express the weight of a set of average goodness, the number expressive of that weight is reduced in practice to 4. The measures when made are registered in the printed skeleton forms, originally proposed by Sir JOHN HERSCHEL, and adopted, I believe, by almost all observers of double stars in this country. A number of these forms were bound together in a volume, and care was taken to fill up the different columns, so far at least as they refer to measures of position and distance. Whenever the weather permitted, a set of measures was obtained of each double star, on three separate days, at each period of the year marked out as the proper one for observation. The common arithmetical mean was used in taking the average of the partial measures, comprising each set, without taking the arbitrary weights into account. At the commencement of the series, the zero of the position circle was frequently determined; but it was found to undergo no alteration that might not be attributed to error of observation; from that time it became the practice to ascertain it about every two months; but it was never found to vary more than about 2′, a quantity very much within the error of observation in determining an angle, as will appear in the sequel; no zero was required for distance, as it was the practice to take an equal number of positive and negative readings.

The observations were commenced on the 15th February 1843, and terminated the 9th October 1849; and they were made almost exclusively by three gentlemen, who acted successively as my astronomical assistants, during the progress of the work. I have always found, that observations are better made when entrusted to one competent individual, who after a short time gains experience, and a facility of manipulation, which cannot be acquired by one who only occasionally observes; and this uniformity in the mode of observing is particularly desirable, where the quantities to be determined depend on comparisons of observations made at different periods; it was a subject of great regret to me, therefore, that I was compelled to make any change in the staff of my observatory during the course of this investigation, and I only observed myself on a few nights at the time of the first appointment of each assistant.

Mr. GODDARD observed from the commencement of the observations to October 2, 1843; Mr. SIMMS from that time to the 11th of June 1844; and Mr. PHILPOTT, my present assistant, from thence to the conclusion.

The results of the observations are embodied in five tables, and it now remains to explain the manner in which those tables have been formed.

The First Table.—In the first table the stars are arranged in the order of their R.A.'s; and the observations of each day, or sets, follow one another in the order of their date.

An asterisk will be found attached to four of the stars in the list; these are stars, which, on comparing their mean positions as ascertained in the course of this inquiry,

with those given by other observers, exhibit changes of such an amount in their angle of position as to afford satisfactory evidence of orbital motion.

This fact being admitted, it is plain that these stars cannot be rendered available in the present inquiry; it was deemed unnecessary therefore to extend to them the method of reduction employed in the case of the other stars; the observations, however, are given with the others in their proper place.

The *third and seventh columns* of the table contain the arithmetical mean of the individual measures, both in position and distance, obtained on the day specified in the first column.

The quantities inserted in *the fourth and fifth columns* headed "probable error" and "computed weight," refer to the positions only, and have been computed from the formulæ

$$P^2 = \frac{0{\cdot}45494}{n(n-1)} \times \text{sum of } \varepsilon^2,$$

$$W = \frac{1}{P^2},$$

where P = probable error of a single set of measures,

n = the number of measures,

ε = the individual errors,

and W = the weight of the result;

numbers proportional to the computed values of W being inserted in the Table.

The average probable error of all the sets, 211 in number, is $8''{\cdot}98$, and the average weight, $0{\cdot}0124$.

The quantities inserted in the *sixth and eighth columns*, headed "assigned weight," represent the sum of the weights, divided by 10, assigned by the observer to the individual measures of position and distance forming the set, or the arbitrary weights of the sets.

The magnitudes and colours of the stars were noted on each night of observation, and are inserted in the *ninth and tenth columns*; where one colour only is named, it is to be understood as applying to both stars; and where two are specified, the colour of the brighter component, or, in cases of equal brightness, of the star called A, is placed first in order.

The *eleventh column* contains the initial of the observer's name.

The results of each epoch and year are for the sake of distinctness separated by a blank space.

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TABLE I.

Year, Month and Day.	Fraction of year.	Position.	Probable error.	Computed weight.	Assigned weight.	Distance.	Assigned weight.	Magnitudes.	Colours.	Observer.
γ Arietis.										
1843. July 11.	·523	179° 44′	21·18	2	5	9·383	5	5—5	White.	G.
Aug. 7.	·597	178 59	14·02	5	7	9·103	7	6—6	Yellowish green.	G.
11.	·608	179 8	9·40	11	8	8·865	8	6—6	Yellowish.	G.
1844. Jan. 13.	·033	179 42	8·66	13	5·3	8·976	5·1	5—5	White.	S.
Feb. 8.	·104	179 43	15·23	4	4·1	8·933	4·2	5—5	White.	S.
1845. Jan. 21.	·055	178 48	13·96	5	3·2	8·720	2·8	5—5	Yellowish.	P.
24.	·063	181 24	8·94	13	2·9	8·943	3·1	5·6—5·6	Yellowish.	P.
Feb. 1.	·085	180 59	11·98	7	1·9	8·837	1·8	5·6—5·6	Yellowish.	P.
17.	·129	179 57	16·02	4	7·1	8·794	7·3	5·6—5·6	Yellow.	P.
1847. Jan. 29.	·077	178 13	7·36	19	5·7	8·796	5·8	5—5	Yellow.	P.
31.	·082	178 15	9·86	10	5·8	8·750	5·9	5·6—5·6	Yellow.	P.
Aug. 3.	·586	179 19	11·04	8	5·6	8·736	5·8	5—5	Yellowish.	P.
19.	·630	178 36	8·14	15	6·1	8·700	6·2	5·6—5·6	Yellowish.	P.
32 Eridani.										
1843. Aug. 18.	·627	346 29	9·91	10	8	6·866	8	6·7—8	White.	G.
Sept. 4.	·674	345 24	7·10	20	6	6·784	6	6—7	White.	G.
7.	·682	345 56	7·58	18	7	6·981	7	6·7—8	White.	G.
1845. Feb. *7.	·101	351 51	9·28	12	2·9	6·677	3·2	6·7—8	Yellow and blue.	P.
16.	·126	350 42	10·85	9	7	6·898	6·8	6·7—8·9	Red.	P.
17.	·129	349 41	11·72	7	6·8	6·758	6·6	6·7—8·9	Orange.	P.
1846. Feb. 10.	·110	348 23	8·44	14	4·1	6·664	4·2	7—9	White and bluish.	P.
1847. Jan. 31.	·082	347 12	6·60	23	4·8	6·642	5	7—9	Yellow and blue.	P.
1849. Feb. 17.	·129	346 39	11·97	7	2·2	6·714	2·1	6—8	Yellow and bluish.	P.
ω Aurigæ.										
1843. Sept. 8.	·684	349 28	11·46	8	7·4	6·588	7·4	7·8—9·10	White.	G.
16.	·706	350 20	14·38	5	7	6·367	7	7·8—9·10	White.	G.
24.	·728	351 19	10·55	9	7	6·343	7	8—10	White.	G.
1844. Sept. 25.	·734	351 37	12·30	7	6·6	6·724	6·7	7·8—10	Yellowish and light blue.	P.
26.	·736	351 6	13·43	6	6·9	6·546	6·6	7·8—10	Yellowish and light blue.	P.
Oct. 10.	·775	351 19	14·92	5	5·5	6·539	5·6	7·8—10	Yellowish and light blue.	P.
1845. Sept. 22.	·723	351 4	11·93	7	5·7	6·229	5·8	7·8—10	Yellowish and light blue.	P.
30.	·745	351 59	23·20	2	3·5	6·384	3·1	7—10	Yellowish and light blue.	P.
1846. Mar. 7.	·178	350 37	9·35	11	4·8	6·457	4·8	7·8—10	Yellow and whitish.	P.
12.	·192	351 37	14·81	5	5·8	6·442	5·9	7—10	Yellow and white.	P.
13.	·194	349 51	13·15	6	4·8	6·292	4·9	7—9·10	Yellow and blue.	P.
1847. Mar. 12.	·192	351 10	12·52	6	5·6	6·250	5·5	8—9·10	White and pale blue.	P.
24.	·225	351 35	20·59	2	2·2	6·297	2·1	7·8—9·10	Pale yellow and pale blue.	P.

* A power of 190 used.

TABLE I. (Continued.)

Year, Month and Day.	Fraction of year.	Position.	Probable error.	Computed weight.	Assigned weight.	Distance.	Assigned weight.	Magnitudes.	Colours.	Observer.
1848. Mar. 31.	·246	351° 47	11·05	8	5	6·233	5	6·7—9	Yellow and pale yellow.	P.
Apr. 1.	·249	352 8	10·61	9	5·8	6·230	5·4	6·7—8·9	Yellow.	P.
1849. Mar. 17.	·205	351 14	12·01	7	5·4	6·260	5·5	7·8—9·10	Yellow and white.	P.
						118 **Tauri.**				
1843. Sept. 24.	·728	196 46	11·31	8	7	4·866	7	8—9	White.	G.
27.	·736	196 36	5·48	33	7	4·612	7	8—9	White.	G.
1844. Mar. 10.	·189	195 20	5·53	33	3·6	5·065	3·4	8—9·10	White.	S.
29.	·241	195 22	8·57	14	5·8	4·967	5·6	8—9·10	Reddish.	S.
31.	·246	196 40	8·73	13	4	5·030	4	8—9·10	Reddish.	S.
Sept. 24.	·731	193 25	10·71	9	5·4	5·129	5·1	8—9	Light blue and bluish.	P.
25.	·734	195 9	8·93	13	7	5·087	7	8—9·10	Light blue and bluish.	P.
29.	·745	195 54	13·20	6	2·2	5·232	2·2	8—9·10	Light blue and bluish.	P.
1845. Feb. 26.	·153	192 1	25·48	2	5·8	4·796	5·5	7·8—9	Yellow and blue.	P.
Mar. 7.	·178	193 30	8·86	13	5·5	4·939	5·7	7·8—9	Light blue.	P.
8.	·181	194 8	15·76	4	3·3	4·984	3·3	7—9	Yellow and light blue.	P.
*13.	·194	191 17	43·84	1	1·4	5·033	1·3	7—9	Blue.	P.
20.	·214	196 49	17·13	3	5	5·097	4·8	7—9	Blue.	P.
Sept. 22.	·723	193 23	13·62	5	6·2	4·835	6·4	7—9	Yellow and light blue.	P.
1846. Feb. 28.	·159	196 7	10·35	9	3·9	5·035	3·6	7—8·9	Bluish and blue.	P.
Mar. 7.	·178	195 11	7·59	17	5·5	5·034	5·7	7—8·9	Bluish and white.	P.
						41 **Aurigæ.**				
1843. Feb. †15.	·123	351 25	16·19	4	4·6	7·856	3·6	7—8	White.	W. and G.
Mar. 1.	·162	352 2	24·41	2	4·2	8·393	4·5	7—8	White.	W. and G.
17.	·205	355 0	9·35	11	7	8·445	7	7·8—8	White.	G.
Sept. 1.	·665	352 0	14·44	5	5·9	7·857	5·9	8—9	White and rather blue.	W. and G.
8.	·684	354 18	4·66	46	7·8	7·930	7·8	8—9	White.	G.
16.	·706	353 9	10·20	10	6·8	7·820	6·8	8—9	White.	G.
1845. Feb. 26.	·153	352 41	11·46	8	5	8·097	5·2	8—9	White.	P.
Mar. 7.	·178	352 22	19·92	3	5·2	7·836	5	8—9	White.	P.
12.	·192	351 57	9·54	11	2·2	8·262	2·3	7·8—8·9	Blue.	P.
17.	·205	351 23	9·20	12	5·8	7·681	5·7	8—9	White.	P.
20.	·214	351 20	14·94	5	6	7·849	6·1	7·8—9	Yellowish.	P.
1846. Feb. 28.	·159	351 34	10·90	8	5·6	7·816	5·7	7·8—9	White and blue.	P.
Mar. 7.	·178	352 3	9·40	11	6	7·949	6·2	7·8—8·9	Whitish.	P.
1847. Mar. 10.	·186	352 42	9·79	10	4·6	7·925	4·4	7·8—9	Pale yellow.	P.
12.	·192	353 11	8·71	13	6·9	7·851	7	7·8—8·9	Pale blue.	P.
1848. Mar. 31.	·246	353 8	4·91	42	5·8	7·843	5·8	7·8—8·9	White.	P.
Apr. 1.	·249	353 18	7·71	17	5·9	7·876	6	7·8—8	White.	P.

* A power of 320 used. † *Ibid.*

TABLE I. (Continued.)

Year, Month and Day.	Fraction of year.	Position.		Probable error.	Computed weight.	Assigned weight.	Distance.	Assigned weight.	Magnitudes.	Colours.	Observer.
						δ Geminorum.					
1843. Apr. 5.	·257	194	29	10·57	9	5·8	7·610	5·8	3—10	White.	G.
10.	·271	195	34	79·59	0·2	0·4	8·400	0·4	3—10	White.	G.
11.	·274	194	12	9·07	12	5·2	7·614	4·9	3—9	White.	G.
1844. Mar. 29.	·241	197	58	12·35	7	5·5	7·310	5·7	3·4—11	Red.	S.
Apr. 1.	·249	198	5	13·45	6	5·2	7·468	5·5	3·4—11	Red.	S.
25.	·315	198	30	7·89	16	4·6	7·344	4·6	3·4—11	Red.	S.
1845. Oct. 31.	·830	197	54	17·95	3	5·8	7·295	5·6	3—11	Yellow and white.	P.
Nov. 12.	·862	198	30	9·14	12	5·8	7·322	5·8	3—11	Orange and light blue.	P.
1846. Apr. 9.	·268	198	42	9·19	12	5·8	7·355	5·8	3·4—10·11	Yellow and white.	P.
						Anon. Cancri.					
1843. Mar. 18.	·208	353	14	24·99	2	8·6	3·802	8·5	8—8·9	White.	W. and G.
29.	·238	354	33	5·25	36	3·6	4·580	3·6	8—8·9	White.	G.
Apr. 11.	·274	352	59	9·65	11	5·8	4·015	6·9	7—7·8	White.	G.
1844. Apr. 1.	·249	351	25	8·40	14	6·6	3·557	6·6	8·9—9·10	White.	S.
May 1.	·331	352	24	11·06	8	3	3·599	3	8·9—10	White.	S.
2.	·334	352	10	19·26	3	1·8	3·598	1·8	8·9—10	White.	S.
1845. Apr. 3.	·252	354	3	10·23	10	6	3·498	6	8·9—10	White.	P.
8.	·266	353	11	19·53	3	5·6	3·361	5·7	8·9—10	White.	P.
20.	·298	353	10	8·47	14	5	3·314	5·3	8·9—10	White.	P.
Oct. 31.	·830	351	29	11·94	7	5·9	3·563	5·7	7·8—9·10	Light blue.	P.
Nov. 12.	·862	352	5	15·23	4	6	3·617	6·1	7·8—9·10	Light blue.	P.
1848. Apr. 3.	·255	353	42	9·58	11	5·8	3·479	5·7	8—9	Deep yellow and pale yellow.	P.
						ϕ^2 Cancri.					
1843. Mar. 24.	·225	30	59	13·26	6	6·2	4·946	6	7—7	White.	G.
28.	·235	31	10	10·52	9	4	5·074	3·2	7—7	White.	G.
Apr. 20.	·298	31	54	7·44	18	6	4·930	6	5·6—5·6	White.	G.
1844. May 2.	·334	33	3	10·75	9	4·2	4·835	4·2	6·7—6·7	Reddish.	S.
5.	·342	32	52	8·56	14	4·3	4·889	4	6·7—6·7	Reddish.	S.
13.	·364	33	16	5·40	34	5	5·062	4·8	6·7—6·7	White.	S.
1845. Mar. 20.	·214	30	49	19·04	3	6	5·011	6	7—7	Yellow.	P.
24.	·225	32	15	11·54	8	6·2	4·696	5·8	7—7	Yellow.	P.
Apr. 2.	·249	33	2	11·21	8	4·8	4·779	4·1	7—7	Yellow.	P.
3.	·252	33	4	11·88	7	6·4	4·642	6·6	7—7	Yellow.	P.
Nov. 12.	·862	34	47	16·35	4	5·1	4·632	4·5	7—7	Light blue.	P.
22.	·890	33	4	11·43	8	7	4·701	6·8	7—7	Light blue.	P.
1846. Mar. 27.	·233	30	46	16·79	4	4	4·763	4	7—7	Bluish.	P.
Apr. 9.	·268	32	34	11·38	8	5	4·680	4·9	7—7	Bluish.	P.

TABLE I. (Continued.)

Year, Month and Day.	Fraction of year.	Position.		Probable error.	Computed weight.	Assigned weight.	Distance.	Assigned weight.	Magnitudes.	Colours.	Observer.
		°	′	″			″				
						2 **Comæ Berenices.**					
1843. May 2.	·331	235	15	38·45	1	0·6	3·939	0·6	6·7—8·9	White.	G.
3.	·334	237	16	14·52	5	5	4·056	5	6·7—8·9	Yellowish and white.	G.
5.	·339	238	47	19·61	3	4	3·947	4	6·7—9	White.	G.
10.	·353	237	8	14·97	5	4	3·777	4	6·7—9	White.	G.
1844. May 13.	·364	243	58	6·72	22	4·6	4·092	3·8	6·7—9	Red.	S.
23.	·392	243	31	14·43	5	4	4·129	4	6·7—8·9	Reddish.	S.
June 10.	·441	237	2	6·56	23	6·4	4·145	6	6·7—8·9	Orange.	S.
Nov. 20.	·887	237	23	10·20	10	3·6	4·043	3·6	7—9	Yellowish blue and blue.	P.
1845. May 5.	·339	238	46	19·60	3	4·2	3·990	4·2	6·7—10	Yellow and light blue.	P.
13.	·361	236	55	20·89	2	5·7	4·052	5·4	6·7—9·10	Yellow and bluish.	P.
19.	·378	236	38	17·58	3	5·8	4·066	5·7	6·7—9·10	Yellow.	P.
Nov. 22.	·890	240	25	9·44	11	4·9	3·817	4·8	7—9	White and yellowish.	P.
1846. May 7.	·345	236	20	13·73	5	1·5	3·829	1·5	7—9	White.	P.
May 8.	·348	236	42	22·49	2	4·6	3·809	4·5	7—9	White.	P.
11.	·356	236	41	12·32	7	5·9	3·795	5·6	7—9	Orange and pale blue.	P.
1847. May 26.	·397	238	10	13·91	5	5·4	3·618	5·2	6·7—8	Pale yellow and blue.	P.
1848. May 3.	·337	239	51	11·68	7	5·7	3·709	5·2	6·7—8	White and pale blue.	P.
5.	·342	239	20	17·40	3	1·7	3·868	1·5	6·7—8·9	Yellow and pale blue.	P.
12.	·361	238	28	5·60	32	4·4	3·618	3·5	6·7—8·9	Yellow and pale blue.	P.
						39 **Bootis** *.					
1843. Mar. *2.	·164	41	11	.....	...	0·9	3·993	0·9	7—7·8	White.	W. and G.
†4.	·170	43	1	.....	...	2	4·080	2	8—8·9	White.	W. and G.
6.	·175	43	18	.....	...	7·6	3·951	7·8	8—8·9	White.	G.
Aug. 18.	·627	43	25	.....	...	6	3·764	6	6—6·7	White.	G.
19.	·630	44	1	.....	...	7	3·922	7	6—6·7	White.	G.
Sept. 4.	·674	44	3	.....	...	7	3·854	7	6—7	White.	G.
						ε **Serpentis** *.					
1843. Aug. 1.	·580	196	8	.....	...	8	3·137	8	4—5	Yellowish.	G.
7.	·597	195	37	.....	...	7	3·003	7	3—4	White.	G.
10.	·605	195	1	.....	...	5	2·950	5	3—4	White.	G.
1844. July 26.	·567	197	15	.....	...	1·5	3·146	1·7	3—5	Yellowish and light blue.	P.
Aug. 4.	·591	192	13	.....	...	4	3·718	4	3—5	Yellowish and light blue.	W. and P.
9.	·605	192	58	.....	...	1·6	3·133	1·6	3—5	Yellowish and light blue.	P.
28.	·657	197	34	.....	...	5	3·310	5	3—5	Yellowish and light blue.	P.
29.	·660	197	16	.....	...	5	3·151	5	3—5	Yellowish and yellowish blue.	P.
1845. Feb. 13.	·118	192	18	.....	...	5·4	3·185	5·2	3—5	Yellow.	P.
17.	·129	190	47	.....	...	7·6	3·135	7·6	3—5	Yellow and yellowish.	P.
17.	·129	192	26	.....	...	6·8	3·105	6·3	3—5	Yellow and yellowish.	P.

* A power of 320 used. † *Ibid.*

TABLE I. (Continued.)

Year, Month and Day.	Fraction of year.	Position.	Probable error.	Computed weight.	Assigned weight.	Distance.	Assigned weight.	Magnitudes.	Colours.	Observer.
		° ′	″			″				
1845. Aug. 7.	·597	194 40	.....	...	5·4	3·105	5·2	4·5—6	Yellow.	P.
16.	·621	194 53	.....	...	5·4	2·910	5·6	4—5·6	Yellow.	P.
Aug. 28.	·654	195 20	.....	...	6·6	2·990	6·6	4—6	Yellow and light blue.	P.
1846. July 25.	·561	194 35	.....	...	4·9	2·829	4·6	3·4—5·6	Yellow and light blue.	P.
30.	·575	195 4	.....	...	1·5	2·970	1·4	3·4—5	Yellow and pale blue.	P.
Aug. 5.	·591	194 19	.....	...	4·9	2·764	4·7	3·4—5	Yellow and pale blue.	P.
1847. Mar. 10.	·186	192 41	.....	...	6	2·903	5·4	3·4—5·6	Yellow and yellowish.	P.
July 29.	·572	193 32	.....	...	4·9	3·021	4·4	3·4—6	Orange and pale yellow.	P.
30.	·575	193 3	.....	...	4·6	2·974	4·2	3·4—5·6	Orange and yellow.	P.
Aug. 1.	·580	193 35	.....	...	6·2	2·805	6·3	3·4—6	Orange and yellow.	P.
1848. Aug. 19.	·632	195 9	.....	...	0·8	2·847	0·7	4·5—6·7	Yellow and pale yellow.	P.
30.	·663	196 3	.....	...	4·3	2·908	4·3	4—6	Yellow.	P.
Sept. 4.	·676	195 11	.....	...	4·9	2·986	4·7	4—6	Yellow and pale yellow.	P.
178 (Bode) Libræ.										
1843. Aug. 1.	·580	7 39	7·92	16	7	12·374	7	8—8	White.	G.
7.	·597	6 49	11·65	7	5·4	11·798	5·4	8—8	White.	G.
11.	·608	8 14	8·47	14	5	12·195	5	8—8	White.	G.
1845. Feb. 14.	·120	7 15	15·12	4	3	12·232	3	8—8	Yellow and white.	P.
1847. Aug. *2.	·583	7 39	5·25	36	4·2	11·706	4·6	7·8—7·8	White.	P.
3.	·586	7 15	7·55	18	5·1	11·844	4·6	7·8—7·8	White.	P.
μ Draconis *.										
1843. Mar. †8.	·181	15 0	.....	...	0·8	2·844	0·9	7—7·8	Bluish.	W.
17.	·205	17 12	.....	...	4·3	3·693	4·3	8—8	White.	G.
24.	·225	17 27	.....	...	5·7	3·402	5·3	7—7·8	Bluish.	W. and G.
Aug. 22.	·638	17 47	.....	...	6	3·793	6	8—8	White.	G.
26.	·649	16 36	.....	...	6	3·233	6	8—8	White.	G.
26.	·649	15 19	.....	...	6·1	3·271	6·1	7·8—7·8	Bluish.	W.
1844. Aug. 28.	·657	17 46	.....	...	4	3·241	4	7·8—7·8	Light blue.	P.
30.	·663	16 40	.....	...	5	3·238	5	7—7	Light blue.	P.
31.	·665	17 5	.....	...	5	3·285	5	7—7	Light blue.	P.
Sept. 2.	·671	17 16	.....	...	4·2	3·110	4·2	7—7	Light blue.	P.
1845. Aug. 24.	·643	13 16	.....	...	6·1	3·202	6·1	7—7	Bluish.	P.
27.	·652	12 27	.....	...	6·8	3·209	6·4	7—7	Bluish.	P.
28.	·654	12 58	.....	...	7	3·199	6·9	6·7—6·7	Bluish.	P.
1846. Mar. 9.	·183	13 16	.....	...	6·9	3·074	6·8	7—7	Blue.	P.
20.	·214	12 29	.....	...	5·3	3·063	4·8	7—7	Blue.	P.
Sept. 1.	·665	11 36	.....	...	1·8	2·943	1·7	6·7—6·7	Pale yellow.	P.
2.	·668	11 35	.....	...	6·5	2·996	6·3	6·7—6·7	Yellow.	P.
4.	·674	11 52	.....	...	6	3·007	5·9	6·7—6·7	Yellow.	P.
1849. Sept. 5.	·676	10 0	.....	...	5·2	2·980	5·3	6—6	Yellow.	P.
8.	·684	9 24	.....	...	4·6	3·023	4·2	6·7—6·7	Pale yellow.	P.

* A power of 190 used.

† The powers of 320 and 450 used.

TABLE I. (Continued.)

Year, Month and Day.	Fraction of year.	Position.	Probable error.	Computed weight.	Assigned weight.	Distance.	Assigned weight.	Magnitudes.	Colours.	Observer.
		° ′	″			″				
					100 **Herculis.**					
1843. Mar. 28.	·235	1 25	10·02	10	6	13·855	6	6·7—7	Bluish.	G.
Apr.* 13.	·279	0 50	18·24	3	4·5	14·471	4·2	5—5·6	Bluish.	W. and G.
18.	·293	2 6	10·87	9	5	14·508	5	6—6	Bluish.	G.
Aug.† 22.	·638	2 52	6·02	28	8	14·276	8	6—6	White.	G.
24.	·643	2 20	2·77	130	8	13·951	8	6—6	White.	G.
30.	·660	2 34	6·62	23	7	14·057	7	7—7	White.	G.
1844. Aug. 30.	·663	2 12	10·24	10	5	13·916	5	6—6	Light blue.	P.
Sept. 7.	·684	2 41	6·33	25	5·7	13·941	5·7	6—6	Bluish.	P.
10.	·693	2 38	4·44	51	5·9	14·031	5·9	6—6	Bluish.	P.
1845. Mar. 11.	·189	1 34	10·94	8	5·3	13·986	5·1	6—6	Blue.	P.
17.	·205	1 44	11·08	8	6	14·031	6	6·7—6·7	Bluish.	P.
19.	·211	1 41	13·06	6	6·3	13·858	6·3	6·7—6·7	Yellowish blue.	P.
Aug. 24.	·643	2 11	3·80	69	7	13·772	6·7	6·7—6·7	Blue.	P.
27.	·652	1 54	3·26	94	6	14·022	6	6·7—6·7	Bluish.	P.
28.	·654	2 1	4·61	47	7	13·900	6·9	6—6	Yellow.	P.
29.	·657	2 18	3·63	76	6	14·015	6	6—6	Yellow.	P.
1846. Aug. 29.	·657	2 32	6·24	26	2·3	13·944	2·4	6·7—6·7	Pale yellow.	P.
Sept. 2.	·668	2 9	6·51	24	6·1	13·893	5·8	7—7	Pale yellow.	P.
4.	·674	2 9	7·13	20	5·6	13·970	5·9	7—7	Pale yellow.	P.
1847. Mar. 18.	·208	2 45	4·73	45	5·1	13·989	5·2	6·7—6·7	Bluish.	P.
28.	·235	2 57	8·09	15	4·2	13·977	4·3	6·7—6·7	Pale yellow.	P.
Sept. 8.	·684	2 45	5·49	33	6·8	13·993	6·5	6·7—6·7	Pale yellow.	P.
27.	·736	2 10	6·00	28	6·6	13·941	6·5	6·7—6·7	Pale yellow.	P.
1848. Sept. 4.	·676	2 30	4·99	40	5·5	13·826	5·7	6·7—6·7	Yellow.	P.
16.	·709	2 25	4·61	47	6·7	13·993	6·6	6·7—6·7	Yellow.	P.
1849. Sept. 8.	·684	1 4	4·52	49	5·3	14·083	5·6	7—7	Yellow.	P.
Oct. 9.	·769	1 41	5·22	37	5·2	14·057	5·3	6·7—6·7	Yellow.	P.
					579 **Struve.**					
1843. Sept. 9.	·687	159 56	6·83	22	7	5·249	7	9—9	White.	G.
16.	·706	159 29	12·44	7	7	5·056	7	9—9	White.	G.
20.	·717	159 25	11·14	8	6	4·850	6	9—9	White.	G.
1844. Oct. 5.	·761	160 8	12·41	7	5·1	5·441	4·9	8—8	Bluish.	P.
7.	·767	159 28	26·85	1	3·4	5·240	3·6	8·9—8·9	Light blue.	P.
10.	·775	160 8	10·14	10	5·7	5·332	5·5	8·9—8·9	Bluish.	P.
15.	·789	159 36	14·68	5	5·9	5·299	5·6	8·9—8·9	Light blue.	P.
1845. Apr. 6.	·260	159 2	20·31	2	6·2	5·188	6	8·9—8·9	Light blue.	P.
Oct. 5.	·758	158 4	14·47	5	5·5	4·981	5·5	8·9—8·9	Bluish.	P.
11.	·775	158 44	21·23	2	5·5	5·092	5·2	8·9—8·9	Bluish.	P.
13.	·780	158 37	10·08	10	5·2	5·111	5·4	8·9—8·9	Bluish.	P.
1847. Oct. 7.	·764	159 22	13·02	6	4·5	4·875	5	8—8	White.	P.
12.	·778	159 20	16·32	4	3·3	4·976	3·1	8—8	Pale blue.	P.

* A power of 140 used. † A power of 320 used.

TABLE I. (Continued.)

Year, Month and Day.	Fraction of year.	Position.		Probable error.	Computed weight.	Assigned weight.	Distance.	Assigned weight.	Magnitudes.	Colours.	Observer.
							287 **h** and **s**.				
1844. Oct. 5.	·761	289	28	19·90	3	0·8	7·775	0·8	7·8—9	Light blue.	P.
15.	·789	291	22	13·60	5	3·4	8·486	3·4	7·8—9	Blue and light blue.	P.
18.	·797	291	2	6·63	23	4·3	8·205	4	7·8—9	Blue and bluish.	P.
23.	·810	291	17	4·72	45	0·8	8·557	0·9	7·8—9	Blue and bluish.	P.
1845. Apr. 6.	·260	290	54	8·44	14	3·5	8·486	3·4	7·8—8·9	Bluish.	P.
Oct. 5.	·758	291	57	18·50	3	4·8	8·080	4·6	7·8—9	Whitish and bluish.	P.
11.	·775	291	37	15·61	4	4·8	8·076	4·9	7·8—9	Whitish and bluish.	P.
16.	·789	290	31	12·92	6	4·2	7·949	4	7·8—8·9	Whitish and bluish.	P.
							ε Draconis.				
1843. Apr. 18.	·293	357	5	24·33	2	4	3·212	5	5—10	Yellowish and bluish.	G.
19.	·296	354	19	43·84	1	3·4	2·986	3·4	4—10	Bluish and white.	W. and G.
28.	·320	355	34	43·64	1	4·2	3·060	4	4—9	White.	W. and G.
1845. Oct. 24.	·810	354	40	38·58	1	1·8	2·802	1·7	5·6—10·11	Yellow and white.	P.
25.	·813	355	18	30·57	1	5·2	2·938	5·4	5—10	Yellow and white.	P.
31.	·830	357	28	16·91	4	5·8	2·982	6·2	5—9·10	Yellow and white.	P.
Nov. 1.	·832	356	9	9·02	12	6	2·972	6	5—9·10	Yellow and white.	P.
							I of H 95.				
1843. May 22.	·386	336	24	26·29	1	1·2	3·380	1·2	6·7—10	White.	G.
24.	·392	338	21	15·83	4	4	3·334	4	6·7—10	White.	G.
25.	·394	336	54	14·29	5	5	3·075	5	6·7—10	White.	G.
June 5.	·424	336	33	28·04	1	4·5	3·002	4·5	6·7—10	White.	G.
Nov. 23.	·893	344	42	44·69	1	1·5	3·523	1·5	6—8	White.	W. and S.
29.	·909	341	24	9·19	12	2·5	3·480	2·4	6—8	White.	S.
Dec. 24.	·977	342	29	25·38	2	3·8	3·149	3·2	6—8	White.	S.
1844. Jan. 2.	·003	344	3	21·48	2	1·2	3·677	1·1	6—8	White.	S.
7.	·016	342	24	12·01	7	5·5	3·115	5·3	6—8·9	White.	S.
June 1.	·416	341	43	9·27	12	5	3·252	4	6—8·9	Orange and white.	S.
3.	·422	341	26	14·52	5	1·4	3·294	1·3	6·7—9	Orange and white.	S.
10.	·441	341	44	17·58	3	4·7	3·286	4·4	6·7—9	Orange and white.	S.
July 29.	·575	339	26	10·39	9	3·2	3·085	3·2	6·7—9	Bluish white.	P.
Aug. 4.	·591	338	12	12·65	6	4·2	3·065	4·2	6·7—9	Bluish white.	P.
14.	·619	341	42	12·49	6	4·4	3·230	4	6·7—9	Bluish and light blue.	P.
24.	·646	341	27	24·86	2	2·4	3·113	2·4	6·7—9	Bluish white.	P.
Nov. 21.	·890	340	16	16·77	4	6·7	3·069	6·7	6·7—9	Bluish white.	P.
25.	·901	339	36	15·45	4	7	3·125	7·5	6·7—9	Bluish white.	P.
26.	·903	340	27	11·75	7	5·8	3·155	5·7	6·7—9	Bluish white.	P.
1845. Oct. 31.	·830	342	36	11·32	8	7	3·179	6·9	7—9	Light blue.	P.
Nov. 3.	·838	339	55	14·47	5	5·2	3·162	5·3	7—9	Bluish.	P.
4.	·841	342	26	22·19	2	4·3	3·163	4·2	7—9	Bluish.	P.
1846. May 10.	·353	342	23	10·17	10	4·8	3·115	4·9	7—9	Pale yellow and pale blue.	P.
29.	·405	342	12	13·47	6	6·7	3·010	6·6	7—9	Pale yellow and pale blue.	P.
Oct. 30.	·827	340	51	12·59	6	4·1	3·049	3·5	7—8·9	Pale yellow and pale blue.	P.

TABLE I. (Continued.)

Year, Month and Day.	Fraction of year.	Position.	Probable error.	Computed weight.	Assigned weight.	Distance.	Assigned weight.	Magnitudes.	Colours.	Observer.
					P XXII 306 *.					
1843. June 25.	·479	143 2	.....	...	6	8·467	6	6·7—9	White.	G.
July 3.	·501	143 56	.....	...	6	8·353	6	6·7—9	White.	G.
6.	·509	144 17	.....	...	5·5	8·461	5·5	6·7—9	White.	G.
Dec. 17.	·958	145 46	.....	...	4·1	8·521	3·5	6·7—8·9	White.	S.
1844. Jan. 11.	·027	145 34	.....	...	4·5	8·618	4·2	6·7—9	Reddish and white.	S.
13.	·033	144 13	.....	...	4·9	8·788	4·7	6·7—9	White.	S.
Nov.* 27.	·906	146 0	.....	...	6·4	8·670	6·5	7—9	Blue.	P.
1846. Dec. 14.	·950	146 18	.....	...	1·1	8·350	1·3	8—10	Pale yellow and pale blue.	P.
22.	·972	146 3	.....	...	4·1	8·335	3·8	7·8—9	Yellow and pale blue.	P.
1847. Dec. 28.	·988	146 16	.....	...	1·7	8·024	1·7	7·8—9	Whitish and pale blue.	P.
1848. Dec. 21.	·972	145 37	.....	...	4·6	8·251	4·7	7·8—9·10	Yellow and pale blue.	P.

The Second Table.—This table contains the means of all the results obtained at the same period of each year, when observations have been obtained on two or more days.

The *second column* contains the mean epoch of the results from which the means are formed.

The positions in the *third column* are computed in the ordinary mode by multiplying each individual set by its computed weight as contained in the fifth column of Table I., adding the products, and dividing by the sum of all the weights.

The *fifth column* contains the sum of the computed weights of the individual sets, from which the probable error of the mean in the preceding column is calculated from the equation

$$P = \frac{1}{\sqrt{W}}.$$

The computed weights inserted in the Tables have been multiplied by 1000.

The mean distances in the *seventh column* are computed in the same manner, using, however, the arbitrary or assigned weights in column 8 of Table I., instead of weights computed as above explained; for it was not deemed necessary, for reasons which will appear in the sequel, to apply the calculus of probabilities to the reduction of any of the observations of the distances.

The quantities in the *sixth and eighth* columns, headed "Assigned weight," are the sums of the weights assigned to the individual sets, from which each mean is formed.

* A power of 190 used.

In the case of the four binary stars, to which an asterisk is attached in the Tables, the means, both of the positions and the distances, given in this table are obtained by employing the assigned or arbitrary weights, as before described.

In deference to the objections which may be urged to this mode of reduction, I had originally intended to use the common arithmetical mean in all these cases; but out of fifty-four means of positions computed strictly by the usual formulæ, and compared with means obtained as last described, and also with the arithmetical means of the same quantities, it was found that the mean obtained by using the arbitrary weights differed only in thirteen cases more than 9′ from the strict mean, and that in eight out of the thirteen cases, the result was rendered more erroneous by using the arithmetical mean, instead of the mean derived from the use of arbitrary weights: there were seven cases out of the fifty-four in which the computed and arbitrary weights gave precisely the same result.

Table II.

Name.	Fraction of year.	Position.	Probable error.	Computed weight.	Assigned weight.	Distance.	Assigned weight.
		° ′	″			″	
γ **Arietis.**	1843. ·576	179 10	7·33	19	20	9·078	20
	1844. ·069	179 42	7·54	18	9	8·957	9
	1845. ·083	180 38	5·92	29	15	8·816	15
	1847. ·080	178 14	5·89	29	12	8·773	12
	1847. ·608	178 51	6·55	23	12	8·717	12
32 **Eridani.**	1843. ·661	345 50	4·59	48	21	6·881	21
	1845. ·119	350 55	6·04	27	17	6·800	17
ω **Aurigæ.**	1843. ·706	350 26	6·84	21	21	6·436	21
	1844. ·748	351 22	7·76	17	19	6·607	19
	1845. ·734	351 16	10·60	9	9	6·283	9
	1846. ·188	350 37	6·77	22	15	6·400	16
	1847. ·209	351 17	10·66	9	8	6·263	8
	1848. ·248	351 58	7·65	17	11	6·231	10
118 **Tauri.**	1843. ·732	196 38	4·93	41	14	4·739	14
	1844. ·225	195 38	4·10	59	13	5·012	13
	1844. ·737	194 45	6·10	27	15	5·124	14
	1845. ·184	193 58	6·73	22	21	4·951	21
	1846. ·169	195 31	6·13	27	9	5·034	9
41 **Aurigæ.**	1843. ·163	353 54	7·69	17	16	8·290	15
	1843. ·685	353 56	4·07	60	21	7·873	21
	1845. ·188	351 52	5·17	37	24	7·899	24
	1846. ·169	351 51	7·13	20	12	7·885	12
	1847. ·189	352 58	6·51	24	12	7·880	11
	1848. ·248	353 11	4·14	58	12	7·860	12
δ **Geminorum.**	1843. ·267	194 24	6·62	23	11	7·640	11
	1844. ·268	198 18	5·96	28	15	7·375	16
	1845. ·846	198 23	8·14	15	12	7·309	11
Anon. Cancri.	1843. ·240	354 10	4·53	49	18	4·027	19
	1844. ·305	351 49	6·31	25	11	3·575	11
	1845. ·272	353 30	6·19	26	17	3·395	17
	1845. ·846	351 43	9·41	11	12	3·591	12
ρ^2 **Cancri.**	1843. ·253	31 32	5·40	33	16	4·967	15
	1844. ·347	33 8	4·20	57	14	4·935	13
	1845. ·235	32 34	6·29	25	23	4·779	23
	1845. ·876	33 37	9·37	11	12	4·674	11
	1846. ·251	32 0	9·45	11	9	4·717	9
2 **Comæ Berenices.**	1843. ·339	237 25	8·94	13	14	3·937	14
	1844. ·399	240 42	4·46	50	15	4·126	14
	1845. ·359	237 24	11·11	8	16	4·040	15
	1846. ·350	236 33	8·48	14	12	3·805	12
	1848. ·347	238 46	4·85	43	12	3·701	10
39 **Bootis** *.	1843. ·170	43 4	.....	.....	11	3·979	11
	1843. ·644	43 51	.....	.....	20	3·851	20
δ **Serpentis** *.	1843. ·594	195 40	.....	.....	20	3·043	20
	1844. ·616	195 46	.....	.....	17	3·326	17
	1845. ·125	191 46	.....	.....	20	3·139	19
	1845. ·624	194 59	.....	.....	17	2·999	17
	1846. ·576	194 32	.....	.....	11	2·819	11
	1847. ·576	193 25	.....	.....	16	2·916	15
	1848. ·657	195 33	.....	.....	10	2·941	10

TABLE II. (Continued.)

Name.	Fraction of year.	Position.	Probable error.	Computed weight.	Assigned weight.	Distance.	Assigned weight.
		° ′	′			″	
178 (**Bode**) **Libræ.**	1843. ·595	7 42	5·19	37	17	12·144	17
	1847. ·585	7 31	4·31	54	9	11·775	9
μ Draconis ∗.	1843. ·204	17 2	.....	.....	11	3·521	11
	1843. ·645	16 34	.....	.....	18	3·431	18
	1844. ·664	17 10	.....	.....	18	3·222	18
	1845. ·650	12 53	.....	.....	20	3·203	19
	1846. ·199	12 56	.....	.....	12	3·069	12
	1846. ·669	11 42	.....	.....	14	2·994	14
	1849. ·680	9 43	.....	.....	10	2·999	10
100 **Herculis.**	1843. ·269	1 36	6·82	22	16	14·240	15
	1843. ·647	2 27	2·35	181	23	14·096	23
	1844. ·680	2 36	3·43	85	17	13·965	17
	1845. ·202	1 39	6·68	22	18	13·955	17
	1845. ·652	2 6	1·87	286	26	13·922	26
	1846. ·666	2 18	3·81	69	14	13·934	14
	1847. ·222	2 48	4·08	60	9	13·984	10
	1847. ·710	2 29	4·05	61	13	13·967	13
	1848. ·693	2 27	3·39	87	12	13·916	12
	1849. ·727	1 20	3·42	86	11	14·070	11
579 **Struve.**	1843. ·703	159 44	5·26	36	20	5·062	20
	1844. ·773	159 59	6·71	22	20	5·333	20
	1845. ·771	158 28	7·72	17	16	5·060	16
	1847. ·771	159 21	10·15	10	8	4·914	8
287 **h** and **s.**	1844. ·789	291 9	3·64	76	9	8·307	9
	1845. ·774	291 11	8·77	13	14	8·040	14
ι Draconis.	1843. ·303	356 17	19·25	3	12	3·101	12
	1845. ·821	356 18	7·54	18	19	2·951	19
I of H 95.	1843. ·399	337 18	9·29	12	15	3·148	15
	1843. ·960	342 6	6·58	23	15	3·279	14
	1844. ·530	340 40	4·82	43	25	3·187	24
	1844. ·898	340 10	8·17	15	20	3·115	20
	1845. ·836	341 42	8·25	15	17	3·169	16
	1846. ·379	342 19	8·11	15	12	3·055	12
P XXII 306∗	1843. ·496	143 44	.....	.....	18	8·420	18
	1844. ·006	145 8	.....	.....	14	8·655	12
	1846. ·961	146 6	.....	.....	5	8·339	5

The Third Table.—This Table contains the general means of all the observations of each star, both in position and distance, reduced to the mean epoch of all the observations, which will be found in *column three.*

The positions in *column four*, and the probable errors and weights in *columns five and six*, are obtained by combining the means and individual sets (where one set only has been obtained at any one period) in accordance with the theory of probabilities, as above mentioned, except in the case of the four binary stars, in which the arbitrary weights are substituted for computed ones, as before explained.

Where two stars are supposed to be of equal brightness, it has been usual to register always the smaller of the two angles, by which the position may be designated, according as the one or the other component be considered as the larger star.

In these cases there is some confusion as to the identity of the two stars, which might be obviated by astronomers agreeing always to designate the northern, or in cases of equal N.P.D., the east star as A.

The *seventh and ninth* columns contain the sums of the assigned or arbitrary weights.

The *eighth column* contains the mean of all the observed distances, obtained in all cases by using the arbitrary weights.

The *tenth column* contains the mean of all the registered magnitudes of each component star, the first No. referring to the brightest of the two components, or in cases of equal brightness, to the star called A.

The *eleventh and twelfth* columns contain the R.A. and N.P.D. of each star, taken from the B. A. C.; and when the star is not found in that Catalogue, from Struve's of 1837.

The *thirteenth column* contains the total number of individual measures in position; the number of measures in distance being equal in every case, except one, that is specified, one column suffices for both position and distance.

In the notes will be found notices of any remarkable differences between the measures contained in this Table, and those given by Struve in his Catalogue of 1837:—they are confined to differences exceeding 1° in position, and 0″·200 in distance.

Table III.

No.	Name.	Mean epoch.	Position.		Probable error.	Computed weight.	Assigned weight.	Distance.	Assigned weight.	Magnitudes.		R.A.		N.P.D.		n.
												h	m	°	′	
1	γ Arietis[a]	1845. ·483	179°	18′	2″·93	117	68	8″·887	68	5·3	5·3	1	45	71	27	118
2	32 Eridani	1846. ·220	347	37	2·90	119	49	6·803	49	6·5	8·2	3	47	93	24	86
3	ω Aurigæ	1846. ·434	351	5	3·14	102	89	6·401	88	7·3	9·7	4	49	52	21	154
4	118 Tauri[b]	1844. ·962	195	27	2·35	182	79	4·955	78	7·6	9·1	5	20	64	59	150
5	41 Aurigæ	1845. ·607	353	5	2·15	216	95	7·947	95	7·6	8·7	6	0	41	16	162°
6	∂ Geminorum[c]	1844. ·912	197	14	3·58	78	44	7·422	44	3·2	10·5	7	11	67	45	82
7	Anon. Cancri[d]	1845. ·384	353	16	2·86	122	64	3·655	65	8·1	9·3	7	56	62	3	113
8	ρ^2 Cancri[e]	1844. ·992	32	36	2·70	137	74	4·823	71	6·8	6·8	8	18	62	35	137
9	2 Comæ Berenicea[f]	1845. ·746	239	4	2·60	148	82	3·916	78	6·6	8·8	11	57	67	42	166
10	39 Bootis *[g]	1843. ·407	43	35	...	...	31	3·896	31	6·8	7·4	14	45	40	40	46
11	∂ Serpentis *[h]	1846. ·119	194	21	...	...	117	3·042	115	3·5	5·3	15	28	78	57	215
12	178 (Bode) Libræ[i]	1845. ·433	7	34	3·24	95	30	12·038	30	7·8	7·8	15	29	98	10	56
13	μ Draconis *[k]	1845. ·673	14	15	...	...	103	3·217	101	7·0	7·1	17	2	35	20	184
14	100 Herculis	1846. ·247	2	14	1·02	959	158	14·005	158	6·4	6·4	18	2	63	55	260
15	579 Struve	1845. ·456	159	29	3·39	87	70	5·131	70	8·5	8·5	18	31	48	49	124
16	287 h and s[l]	1845. ·274	291	7	3·12	103	27	8·192	26	7·5	8·9	18	58	83	7	56
17	ε Draconis[m]	1844. ·562	356	18	7·02	20	30	3·010	32	4·8	9·8	19	49	20	7	64
18	I of H 95	1845. ·118	340	53	2·78	129	106	3·157	103	6·5	8·8	20	14	35	9	211
19	P XXII 306 *[n]	1846. ·055	144	56	...	...	49	8·475	48	7·0	9·1	22	59	58	7	96

[a] γ Arietis. Distance, Σ−W = −0″·256, period elapsed = 14·64 years.

[b] 118 Tauri. Position, Σ−W = +1° 20′, period = 15·33 years.

[c] ∂ Geminorum. Distance, Σ−W = −0″·277, period = 15·19 years.

[d] Anon. Cancri. This star is erroneously termed 11 Cancri in Herschel and South's Catalogue. See Smyth's Cycle: it is 2688 B.A.C.

[e] ρ^2 Cancri. Distance, Σ−W = −0″·260, period = 15·54 years.

[f] 2 Comæ Berenices. Position, Σ−W = +1° 33′, period = 16·21 years.

[g] 39 Bootis. Comparing the place of this star in position with that given by Herschel and South and by Struve, its mean yearly motion by

Sh−W = −4′·39, period = 20·48 years.
Σ−W = −2′·39, period = 13·39 years.

Mean = −3′·39

[h] ∂ Serpentis. Comparing the place of this star in position with that given by Herschel and South and by Struve, its mean yearly motion by

Sh−W = −12″·18, period = 24·80 years.
Σ−W = −13″·49, period = 13·05 years.

Mean = −12″·84

Distance, Σ−W = −0″·380

[i] 178 (Bode) Libræ. Distance, Σ−W = −0″·225, period = 14·89 years.

[k] μ Draconis. The mean yearly motion of this star in position by

Sh−W = −35′, period = 24·30 years.
Σ−W = −49′, period = 13·45 years.

Mean = −42′

[l] 287 h and s. Position, Σ−W = +1° 13′, period = 15·47 years.

[m] ε Draconis. Position, Σ−W = −1° 46′ } , period = 12·12 years.
Distance, Σ−W = −0″·220 }

[n] P XXII 306. The mean yearly motion of this star in position by

Sh−W = −9″·10, period = 22·31 years.
Σ−W = −4″·78, period = 15·47 years.

Mean = −6″·94

* The measures of distance are one less in number than those of position in the case of this star.

The Fourth Table.—This Table has been formed for the purpose of exhibiting the results from which, in conformity with the principles explained by Sir John Herschel, the existence or non-existence of parallax in the stars observed must be inferred.

These results are the differences between two angles of position, the one observed at one period of the year, say the early one, and the other at another period (the late) distant from the former about six months.

For reasons which will appear hereafter, it has not been deemed necessary to include the distances in this table.

In so delicate an inquiry as that of parallax, it may not perhaps be considered a legitimate course to pursue, to compare any results except those distant from one another by an interval of about six months, or in other words, results which follow one another immediately at opposite seasons. It is on differences so obtained that many therefore may be disposed to rely exclusively in discussing the questions raised by this investigation; and such differences, when furnished by the observations, are accordingly placed first in the table; but it is in truth only a choice of difficulties; it has been found impossible in the case of some of the stars to obtain such differences at all; and in others, the observations are so few, and their weight consequently so small, at one at least of the periods compared, that little reliance can be placed on the results; it has been thought better therefore to combine in the case of each star, and in the manner already explained, all the observations made in all the years at the early period into one mean result, and all those made at the late period into another, and to insert their difference, leaving it to astronomers to attach whatever importance to the comparison they may think due to it. This difference follows in every case the more legitimate comparisons at intervals of six months.

The four binary stars are excluded from this Table.

The *second column* of this fourth Table contains the mean epoch:—in the case of the *final* means, that is, the means of all the *early* and all the *late* periods, the decimal, following an enumeration of several years, is the mean of all the decimals attached to the years of the individual means or sets, from which the final means are formed.

The *third column* contains the positions, obtained by using the computed weights of the results employed, as before explained.

The *fourth* and *fifth columns* contain the probable errors and weights of these positions.

The *sixth column* contains the differences between the positions, which follow one another in succession, and which have been obtained at opposite periods of the year; or, in other words, the quantity, on which parallax depends, and to which the subjoined remarks on the results of the observations refer.

The *plus* sign is attached to the difference when the change in the angle takes place in that direction, which is in conformity with the hypothesis of a parallax in the stars observed; and the *minus* sign when the change is in the contrary direction.

The *eighth column* contains the weight of the difference computed from the formula

$$W'' = \frac{WW'}{W+W'},$$

where W and W′ are the weights of the two positions compared, and W″ the weight of their difference:—from the value of W″ so obtained, that of the probable error of the difference, inserted in the *seventh column*, is computed by the above equation.

The *ninth column* contains the distance, taken from the third Table to the first place of decimals.

All the computations have been very carefully examined.

TABLE IV.

Name.	Epoch.	Position.	Probable error.	Weight.	Difference.	Probable error of Δ.	Weight of Δ.	Distance.
		° ′	′		′	′		″
γ **Arietis.**	1843. ·576	179 10	7·33	19				8·9
	1844. ·069	179 42	7·54	18	− 32	10·52	9	
	1847. ·080	178 14	5·89	29				
	1847. ·608	178 51	6·55	23	+ 37	8·81	13	
	1843, 7. ·592	178 59	4·89	42				
	1844, 5, 7. ·077	179 29	3·65	75	− 30	6·10	27	
32 **Eridani.**	1843. ·661	345 50	4·59	48				6·8
	1845, 6, 7, 9. ·110	348 48	3·74	72	+178	5·92	29	
ω **Aurigæ.**	1845. ·734	351 16	10·60	9				6·4
	1846. ·188	350 37	6·77	22	− 39	12·58	6	
	1843, 4, 5. ·729	350 55	4·62	47				
	1846, 7, 8, 9. ·213	351 13	4·28	55	+ 18	6·30	25	
118 **Tauri.**	1843. ·732	196 38	4·93	41				5·0
	1844. ·225	195 38	4·10	59	+ 60	6·42	24	
	1844. ·737	194 45	6·10	27	− 53	7·35	19	
	1845. ·184	193 58	6·73	22	+ 47	9·08	12	
	1845. ·723	193 23	13·62	5	− 35	15·18	4	
	1846. ·169	195 31	6·13	27	−128	14·93	5	
	1843, 4, 5. ·731	195 42	3·69	73				
	1844, 5, 6. ·193	195 16	3·04	108	+ 26	4·78	44	
41 **Aurigæ.**	1843. ·163	353 54	7·69	17				7·9
	1843. ·685	353 56	4·07	60	− 2	8·22	15	
	1843, 5, 6, 7, 8. ·191	352 45	2·53	156				
	1843. ·685	353 56	4·07	60	− 71	4·79	44	
δ **Geminorum.**	1845. ·846	198 23	8·14	15				7·4
	1846. ·269	198 42	9·19	12	− 19	12·29	7	
	1843, 4, 6. ·268	196 58	3·99	63				
	1845. ·846	198 23	8·14	15	+ 85	9·06	12	
Anon. Cancri.	1845. ·272	353 30	6·19	26				3·7
	1845. ·846	351 43	9·41	11	+107	11·26	8	
	1843, 4, 5, 8. ·268	353 26	3·00	111				
	1845. ·846	351 43	9·41	11	+103	9·88	10	

TABLE IV. (Continued.)

Name.	Epoch.		Position.	Probable error.	Weight.	Difference.	Probable error of Δ.	Weight of Δ.	Distance.
			° ′	′		′	′		″
φ^3 **Cancri.**	1845.	·235	32 34	6·29	25				4·8
	1845.	·876	33 37	9·37	11	− 63	11·28	8	
	1846.	·251	32 0	9·45	11	− 97	13·30	6	
	1843, 4, 5.	·278	32 30	2·82	126				
	1845.	·876	33 37	9·37	11	− 67	9·78	11	
2 **Comæ Berenices.**	1844.	·399	240 42	4·46	50				3·9
	1844.	·887	237 23	10·20	10	−199	11·14	8	
	1845.	·359	237 24	11·11	8	− 1	15·09	4	
	1845.	·890	240 25	9·44	11	+181	14·59	5	
	1846.	·350	236 33	8·48	14	+232	12·70	6	
	1843, 4, 5, 6, 7, 8.	·365	239 4	2·80	127				
	1844, 5.	·889	239 1	6·93	21	− 3	7·48	18	
178 (**Bode**) **Libræ.**	1843, 7.	·590	7 35	3·32	91				12·0
	1845.	·120	7 15	15·12	4	+ 20	15·44	4	
100 **Herculis.**	1843.	·269	1 36	6·82	22				14·0
	1843.	·647	2 27	2·35	181	+ 51	7·21	19	
	1844.	·680	2 36	3·43	85				
	1845.	·202	1 39	6·68	22	− 57	7·51	18	
	1845.	·652	2 6	1·87	286	+ 27	6·94	21	
	1846.	·666	2 18	3·81	69				
	1847.	·222	2 48	4·08	60	− 30	5·58	32	
	1847.	·710	2 29	4·05	61	− 19	5·75	30	
	1843, 5, 7.	·231	2 18	3·10	104				
	1843, 4, 5, 6, 7, 8, 9.	·682	2 14	1·08	855	− 4	3·28	93	
579 **Struve.**	1844.	·773	159 59	6·71	22				5·1
	1845.	·260	159 2	20·31	2	− 57	21·49	2	
	1845.	·771	158 28	7·72	17	+ 34	21·82	2	
	1843, 4, 5, 7.	·755	159 30	3·43	85				
	1845.	·260	159 2	20·31	2	− 28	20·70	2	
287 **h** and **s.**	1844.	·789	291 9	3·64	76				8·2
	1845.	·260	290 54	8·44	14	+ 15	9·20	12	
	1845.	·774	291 11	8·77	13	+ 17	12·18	7	
	1844, 5.	·782	291 9	3·36	89				
	1845.	·260	290 54	8·44	14	+ 15	9·10	12	
ε **Draconis.**	1843.	·303	356 17	19·25	3				3·0
	1845.	·821	356 18	7·54	18	+ 1	20·67	2	
I of **H** 95.	1843.	·399	337 18	9·29	12				3·2
	1843.	·960	342 6	6·58	23	+288	11·38	8	
	1844.	·530	340 40	4·82	43	+ 86	8·15	15	
	1844.	·898	340 10	8·17	15	− 30	9·48	11	
	1845.	·836	341 42	8·25	15				
	1846.	·379	342 19	8·11	15	− 37	11·57	8	
	1846.	·827	340 51	12·59	6	− 88	14·98	5	
	1843, 4, 6.	·436	340 28	3·78	70				
	1843, 4, 5, 6.	·880	341 23	4·11	59	+ 55	5·59	32	

Results of the observations.

Distances.—In discussing the results of the observations, little need be said of the measures of distance: it was impracticable, consistently with the due prosecution of the more important task of measuring the angles, to take these measures of distance at shorter intervals, and at the epochs proper for eliminating the changes, if any, due to parallax: the small differences between the measures taken at opposite seasons must be ascribed to errors of observation.

The following Table exhibits these differences; it contains the means of the measures of distance obtained at the early and late periods, with the sums of the assigned weights used in the reduction, and which means correspond, in respect to epoch, with the *final* means of the angles contained in the fourth Table; the mean epochs themselves therefore have been already given in that Table.

The four binary stars are excluded from the following Table.

Table V.

Name.	Mean distance.	Assigned weight.	Δ.	Name.	Mean distance.	Assigned weight.	Δ.
	″				″		
γ Arietis	8·943	32	·105	2 Comæ Berenices	3·917	69·7	·003
	8·838	36			3·914	8·4	
32 Eridani	6·881	21	·136	178 (Bode) Libræ	12·016	26·6	·216
	6·745	27·9			12·232	3·0	
ω Aurigæ	6·474	49·2	·165	100 Herculis	14·064	42·1	·081
	6·309	39·1			13·983	115·5	
118 Tauri	4·915	34·7	·072	579 Σ	5·126	63·8	·062
	4·987	42·9			5·188	6·0	
41 Aurigæ	7·967	74·5	·094	287 h and s	8·148	22·6	·338
	7·873	20·5			8·486	3·4	
δ Geminorum	7·461	32·7	·152	ε Draconis	3·101	12·4	·150
	7·309	11·4			2·951	19·3	
Anon. Cancri	3·669	53·1	·078	I of H 95	3·145	49·7	·023
	3·591	11·8			3·168	53·3	
φ^2 Cancri	4·851	59·6	·177				
	4·674	11·3					

Angles of position.—In the discussion of the measures of position, it must be remembered that four of the stars observed, viz. 39 Bootis, δ Serpentis, μ Draconis, and P XXII 306, exhibit differences in their mean positions, as compared with those given by other observers, of an amount sufficient to induce the belief that they are binary systems: the observations of these stars may therefore be at once dismissed with the remark, that they are useless for the purposes of this inquiry. Again, in the case of five other stars, viz. γ Arietis, φ^2 Cancri, 178 (Bode) Libræ, 100 Herculis and 579 Struve, the components are of equal magnitude, and parallax is in this case not only *à priori* highly improbable, but there is nothing in the actual observations themselves of these stars, which can lead us to any definite conclusion as to its existence in fact. It is proper, however, to direct attention to the large differences in the case of φ^2 Cancri, accordant in respect of sign.

In order to arrive at any decision on the question, whether the changes in the angles of position of the remaining stars, exhibited in the sixth column of the fourth

Table headed "Difference," should induce us to suspect the existence of a sensible parallax in any of the objects observed, two points are to be considered,—first, the *amount* of those changes, as compared with the probable error of the results themselves; and secondly, their *direction*, considered with reference to the motion of the earth in her orbit.

As to the first point, it would appear, that except the change observed bear a very large proportion to the probable error of the determination, it may be justly treated with little respect, so far as the evidence to be derived from *amount* only be concerned.

This conclusion is supported, first, by the great discordances between the partial differences, in the instances of the three stars, 118 Tauri, 100 Herculis, and I of H 95, the only objects that can be said to have been satisfactorily observed, if the delicate nature of the investigation be considered.

2ndly. These discordances are the more remarkable, when we view them in connection with the large ratios which some of the differences bear to their probable errors. Indeed a bare inspection of the measured angles of position given in Table I. in connexion with the computed weights and errors, will convince us that the differences of the results obtained on different evenings are greater in many instances than those of the separate measures obtained on the same evening. This circumstance, which, it is believed, is not unfrequent in observations requiring the same delicacy of estimation as those under consideration, renders the application of the calculus of probabilities embarrassing and its conclusions uncertain. It is certain, in fact, that in the greater number of instances given in this paper, the computed probable errors of the results on which the parallaxes depend, form no probable criterion of the magnitude or even of the existence of parallax. The only probable explanation that can be given in such cases, where purely instrumental errors can scarcely have place, must arise from an unsuspected bias of the observer in the selection of the line of direction passing through the centres of the stars, which causes an estimation constant on the same evening, but differing on different nights. The weight of each evening's result, calculated according to the amount of discordance of the individual measures, may still be an adequate representative of the degree of dependence to be placed on it, as far as the steadiness of the atmosphere and other favourable circumstances are concerned; but the probable errors of combined and final results, calculated on ordinary principles, will not form a good criterion of the accuracy of such results. Supposing, however, the bias of the observer on different evenings likely to be as often in excess as in defect, or not to be of the nature of a constant error, a better estimation of the probable error of the final means would be obtained by considering each evening's set of measures as a single result without regard to their number, and then comparing the separate results in the usual way*. If the results in the present paper had been of a more positive character, a re-reduction of some of them on this principle might have been desirable, but as the case stands this explanation is sufficient. It is proper to add, that I have no reason to

* Putting p for the probable error of 10 *sets* of 118 Tauri for the early period, treated as *individual* measures, and p' for the mean of the probable errors, as given in Table I., of the same sets, $p : p' :: 24 : 15$. In the case of 10 sets of I of H 95 for the late period, $p : p' :: 11 : 9$.

believe, that a single *individual* measure, either in position or distance, however discordant, has been ever rejected on that or any other ground; but all *sets* containing less than four measures have been invariably rejected.

It would seem, then, that though, in one point of view, comparisons at opposite seasons must be treated as possessing value, yet we are after all driven to rely in a great measure on the final means; for notwithstanding the well-founded objections that may be urged to combining observations made in different years, the discordances above alluded to show, that a greater number of measures are required to eliminate error, than can possibly be obtained in any one, or even two years.

Now if we examine the final means of the two periods in Table IV., we shall find five stars only, viz. 32 Eridani, 41 Aurigæ, δ Geminorum, Anon. Cancri, and I of H 95, of which we can affirm that they have any pretensions whatever to be considered as fulfilling the condition above stated:—with respect to δ Geminorum and Anon. Cancri, the small weights of the results of the late period render any conclusions drawn from them extremely liable to doubt: this is the more to be regretted, in the case of the last-mentioned star, inasmuch as the smallness of the distance of its components renders it a very eligible object for observation.

When however we examine the results obtained with reference to the *direction* of the changes observed, they seem to be entitled to rather more consideration.

In the *nf* and *np* quadrants an apparent motion to the westward of the larger star, assumed to be the nearest to the earth, must increase the angle of position, and in the *sf* and *sp* quadrants the contrary effect must take place:—therefore at that epoch of observation when the sun is to the west, and therefore the earth to the east, of the star observed, the *maximum* angle may be expected to occur in the two first named quadrants, and the *minimum* in the others; in the case of double stars, whose components are equal, it is plain that two different conclusions may be drawn from the periods at which the maxima occur, according as the one or the other be taken as the nearest star.

Omitting therefore the five stars, whose components are of equal magnitude, omitting also the four binary stars, and 2 Comæ Berenices and ε Draconis (which exhibit differences less than their probable errors), this interesting result appears, that of the eight that remain, there is only one, that is, 41 Aurigæ, the changes in whose angles, however small and little entitled to confidence they may be, do not conform in direction to those, which would take place, were a sensible parallax admitted in the brighter of the stars themselves.

This is most probably only an accidental coincidence, and I am very far from wishing to estimate it at more than its real worth; but in the case of 32 Eridani and I of H 95, where large, and to a certain extent trustworthy, differences concur with normal directions of motion, it may not perhaps be too much to assert, that this constitutes them objects of interest to the astronomer, possessed of adequate means to prosecute an inquiry, which, I fear, I must be said rather to have attempted than to have succeeded in.

Wrottesley, October 22, 1850.

XIV. *Researches into the Molecular Constitution of the Organic Bases.*

By Dr. A. W. Hofmann, *F.C.S., Professor of the Royal College of Chemistry of London.*

(*Second Memoir.*) *Communicated by Sir* James Clark, *Bart., F.R.S.*

Received March 12,—Read April 3, 1851.

ABOUT twelve months ago I had the honour of presenting to the Royal Society an account of a series of researches into the molecular constitution of the *volatile* organic bases*: at present I beg to submit to the consideration of the Society the history of a new group of alkaloids, which, although intimately connected with the former by their origin, differ from them altogether by their properties, and especially in *not being volatile*.

The members of this new group of alkaloids are so numerous, their deportment is so singular, and their derivatives ramify in so many directions, that I have not as yet been able to complete the study of these substances in all their bearings; nor is it my intention to go fully into the chemistry of the subject in the present communication, my object being merely to establish the existence of these bodies, and to give a general outline of their connection with the *volatile* bases, and of their most prominent chemical and physical properties, reserving a detailed description of their salts and derivatives to a future memoir.

In the paper above referred to, I advanced the position that the majority of the volatile organic bases might be represented by the general expression

$$\left.\begin{matrix} X \\ Y \\ Z \end{matrix}\right\} N,$$

a formula which coincides with that of ammonia, if X=Y=Z=H. I proved by experiment that the terms X, Y and Z may represent, not only hydrogen, but a variety of hydrocarbons, especially the so-called alcohol-radicals ($C_n H_{(n+1)}$), and endeavoured, on the ground of these experiments, to classify the volatile organic bases according to the amount of basic† hydrogen retained in the several alkaloids. I thus arrived at three groups of bases, for which, in accordance with traditional names, I proposed the terms amidogen-bases, imidogen-bases and nitrile-bases.

The ideas enunciated regarding the constitution of the volatile organic bases, were, it is true, to a certain extent, the result of theoretical speculation; but they were so

* Philosophical Transactions, Part I. 1850, p. 93. † Ibid. p. 111.

perfectly in accordance with all the observations recorded, and had been submitted to the test of experiment in so many directions, that no doubt was left in my mind regarding their correctness. Still, the former inquiry had elicited several facts which I was unable to explain by the view which I had taken. These facts I have since endeavoured to elucidate by additional experiments; the results detailed in the following pages, I am happy to say, are in perfect accordance with my former observations, the extension of which has led me, at the same time, to a still more general conception of the question.

After having succeeded in replacing, step by step, one, two or three equivalents of hydrogen in ammonia by a corresponding number of compound atoms, the question arose whether the agent by which these changes were affected would exert a still farther influence upon the last product of the reaction. In order to decide this question by experiment, I submitted two nitrile-bases, diethylaniline and triethylamine, to a prolonged action of bromide of ethyl. These experiments, of which a full account is given in my preceding memoir (pages 106 and 122), did not lead to perfectly decisive results. It was evident that in these cases a reaction ensued; even on using both the base under examination, and the bromide in the absolutely dry state, a gradual change was observed to take place, a small but unmistakeable quantity of hydrobromates being invariably formed. No doubt, however, could be entertained as to this reaction being altogether different from the preceding stages of the process, for instance, the conversion of ammonia into ethylamine, of ethylamine to diethylamine, &c.; it was proved, moreover, by careful and frequently-repeated experiment, that this process, whatever its nature might be, in no case gave rise to the formation of any other *volatile* bases containing a larger number of replacing radicals than the nitrile-base submitted to investigation.

I have been again studying these reactions during the past year on a much larger scale, whereby the experiments were greatly facilitated; and here I gladly avail myself of an opportunity of expressing my sincere thanks to the Royal Society for the kind and encouraging interest they have taken in this inquiry, the new results of which are mainly attributable to their liberal and munificent support.

Action of Bromide and Iodide of Ethyl upon Triethylamine.

In the paper repeatedly referred to, I have stated that a mixture of an aqueous solution of triethylamine and bromide of ethyl, sealed up in a tube, solidified after several hours' ebullition, and that the crystals consisted chiefly of the fibrous hydrobromate of triethylamine, with which however a certain quantity of white opake granular crystals invariably was mixed. I have endeavoured to increase the quantity of the latter by using both the bromide of ethyl and the triethylamine in the anhydrous state. But by thus changing the conditions of the formation, the progress of the reaction is retarded in the most remarkable manner, days elapsing before the con-

version is completed. While engaged in these experiments, I found that the result intended may be obtained almost instantaneously by substituting the iodide for the bromide of ethyl; and as this observation was immediately followed by a perfect elucidation of the whole question, I have scarcely returned again to the use of the bromide.

On adding perfectly anhydrous iodide of ethyl to triethylamine dried over hydrate of potassa, the mixture becomes slightly turbid, heat being evolved at the same time. At the common temperature the action goes on very gradually, and after the lapse of several days, the mixture is converted into a solid mass. On the other hand, if the mixture be exposed for a minute or two to the temperature of boiling water, a very powerful reaction ensues; the liquid remains, even after the removal of the source of heat, for some time in a state of lively ebullition, and solidifies after cooling, into a hard mass of crystals, which is snow-white or of a yellowish tint, according as either the triethylamine or the iodide of ethyl has been in excess. To avoid losing iodide of ethyl, and especially triethylamine, which is the result of a long series of troublesome and expensive processes, I have usually performed the reaction in strong glass tubes of between three and four feet in length, which were sealed before the blowpipe after the substances had been introduced.

The crystalline mass produced in this reaction readily dissolves even in cold water. If the iodide was in excess this substance separates in heavy oily globules, which may be readily separated and recovered by distillation, and the solution is inodorous, and either neutral or very slightly acid; in the latter case it has always a marked yellowish tint. On the other hand, if the base was in excess, the solution of the crystals was colourless and alkaline; the alkaline reaction disappears however on ebullition, triethylamine being disengaged. The crystals are likewise soluble in alcohol, but almost insoluble in ether.

The purification of the crystals is not attended with any difficulties; it is in fact sufficient to dissolve them in cold water and to allow the solution to evaporate spontaneously, when beautiful well-defined white crystals of considerable size are deposited, which may be easily separated by mechanical means, from small quantities of a reddish iodine-compound, which is sometimes formed by the action of the air. This latter substance is formed in much larger quantity at higher temperatures, wherefore it is advisable to avoid the use of boiling water in the purification.

The crystals are anhydrous; when exposed to the temperature of boiling water they exhibit no change in weight. When submitted to analysis they gave the following results:—

I. 0·4385 grm. of substance gave 0·6015 grm. of carbonic acid and 0·3040 grm. of water.

II. 0·5538 grm. of substance gave 0·5050 grm. of iodide of silver.

III. 0·5205 grm. of substance gave 0·4752 grm. of iodide of silver.

IV. 0·5000 grm. of substance gave 0·4552 grm. of iodide of silver.

These numbers lead to the following percentage composition:—

	I.	II.	III.	IV.
Carbon	37·41	——	——	——
Hydrogen	7·71	——	——	——
Iodine	——	49·29	49·35	49·21

The simplest formula into which these results may be translated is

$$C_{16}H_{20}NI,$$

the theoretical values of which I subjoin for comparison with the experimental numbers.

	Theory.		Mean of Experiments.
16 equivs. of Carbon . . .	96·0	37·34	37·41
20 equivs. of Hydrogen . .	20·0	7·78	7·71
1 equiv. of Nitrogen . . .	14·0	5·45	——
1 equiv. of Iodine	127·1	49·43	49·28
1 equiv. of Iodine-compound	257·1	100·00	

Hence it would appear that the formation of the white crystals simply depends upon a direct combination of triethylamine with iodide of ethyl, for—

$$\underbrace{C_{12}H_{15}N}_{\text{Triethylamine.}}+\underbrace{C_4H_5I}_{\text{Iodide of Ethyl.}}=\underbrace{C_{16}H_{20}NI.}_{\text{New crystalline compound.}}$$

In perfect accordance with this formation is the change which the iodine-compound undergoes when subjected to the action of heat; when rapidly heated the crystals fuse and are decomposed, the products being triethylamine and iodide of ethyl, which form two layers in the receiver, but rapidly unite again to the original compound. In fact this recombination takes place to a considerable extent in the neck of the retort; so that when first I became acquainted with this substance, I was for some time under the impression that the iodine-compound might be volatilized without decomposition.

I confess I had not anticipated the possibility of the existence of a compound having the above composition; it appeared at the first glance in direct opposition to the theoretical view repeatedly referred to. For if it was possible to decompose the new iodide by potassa just as the preceding iodides, obtained respectively by the action of bromide (or iodide) of ethyl upon ammonia, ethylamine and diethylamine, if it was possible to separate from this substance a new volatile base analogous to the preceding ethylated alkaloids, there was no reason why the process of ethylation should cease with the fourth equivalent of ethyl. On the contrary, it then appeared more than probable, that the formations of ethylamine, diethylamine and triethylamine, far from being due to a successive replacement of the various equivalents of hydrogen in ammonia, were rather to be considered as special instances of a far more general tendency of carbon and hydrogen to accumulate in organic substances.

The eagerness with which I submitted this question to the test of experiment may be readily imagined. On adding potassa to a solution of the iodine-compound in water, the liquid solidified at once into a crystalline mass, which I took of course for the base existing in the iodide; analysis however soon showed that the substance thus precipitated is nothing but the iodide itself, which is less soluble in an alkaline solution than in pure water. In fact, Analysis IV. was made with a specimen which had thus been treated with potassa. The iodide was now distilled with a very concentrated solution of potassa for several hours; but during this time nothing but pure water passed over; the solution in the retort solidified on cooling to crystals of the unchanged iodide. When the solution of the iodide in potassa had become very concentrated, it separated into two layers, an aqueous one and an oily one, which floated on the surface. But even now no decomposition had been induced, the oily stratum solidifying on cooling into a crystalline cake of iodide perfectly unaltered. It was only when the liquid was evaporated to dryness that a change took place; but then the change was no longer due to the action of the potassa, but to the influence of heat, and in no way differed from the decomposition which the crystals undergo when heated alone.

The deportment with potassa of the new iodine-compound altogether precludes the idea of considering it as an analogue of the hydriodides of ethylamine, diethylamine, and triethylamine, all of which are readily decomposed under the influence of the alkalies.

However, the iodine, though it cannot be eliminated from the crystals by the alkalies, may be separated with the greatest facility by silver-salts; in fact, the nitrate, the sulphate, the oxide of silver, behave with the iodine-compound exactly as with iodide of potassium or sodium. On adding nitrate of silver to a solution of the iodide, iodide of silver is at once precipitated and a nitrate remains in solution, which on evaporation crystallizes in deliquescent needles. With sulphate of silver, a perfectly similar change takes place, an analogous sulphate remaining in solution. During these interchanges the liquids remain perfectly neutral. If the solution of the iodide be digested with freshly precipitated protoxide of silver, the solution at once assumes a powerfully alkaline reaction, iodide of silver being formed: the same effect is produced by the action of caustic baryta upon the sulphate, sulphate of baryta being precipitated. These reactions, when viewed collectively, exhibit a striking analogy between the new iodide and the metallic iodides, especially those of the alkali-metals. In fact the molecular group combined with iodine, in the compound under examination, behaves like sodium or potassium; it is a true organic metal in all its bearings. For this metal I propose, on the ground of its formation and composition, the name Tetrethylammonium (τέτρα), which implies that it is built up by the intimate union of nitrogen with four equivalents of the hypothetical hydrocarbon called ethyl, and that it may be viewed as ammonium, in which the whole of the hydrogen is replaced by an equivalent proportion of the above hydrocarbon.

Ammonium. Tetrethylammonium.

$$\left.\begin{matrix}H\\H\\H\\H\end{matrix}\right\}N. \qquad \left.\begin{matrix}C_4H_5\\C_4H_5\\C_4H_5\\C_4H_5\end{matrix}\right\}N=C_{16}H_{20}N.$$

The new crystalline compound, according to this mode of viewing it, must be considered as iodide of tetrethylammonium, and its formation takes place in consequence of a transposition of the elements, which is perfectly analogous to that assumed by the followers of the ammonium-theory, in the formation of iodide of ammonium from ammonia and hydriodic acid.

$$\left.\begin{matrix}H\\H\\H\end{matrix}\right\}N+\quad H\ I=\left\{\left.\begin{matrix}H\\H\\H\\H\end{matrix}\right\}N\right\}I.$$

$$\left.\begin{matrix}C_4H_5\\C_4H_5\\C_4H_5\end{matrix}\right\}N+C_4H_5\,I=\left\{\left.\begin{matrix}C_4H_5\\C_4H_5\\C_4H_5\\C_4H_5\end{matrix}\right\}N\right\}I.$$

I will not enter here into the merits of the ammonium-theory; the question whether this mode of representing the constitution of the ammonia-salts be correct or not, may be left out of consideration for the present. The view I have taken of the new compound, and which I readily admit must stand and fall with the ammonium-theory, has the great advantage of closely assimilating these substances to the compounds of mineral chemistry, whereby the nomenclature is essentially facilitated.

I will now proceed to give a sketch of the tetrethylammonium-compounds, as far as this is necessary, in order to convey a clear perception of the character and the peculiarities of this group of bodies. I reserve a detailed account of the derivatives of this group to a special memoir, in which I intend to give moreover a fuller description of diethylamine and triethylamine, which were only briefly noticed in my former paper.

Among the various substances derived from the new iodide, the corresponding oxide, the compound corresponding to oxide of ammonium, attracted my attention in a pre-eminent degree.

Oxide of Tetrethylammonium.

I have already briefly stated how this substance is prepared; on decomposing the iodide with an excess of sulphate of silver, a sulphate is obtained, from which sulphuric acid and excess of silver may be separated by baryta. This was in fact the first process by which I separated the base; it is, however, difficult to obtain the solution in this manner free from either baryta or sulphuric acid; nor have I ever

used this process again, after I had found that digestion with freshly precipitated protoxide of silver produces at once the same effect. The oxide is added in small proportions to the solution of the iodide, which is gently heated; it is immediately converted into yellow iodide of silver, which assumes a white colour on stirring, as long as any undecomposed iodide is in solution. Gradually the yellow colour becomes permanent, and the decomposition is complete as soon as an excess of unchanged oxide becomes perceptible by its colour.

On filtering off the silver-precipitate, a clear and colourless liquid is obtained, which contains the isolated base in solution. It is of a powerfully alkaline reaction, which manifests itself, not only in its deportment with vegetable colours, but also in its possessing the pungent bitterness of quinine; the solution, when strongly concentrated, not only burns the tongue, but it acts upon the epidermis, which it destroys like caustic potassa or soda. On rubbing a solution of the oxide between the fingers, the well-known sensation produced by the fixed alkalies under the same circumstances is felt, and we perceive moreover the same peculiar odour.

Oxide of tetrethylammonium saponifies the fats as readily as potassa. The experiment was made with cocoa-nut oil, which after half an hour's ebullition with the new alkali was converted into a beautiful soft soap, having the appearance of an ordinary potassa-soap. This soap washes very well.

I have traced the analogy of the new body with potassa in many other directions. Chemists recollect the remarkable effect produced by ebullition with potassa in certain nitrogenous substances. It was by this process that Dr. Fownes, the discoverer of this reaction, converted furfuramide into the isomeric alkaloid, *furfurine*. I found that boiling with oxide of tetrethylammonium effects this molecular change as rapidly as potassa.

Like potassa it decomposes oxalic ether into oxalic acid and alcohol, and evolves ammonia from ammonia-salts, even in the cold. It may be substituted for potassa in Trommer's well-known sugar-test. Copper-solutions, mixed with either cane- or grape-sugar, yield on addition of oxide of tetrethylammonium, a light-blue precipitate of the hydrate of the protoxide, which redissolves in an excess of the base, forming a deep blue solution, with a shade of green; on ebullition the cane-sugar solution deposits a greenish precipitate, which is only slowly and always very incompletely converted into suboxide of copper. In the presence of grape-sugar the reduction ensues instantaneously on application of heat.

The reactions of the new base with metallic oxides assimilate this compound as nearly as possible to the fixed caustic alkalies. The following table exhibits the reactions of this body.

Barium-salts	White precipitate of the hydrate of the earth, insoluble in an excess of the base.
Strontium-salts . . .	White precipitate of the hydrate of the earth, insoluble in an excess of the base.

Calcium-salts	White precipitate of the hydrate of the earth, insoluble in an excess of the base.
Magnesium-salts	White precipitate of the hydrate of the earth, insoluble in an excess of the base.
Aluminum-salts	White gelatinous precipitate, soluble in a considerable excess of the base.
Chromium-salts	Greenish precipitate of the hydrated sesquioxide, insoluble in an excess of the base.
Nickel-salts	Apple-green hydrate of the protoxide, insoluble in excess.
Cobalt-salts	Pink hydrate of the protoxide, insoluble in excess.
Manganese-salts	Whitish precipitate of the hydrated protoxide, insoluble in excess.
Iron-salts.	
Protoxide	Green precipitate of the hydrated protoxide, insoluble in excess.
Sesquioxide	Brown precipitate of the hydrated sesquioxide, insoluble in excess.
Zinc-salts	White hydrate of the protoxide, soluble in excess.
Lead-salts	White hydrate of the protoxide, soluble in excess.
Silver-salts	Brown protoxide, insoluble in excess.
Mercury-salts.	
Suboxide	Black suboxide, insoluble in excess.
Protoxide	Red precipitate, probably of a double salt, becoming yellow protoxide by addition of excess.
Copper-salts	Blue hydrated protoxide, insoluble in excess, becoming black by ebullition.
Cadmium-salts	White hydrated protoxide, insoluble in excess.
Bismuth-salts	White hydrated teroxide, insoluble in excess.
Antimony-salts (Teroxide)	White hydrated teroxide, soluble in excess.
Tin-salts.	
Protoxide	White hydrated protoxide, soluble in excess.
Binoxide	White hydrated binoxide, soluble in excess.
Gold-salts	Yellow precipitate of the double compound.
Platinum-salts	Yellow precipitate of the double compound.

It is evident that the salts of the metals, with the exception of those of chromium, whose oxide does not redissolve in an excess of the base, exhibit the same reactions with potassa and oxide of tetrethylammonium.

Although I did not anticipate great results from the experiment, I have submitted the new base to the action of the galvanic current, the pile being disposed as in Berzelius's original experiment on the formation of the ammonium-amalgam.

However, no phenomenon, other than the decomposition of water, was observed. This decomposition certainly appeared to be facilitated, especially if the base was employed in the form of sulphate. Nor had potassium-amalgam the slightest effect upon oxide of tetrethylammonium.

I must still mention a peculiar reaction, somewhat singular at the first glance, which takes place on adding the concentrated oxide to alkaline solutions of iodide of potassium, when the crystalline iodide of tetrethylammonium is at once precipitated. I need scarcely remark that this phenomenon is due to the insolubility of the iodide in alkaline solutions, and not to the new base possessing a greater affinity for iodine than potassium, whose oxide is thus actually liberated.

A moderately concentrated solution of oxide of tetrethylammonium may be boiled without alteration; it is only in a very advanced stage of evaporation that decomposition gradually sets in. The change occurs even in the water-bath, as soon as the base becomes nearly dry. On evaporating, however, under the bell of the air-pump in the presence of sulphuric acid and lime, the base may be obtained in the dry state. I have endeavoured to obtain it in this manner in a condition fit for analysis, but all my attempts have been hitherto in vain; a concentrated solution of the oxide, when placed in a vacuum, deposits, after some days' exposure, long hair-like needles, which are, however, deliquescent in the extreme, and attract carbonic acid with great avidity. In both respects they are scarcely inferior to potassa. By remaining longer *in vacuo*, these crystals disappear again, and the whole liquid dries up to a semisolid mass, which also deliquesces rapidly, and becomes carbonated on exposure to the air. As there was not the slightest chance of getting this substance into a combustion-tube, I endeavoured to ascertain its composition, by decomposing a known quantity of the iodide with protoxide of silver, evaporating *in vacuo* in a glass dish, which on removing from the air-pump, could be covered with a glass plate, and determining the weight of the solid mass remaining. These experiments, however, have not led to any decisive results. Soon after the formation of the crystals, the liquid becomes coated with a thin pellicle, which retards the evaporation to such an extent, that even after a fortnight's exposure the mass still continues to lose in weight; but by that time, however carefully the air was excluded, the base has partly been converted into carbonate. Some similar experiments, made in the amyl-series, which I shall describe in another part of this paper, have been attended with more success. From these experiments it may be inferred by analogy, that the residue remaining on evaporating a solution of oxide of tetrethylammonium is the hydrate

$$\left.\begin{matrix} C_4\,H_5 \\ C_4\,H_5 \\ C_4\,H_5 \\ C_4\,H_5 \end{matrix}\right\} NO, HO$$

corresponding to hydrate of potassa. The hair-like crystals contain evidently in

addition several equivalents of water of crystallization; they may possibly be analogous to the well-known crystallized hydrate of potassa KO, HO+4 Aq, which is formed at low temperatures.

It was above stated that the residuary substance left on evaporating a solution of the oxide of tetrethylammonium is decomposed, even at the temperature of boiling water. During this change a powerful ammoniacal odour is evolved, the substance strongly intumesces, and is by degrees perfectly volatilized. If this process be conducted in an appropriate apparatus, it is found that the base, under these circumstances, yields three products of decomposition, namely, water, an highly alkaline oil, rather soluble in water, and a colourless inflammable gas. The oily base has all the properties of triethylamine; it was identified moreover by the analysis of its characteristic platinum-salt.

0·2257 grm. of platinum-salt gave on ignition 0·0725 grm. of platinum.

	Theoretical percentage.	Experimental percentage.
Platinum	32·12	32·12

The inflammable gas evolved was pure olefiant gas; it was perfectly absorbed by bromine water, proving the absence of hydrocarbons not belonging to the family $C_n H_n$, and the oily liquid formed had all the properties of the Dutch liquid. It readily solidified into a crystalline mass when exposed to a freezing mixture, a deportment whereby the olefiant gas compound may be easily distinguished from the corresponding compounds of propylene, butylene and amylene.

Hence the metamorphosis of oxide of tetrethylammonium under the influence of heat is represented by the following equation:—

$$\left.\begin{matrix} C_4\,H_5 \\ C_4\,H_5 \\ C_4\,H_5 \\ C_4\,H_5 \end{matrix}\right\} NO,\ HO = 2HO + \left\{\begin{matrix} C_4\,H_5 \\ C_4\,H_5 \\ C_4\,H_5 \end{matrix}\right\} N + C_4H_4.$$

This change is not only interesting as an additional corroboration of the formula of the base, but likewise as affording a new source from which triethylamine may be obtained in a state of perfect purity; for as I shall show presently, the oxide may be also prepared from very impure mixtures of the lower ethyl-bases, and even from ammonia directly.

This, too, deserves to be noticed, as a new instance of the failure of the analogy between iodide and oxide of ethyl (ether)—a fact which was pointed out long ago by MM. Laurent and Gerhardt, and most indubitably demonstrated by the recent investigations of Dr. Williamson. If the decomposition by heat of oxide of tetrethylammonium was perfectly analogous to that of the iodide, we should expect to see the former split into triethylamine and oxide of ethyl (ether).

$$\left.\begin{matrix} C_4\,H_5 \\ C_4\,H_5 \\ C_4\,H_5 \\ C_4\,H_5 \end{matrix}\right\} N\,I = \left\{\begin{matrix} C_4\,H_5 \\ C_4\,H_5 \\ C_4\,H_5 \end{matrix}\right\} N + C_4\,H_5\,I.$$

$$\left.\begin{matrix}C_4H_5\\C_4H_5\\C_4H_5\\C_4H_5\end{matrix}\right\}NO=\left\{\begin{matrix}C_4H_5\\C_4H_5\\C_4H_5\end{matrix}\right\}N.+C_4H_5O.$$

But, as I have stated above, this decomposition yields, instead of ether, water and olefiant gas.

I have prepared several of the salts of tetrethylammonium. The sulphate, the nitrate, the phosphate, the carbonate, the hydrochlorate and the hydrobromate, all crystallize. But these salts are extremely soluble, and, with the exception of the carbonate, deliquescent, and but little adapted to analysis. They may be obtained either by directly saturating the oxide with the respective acids, or by double decomposition. The phosphate is conveniently prepared by gently heating a solution of the iodide with an excess of phosphate of silver, when, according to the silver-salt employed, either the common phosphate, or the pyrophosphate, &c. may be obtained. The phosphate thus obtained is strongly alkaline, like the tribasic soda-salt.

Tetrethylammonium forms a great number of beautiful double salts, of which I have examined a few.

Platinum-salt.—On adding a solution of bichloride of platinum to the chloride, an orange-yellow precipitate appears at once, which in every respect resembles the corresponding potassium- or ammonium-compound.

Like the latter, it is slightly soluble in water, and less so in alcohol and in ether; like the latter, it may be crystallized in beautiful octahedrons. The salt was repeatedly prepared. The specimens used for analysis I., II. and III., were precipitated from the pure chloride obtained by saturating the oxide with hydrochloric acid; specimens IV. and V., simply by removing the iodine from the iodide by nitrate of silver, acidifying with hydrochloric acid, and adding bichloride of platinum to the mixture of nitrate and chloride thus obtained. The latter circumstance may have caused the trifling discrepancy, which will be observed in the numbers below.

I. 0·3837 grm. of platinum-salt gave 0·4065 grm. of carbonic acid, and 0·2100 grm. of water.

II. 0·3586 grm. of platinum-salt gave 0·1056 grm. of platinum.

III. 0·2784 grm. of platinum-salt gave 0·0816 grm. of platinum.

IV. 0·5228 grm. of platinum-salt gave 0·1526 grm. of platinum.

V. 0·6782 grm. of platinum-salt gave 0·1977 grm. of platinum.

	Percentage-composition.				
	I.	II.	III.	IV.	V.
Carbon . . .	28·89	——	——	——	——
Hydrogen . .	6·08	——	——	——	——
Platinum . .	——	29·48	29·35	29·19	29·16

The formula

$$C_{16}\,H_{20}\,N\,Cl,\,Pt\,Cl_2 = \left\{\begin{matrix} C_4\,H_5 \\ C_4\,H_5 \\ C_4\,H_5 \\ C_4\,H_5 \end{matrix}\right\} N\,Cl,\,Pt\,Cl_2$$

requires the following values, which I collate with the experimental numbers.

	Theory.		Mean of Experiment.
16 equivs. of Carbon	96	28·63	28·89
20 equivs. of Hydrogen . . .	20	5·95	6·08
1 equiv. of Nitrogen	14	4·20	——
3 equivs. of Chlorine	106·5	31·78	——
1 equiv. of Platinum	98·68	29·44	29·27
1 equiv. of Platinum-salt . .	335·18	100·00	

Gold-Salt.—It is formed by mixing the solution of the two chlorides, when a lemon-yellow precipitate takes place, which is but slightly soluble in cold water or hydrochloric acid. Before analysis it was recrystallized from boiling water. I was satisfied with a determination of the metal in this compound.

I. 0·2555 grm. of gold-salt gave 0·1075 grm. of gold.

II. 0·4344 grm. of gold-salt gave 0·1822 grm. of gold.

The formula

$$\left\{\begin{matrix} C_4\,H_5 \\ C_4\,H_5 \\ C_4\,H_5 \\ C_4\,H_5 \end{matrix}\right\} N\,Cl,\,Au\,Cl_3$$

requires the following values:—

	Theory.		Mean of Experiment.
1 equiv. of Chloride of Tetrethylammonium	165·50	35·32	——
3 equivs. of Chlorine	106·50	22·72	——
1 equiv. of Gold	196·66	41·96	42·00
1 equiv. of Gold-salt	468·66	100·00	

Mercury-Salts.—a. *Chlorine-Compound.*—On adding an excess of protochloride of mercury to a nearly neutral solution of the chloride, a beautiful crystalline white precipitate is formed, which is soluble in water and in hydrochloric acid, especially on ebullition. From the boiling solution it is deposited on cooling in plates of a greasy appearance. The analysis refers to two different preparations, the same specimen being used for I. and II. The mercury was determined as protosulphide.

I. 0·4595 grm. of substance gave 0·3147 grm. of protosulphide of mercury.

II. 0·3960 grm. of substance gave 0·4020 grm. of chloride of silver.

III. 0·5530 grm. of substance gave 0·5580 grm. of chloride of silver.

	Percentage.		
	I.	II.	III.
Mercury	59·01	——	——
Chlorine	——	25·11	25·00

The only expression into which these numbers may be translated is the formula

$$\left.\begin{matrix} C_4\,H_5 \\ C_4\,H_5 \\ C_4\,H_5 \\ C_4\,H_5 \end{matrix}\right\} N\,Cl,\ 5Hg\,Cl,$$

which requires the following values:—

	Theory.		Mean of Experiment.
1 equiv. of Tetrethylammonium . .	130	15·49	——
6 equivs. of Chlorine	213	25·20	25·05
5 equivs. of Mercury	500	59·31	59·01
1 equiv. of Mercury-salt	843	100·00	

This somewhat peculiar composition, though chloride of mercury is known to join other compounds in rather unusual proportions, induced me to prepare an analogous iodine-compound, which on analysis was found to have a corresponding formula.

b. *Iodine-Compound.*—This substance was prepared in two different ways. Protiodide of mercury, when boiled with a solution of iodide of tetrethylammonium, loses at once its red colour, and is converted into a yellow compound, which fuses and collects as a transparent layer at the bottom of the vessel. On cooling it solidifies to a brittle mass, with crystalline fracture (I.). The same substance is obtained by adding a large excess of protochloride of mercury to a solution of iodide of tetrethylammonium. A whitish crystalline precipitate is produced, which consists of a mixture of the double salt just mentioned, with a large amount of the corresponding chlorine-compound*. By boiling with water the latter is removed, the former remaining behind (II.).

I. 0·7305 grm. of salt gave 0·7402 grm. of iodide of silver.

II. 0·8258 grm. of salt gave 0·8270 grm. of iodide of silver.

	Percentage.	
	I.	II.
Iodine	54·34	53·99

The formula

$$\left.\begin{matrix} C_4\,H_5 \\ C_4\,H_5 \\ C_4\,H_5 \\ C_4\,H_5 \end{matrix}\right\} N\,I,\ 5Hg\,I$$

* 6 equivs. of iodide of tetrethylammonium and 30 equivs. of protochloride of mercury, contain the elements of 1 equiv. of the iodide of mercury double salt and of 5 equivs. of the chloride of mercury compound.

requires the following values:—

	Theory.		Mean of Experiment.
1 equiv. of Tetrethylammonium . . .	130·00	9·37	——
6 equivs. of Iodine	758·16	54·61	53·76
5 equivs of Mercury	500·00	36·02	——
1 equiv. of Mercury-salt	1388·16	100·00	

The new oxide forms several other double compounds; but as their analysis would have scarcely given additional elucidation to the subject, I have omitted to study them in detail.

The action of various chemical agents upon the compounds of tetrethylammonium gives rise to a series of very remarkable substances. Chlorine, bromine and iodine, convert the base into substitution-products, in which the basic character of the original atom has disappeared; of these the bromine-compound is distinguished by its splendid appearance, crystallizing as it does from alcohol in long magnificent orange-yellow needles. The iodine-compound, too, is very beautiful; it forms either on addition of iodine-solution to the base, or on evaporating a solution of the iodide exposed to the action of the air. In fact it is difficult to avoid the formation of this substance in recrystallizing iodide of tetrethylammonium. Cyanic acid yields with the base a crystalline compound, a sort of urea, which is remarkable for its composition, inasmuch as it may be viewed as ordinary urea, in which the four equivalents of hydrogen are replaced by ethyl. I hope to communicate shortly a full account of these several substances.

The preceding sketch, incomplete as it is, sufficiently exhibits the leading features of the new class of substances, of which tetrethylammonium and its compounds are the prototypes. It is at once evident that there is a marked difference between these latter and the bases which I have described in my former memoir. Irrespectively of the non-volatility of the new basic oxide, which forms in itself a line of demarcation, there are many other properties not less distinctive which attract our attention. The consecutive replacement of the several hydrogen-equivalents of ammonia induces a gradual change in the properties of the original atom; ethylamine is almost as soluble in water as ammonia itself; in diethylamine this property is less marked; and finally, triethylamine is still less soluble. But another equivalent of ethyl being added, the substance changes altogether in property; it becomes soluble in water in all proportions, in fact so soluble that it can scarcely be obtained in the dry state. A perfectly analogous, although reverse deportment, is exhibited by the platinum-salts of the various bases; here we find that the solubility of the salts augments with the degree of ethylation, the salt of triethylamine being soluble in the extreme, while suddenly the tetrethylammonium-compound becomes not more soluble than the ammonium-compound itself. It is evident that ethylamine, diethylamine and triethylamine are compound ammonias; the tetrethylated base is a compound oxide of ammonium.

It now remained only to submit the new base again to the action of iodide of ethyl. An experiment on a small scale showed that the two compounds do not fail to act promptly upon one another. The mixture, after an hour's ebullition in a sealed tube, solidified into a beautiful crystalline mass. In order to obtain a precise notion of this reaction, I repeated this experiment upon a grand scale. Two ounces of iodide of tetrethylammonium were caustified by protoxide of silver, and then submitted to the action of about half a pound of iodide of ethyl. The mixture was kept boiling in a glass flask, provided with a long tube, in which the volatilized substances were condensed and returned to the seat of the reaction. During the whole process not a trace of permanent gas was disengaged. After a day's ebullition the solution had become neutral, and deposited on cooling magnificent crystals of iodide of tetrethylammonium, which were identified by analysis. In fact, for the determinations III. and V., mentioned at the analysis of the platinum-compound, the salts had been prepared with the base obtained in this manner. On submitting the mother-liquor of the crystals to distillation in such a manner as to collect only the most volatile products, a considerable amount of perfectly pure alcohol was obtained, which was identified by a careful comparison of all its properties. The action of iodide of ethyl upon oxide of tetrethylammonium is therefore represented by the following equation—

$$\left.\begin{matrix}C_4H_5\\C_4H_5\\C_4H_5\\C_4H_5\end{matrix}\right\}NO,HO+C_4H_5I=C_4H_5O,HO+\left\{\begin{matrix}C_4H_5\\C_4H_5\\C_4H_5\\C_4H_5\end{matrix}\right\}NI.$$

I have since found that metallic oxides, such as protoxide of silver, in the presence of water, all exert the same action on iodide of ethyl; in this case a metallic iodide is formed, alcohol being reproduced.

The experiment just described shows that, as might have been expected, the ethylation cannot be carried on *ad infinitum*, and that the oxide of tetrethylammonium is the highest term attainable in the ethyl-series. The latter substance continues to act upon iodide of ethyl, but the change affects solely the iodide of ethyl, while the base is no longer altered.

The preceding researches are, in themselves, sufficient to establish the point of theory which is here in question; still I hoped to gain a broader basis for raising general conclusions by collecting some additional facts. The repetition in the methyl- or amyl-series of an experiment already made in the ethyl-series, resembles, it is true, the working of a design in blue or green, which has been previously printed in red. Nevertheless, irrespectively of the farther elaboration of the theory of homologues, which is still wanted, such experiments lead frequently to new and unexpected observations, which often amply repay the trouble and tediousness of such inquiries. I therefore made the experiments which yielded the following results:—

Action of Iodide of Methyl upon Triethylamine.

A mixture of triethylamine and iodide of methyl solidifies, after some time, to a crystalline mass, even at the common temperature. On ebullition the change takes place instantaneously. The crystalline compound thus produced may be viewed as the iodide of an organic metal, containing, together with nitrogen, one equivalent of methyl, and three of ethyl.

$$\left.\begin{matrix}C_4\,H_5\\C_4\,H_5\\C_4\,H_5\end{matrix}\right\}N+C_2\,H_3\,I=\left\{\begin{matrix}C_2\,H_3\\C_4\,H_5\\C_4\,H_5\\C_4\,H_5\end{matrix}\right\}N\,I.$$

In accordance with the nomenclature I have proposed for the preceding substance, this compound may be called iodide of methylotriethylammonium. And here is perhaps the place to ask for indulgence, for the *sesquipedalia verba*, which I am using in the account of this investigation, and of which the worst, I fear, are still to come. However, the use of these barbarous names has the advantage of conveying an idea of the composition of the substances under examination in the shortest and most impressive manner. Moreover, none of these names are likely to remain in science; all these substances, which are prepared solely for the elucidation of a principle, will be forgotten as soon as the object is gained; they resemble paving-stones, each of which is very important in laying down a street, though no interest is attached to the blocks themselves, as soon as we are passing along the level road.

Iodide of methylotriethylammonium has all the properties which I have enumerated when speaking of the preceding compound. It is extremely soluble, forming a perfectly neutral, very bitter solution. On adding potassa to this solution, a heavy oily liquid is precipitated, which gradually solidifies. This is the unchanged iodide, which, like the corresponding tetrethylammonium-compound, is less soluble in alkaline solutions than in water. The iodide is readily decomposed by protoxide of silver, whereby the oxide is obtained in solution, which is caustic and bitter in the extreme. *In vacuo* it dries up to a crystalline compound, analogous to oxide of tetrethylammonium, the properties of which present the same difficulties to analysis. I have fixed the composition of this compound by the analysis of the beautiful platinum-precipitate, which is formed on adding the bichloride to a solution of the base saturated with hydrochloric acid.

0·3530 grm. of platinum-salt gave 0·1075 grm. of platinum.

To the formula

$$C_{14}\,H_{18}\,N\,Cl,\,Pt\,Cl_2=\left\{\begin{matrix}C_2\,H_3\\C_4\,H_5\\C_4\,H_5\\C_4\,H_5\end{matrix}\right\}N\,Cl,\,Pt\,Cl_2$$

correspond the following values:—

	Theory.		Experiment.
1 equiv. of Methylotriethylammonium	116·00	36·15	——
3 equivs. of Chlorine	106·50	33·15	——
1 equiv. of Platinum	98·68	30·70	30·48
1 equiv. of Platinum-salt	321·18	100·00	

I have not studied this substance very minutely; it forms crystalline salts with sulphuric, nitric, oxalic and hydrochloric acids.

Action of Iodide of Amyl upon Triethylamine.

The experiment was conducted in a perfectly similar manner, and the results obtained are perfectly analogous. The action however is by no means as rapid as in the preceding cases, the amyl-substances being invariably much more inert than the corresponding terms in the ethyl- and methyl-series. In order to complete the process, it is necessary to heat the mixture in sealed tubes for two or three days. The iodide of amylotriethylammonium, for this is the new compound, forms remarkably fine crystals, which exhibit the fatty lustre and touch peculiar to the amyl-compounds. It is extremely soluble in water and alcohol, insoluble in ether. The solutions have the bitter taste of quinine. On adding potassa or carbonate of potassa, the iodide separates as an oil, which rapidly shoots into brilliant needles. The salt when slightly moist fuses at 100° C. to a clear oily liquid, which becomes solid on cooling and perfect desiccation, and gradually assumes a yellow tint.

The iodine-determination gave the following result:—

0·4210 grm. of iodide gave 0·3310 grm. of iodide of silver.

The formula

$$C_{22}\,H_{24}\,N\,I = \left\{\begin{matrix} C_4\ H_5 \\ C_4\ H_5 \\ C_4\ H_5 \\ C_{10}\ H_{11} \end{matrix}\right\} N\,I$$

requires the following values:—

	Theory.		Experiment.
1 equiv. of Amylotriethylammonium	172·0	57·51	——
1 equiv. of Iodine	127·1	42·49	42·51
1 equiv. of Iodine-compound	299·1	100·00	

When boiled with protoxide of silver the base is liberated and dissolves, forming an alkaline liquid of extremely bitter taste. The alkaline properties are however less marked than in the preceding bases. On evaporation the oxide of amylotriethylammonium remains in form of a syrup, which I have never seen assuming the crystalline form, perhaps because I have not allowed it to stand for a sufficiently long time. The sulphate and oxalate of this base, when evaporated in the bell of the air-pump,

leave a syrupy or gummy residue like the base itself. The nitrate, however, and the hydrochlorate form beautiful crystals, the former hard, permanent needles of a cooling taste; the latter, leaves which are extremely deliquescent. The solution of the hydrochlorate is not precipitated by bichloride of platinum unless very concentrated. The salt once formed however is far less soluble. From a boiling solution in water it shoots into most magnificent orange-yellow needles, sometimes half an inch in length.

On ignition, 0·2620 grm. of platinum-salt gave 0·0675 grm. of platinum.

The values corresponding to the formula

$$C_{22}\,H_{26}\,N\,Cl,\,Pt\,Cl_2 = \left\{\begin{matrix} C_4\ H_5 \\ C_4\ H_5 \\ C_4\ H_5 \\ C_{10}\ H_{11} \end{matrix}\right\} N\,Cl,\,Pt\,Cl_2$$

are

	Theory.		Experiment.
1 equiv. of Amylotriethylammonium . . .	172·00	45·60	——
3 equivs. of Chlorine	106·50	28·24	——
1 equiv. of Platinum	98·68	26·16	25·76
1 equiv. of Platinum-salt	377·18	100·00	

The percentage of the platinum is unusually low when compared with the theoretical value, a fact which I suppose is due to the recrystallization of the salt. In several cases I have observed that the platinum-salts of these bases undergo a slight decomposition under these circumstances, which is not indicated by the appearance of the recrystallized salt, but which is invariably pointed out by a diminution of the amount of platinum. I am not as yet acquainted with the mode of this change. In the present instance I have not repeated the experiment, because the mode of formation, the analysis of the iodide, and moreover the decomposition of the base, preclude all doubts respecting the formula of the compound under examination.

The remarkable change exhibited by oxide of tetrethylammonium, when submitted to the action of heat, necessarily induced me to study the deportment of the amylotriethylated base under similar circumstances. This study appeared to promise some farther revelations respecting the molecular constitution of this body, a hope in which I was not deceived. I had no doubt in my mind that the process alluded to would give rise to the formation of a nitrile-base, but there were two alternatives which presented themselves as to the nature of this base. The question arose, Will an equivalent of ethyl or amyl be broken up in this decomposition? This question had to be decided by experiment.

Action of Heat on Oxide of Amylotriethylammonium.

Oxide of amylotriethylammonium, when subjected to distillation, begins to froth as soon as the solution becomes concentrated, after which decomposition rapidly sets in. The products of this decomposition are water, an oily substance, and a transparent inflammable gas. The latter, by the method I have indicated when speaking of the decomposition of oxide of tetrethylammonium, was proved to be perfectly pure olefiant gas, when the composition of the oily liquid was no longer doubtful. It was evident that this substance must be a nitrile-base containing two equivalents of ethyl and one equivalent of amyl.

$$\left.\begin{matrix} C_4\ H_5 \\ C_4\ H_5 \\ C_4\ H_5 \\ C_{10}\ H_{11} \end{matrix}\right\} NO,\ HO = 2HO + \left\{\begin{matrix} C_4\ H_5 \\ C_4\ H_5 \\ C_{10}\ H_{11} \end{matrix}\right\} N + C_4\ H_4.$$

Experiment has entirely confirmed this view. Diethylamylamine—for this is the name which appertains to the new compound—has a peculiar, not unpleasant aromatic odour, and an analogous somewhat bitter taste, is lighter than water, in which it scarcely dissolves, imparting to it a slight but decided alkaline reaction. It combines with acids less readily than do the bases which occupy a lower position in the system. The salts with sulphuric, nitric, oxalic and hydrochloric acids, crystallize beautifully, but are deliquescent. Diethylamylamine, when dried over hydrate of potassa and rectified, exhibits a perfectly constant boiling-point at 154° C.

I have fixed the composition of this base, as usual, by the analysis of the platinum-salt. This compound is obtained by concentrating a mixture of the hydrochlorate with bichloride of platinum, when it shoots into long magnificent orange-yellow prisms.

I. 0·3405 grm. of platinum-salt gave 0·0955 grm. of platinum.

II. 0·3805 grm. of platinum-salt gave 0·1065 grm. of platinum.

	Percentage.	
	I.	II.
Platinum	28·04	28·02

The formula

$$C_{18}\ H_{21}\ N,\ HCl,\ Pt\ Cl_2 = \left\{\begin{matrix} C_4\ H_5 \\ C_4\ H_5 \\ C_{10}\ H_{11} \end{matrix}\right\} N,\ H\ Cl,\ Pt\ Cl_2$$

requires

	Theory.		Experiment.
1 equiv. of Hydrochlorate of Diethylamylamine .	179·50	51·40	——
2 equivs. of Chlorine	71·00	20·34	——
1 equiv of Platinum	98·68	28·26	28·03
1 equiv. of Platinum-salt	349·18	100·00	

The formula of diethylamylamine is moreover supported by the boiling-point. This compound differs from diethylamine by $C_{18}\,H_{21}\,N - C_8\,H_{11}\,N = C_{10}\,H_{10} = 5C_2\,H_2$. I have lately prepared diethylamine on a larger scale, and found its boiling-point to be 57° C.; on starting from this number, the boiling-point of diethylamylamine should be $57 + (5 \times 19) = 152°$. The experimental boiling-point is 154°.

Having seen by the preceding experiments that ethyl, as it were, yields its place to amyl, the idea naturally suggested itself to inquire into the mutual relations of ethyl and methyl under similar circumstances. For this purpose I might have studied the action of heat upon oxide of methylotriethylammonium; however, as I had some grammes of diethylamylamine at my disposal, I preferred to methylate this compound, and examine the new body thus obtained.

Action of Iodide of Methyl upon Diethylamylamine.

On adding iodide of methyl to diethylamylamine, the phenomena which I have already several times described are repeated in turn. The action in this case however is so violent that the iodide must be added very gradually, for on suddenly mixing, the liquid is projected from the vessel with a sort of explosion; on account of the low boiling-point of the methyl-compound, it is advisable to mix in a tubulated retort provided with a condenser.

On cooling the mixture solidifies into a hard white crystalline mass, consisting of a new iodide, for which, in accordance with the nomenclature which I have adopted, I propose the name iodide of methylodiethylamylammonium.

$$\left.\begin{matrix} C_4\ H_5 \\ C_4\ H_5 \\ C_{10}\ H_{11} \end{matrix}\right\} N + C_2\,H_3\,I = \left\{\begin{matrix} C_2\ H_3 \\ C_4\ H_5 \\ C_4\ H_5 \\ C_{10}\ H_{11} \end{matrix}\right\} N\,I.$$

The new iodide, like the preceding analogues, is extremely soluble in water, forming a very bitter solution, from which it is reprecipitated by potassa in oily globules, which solidify again but slowly.

When digested with protoxide of silver, the iodide yields the oxide in form of a powerful alkaline solution, which forms crystalline salts with sulphuric, nitric and hydrochloric acid. The hydrochlorate of methylodiethylamylammonium gives with bichloride of platinum a beautiful salt, by the analysis of which the composition of the substance was established.

0·2015 grm. of platinum-salt gave 0·0550 grm. of platinum.

The values corresponding to the formula

$$C_{20}\,H_{24}\,N\,Cl,\,Pt\,Cl_2 = \left\{\begin{matrix} C_2\ H_3 \\ C_4\ H_5 \\ C_4\ H_5 \\ C_{10}\ H_{11} \end{matrix}\right\} N\,Cl,\,Pt\,Cl_2$$

are the following:—

	Theory.		Experiment.
1 equiv. of Methylodiethylamylammonium . .	158·00	43·50	——
3 equivs. of Chlorine	106·50	29·33	——
1 equiv. of Platinum	98·68	27·17	27·29
1 equiv. of Platinum-salt	363·18	100·00	

Action of Heat upon Oxide of Methylodiethylamylammonium.

When submitted to distillation this compound is decomposed like the analogous substances, the product being water, pure olefiant gas, which was identified as formerly, and a very remarkable nitrile-base, containing the three different alcohol-radicals, an ammonia, in fact, in which the first equiv. of hydrogen is replaced by methyl, the second by ethyl, the third by amyl, and which receives the name methylethylamylamine.

$$\left.\begin{matrix} C_2\ H_3 \\ C_4\ H_5 \\ C_4\ H_5 \\ C_{10}\ H_{11} \end{matrix}\right\} NO,\ HO = 2HO + \left\{\begin{matrix} C_2\ H_3 \\ C_4\ H_5 \\ C_{10}\ H_{11} \end{matrix}\right\} N + C_4H_4.$$

This reaction shows that ethyl yields its place even to methyl, a fact which could not have been anticipated from analogy, methyl occupying a lower position than ethyl in the system. The above deportment is moreover remarkable, when considered in another point of view. The elimination of methyl in the case under examination would have given rise to the formation of methylene* C_2H_2, a substance the existence of which is still very problematical.

Methylethylamylamine is a transparent oil of a fragrant odour and an analogous taste. Both these properties are much more marked than those of diethylamylamine, which it resembles in most respects. It is somewhat more soluble in water than the latter, and of a more decided alkaline character. When dried over potassa and rectified, the new base boils constantly at 135°. Hence its boiling-point is 154 − 135 = 19° lower than that of diethylamylamine, a difference which is in perfect accordance with H. Kopp's rule.

Methylethylamylamine dissolves but slowly in the acids, forming salts which resemble those of the diethylamylated bases. The hydrochlorate gives with bichloride of platinum a beautiful double salt, which is extremely soluble in water. It is usually precipitated on mixing the highly concentrated solutions, or on evaporating the mixture, in oily globules of a deep orange-yellow, which gradually solidify into magnificent needles.

* In their paper on Methylalcohol, Dumas and Peligot (Ann. Ch. Phys. [2] lviii. 5) state that methylene is formed by passing the vapour of chloride of methyl through a red-hot tube. The gas obtained by this process did not however very accurately exhibit the composition $C_n\ H_n$, nor are any experiments for the determination of the value of n recorded.

On analysis 0·4155 grm. of platinum-salt gave 0·1212 grm. of platinum.

The formula

$$C_{16}\,H_{19}\,N,\,H\,Cl,\,Pt\,Cl_2=\left\{\begin{matrix}C_2\;\;H_3\\C_4\;\;H_5\\C_{10}\,H_{11}\end{matrix}\right\}N,\,H\,Cl,\,Pt\,Cl_2$$

requires

	Theory.		Experiment.
1 equiv. of Hydrochlorate of Methylethylamylamine	165·50	49·38	——
2 equivs. of Chlorine	71·00	21·18	——
1 equiv. of Platinum	98·68	29·44	29·36
1 equiv. of Platinum-salt	335·18	100·00	

In the preceding experiments, the radicals of the alcohols $C_n\,H_{(n+2)}\,O_2$ have been exclusively used as replacing materials. However, as might have been expected, nitrile-bases containing a radical which is not homologous but only analogous to the two others, exhibit exactly the same deportment when submitted to the action of iodide of methyl, of ethyl or of amyl. In order to establish this point, I had to return once more to the aniline-series.

Action of Iodide of Ethyl upon Diethylaniline.

In my former paper* I have recorded a few experiments on the action of bromide of ethyl upon diethylaniline. The results, unintelligible to me at that period, are now perfectly clear. I have since repeated these reactions, substituting the iodide for the bromide on account of its more prompt action. Diethylaniline is perfectly soluble in iodide of ethyl; on exposing the mixture, sealed in tubes, to the temperature of boiling water, the liquid column gradually separates into two layers, the lower one increasing in bulk with the time of exposure, and solidifying on cooling into a semi-solid crystalline mass. The reaction is terminated, when, after half a day's ebullition, the volume of the lower layer, which is the new iodide in the fused state, ceases to increase.

The mixture is now distilled with water in order to separate an excess of either diethylaniline or iodide of ethyl, when a strongly acid solution of iodide of phenylotriethylammonium is obtained, which usually contains a small quantity of hydriodate of diethylaniline, as may be inferred from the oily precipitate produced in the solution by potassa, which does not redissolve on addition of more water.

Digestion with protoxide of silver separates the two bases, one of which, diethylaniline, being insoluble in water, remains mixed with the oxide and iodide of silver; the other, the oxide of phenylotriethylammonium, forming a powerfully alkaline solution of a strongly bitter taste.

* Philosophical Transactions, 1850, Part I. p. 107.

After having ascertained by treatment with ether that the solution did not retain a trace of diethylaniline, it was saturated with hydrochloric acid and mixed with bichloride of platinum, when at once a pale-yellow, apparently amorphous precipitate was thrown down, which was scarcely soluble in water and insoluble in alcohol and ether.

0·4055 grm. of platinum-salt gave 0·1045 grm. of platinum.

To the formula

$$C_{24}H_{20}NCl,PtCl_2=\left\{\begin{matrix}C_4\ H_5\\C_4\ H_5\\C_4\ H_5\\C_{12}H_5\end{matrix}\right\}NCl,PtCl_2$$

correspond the following values:—

	Theory.		Experiment.
1 equiv. of Phenylotriethylammonium . .	178·00	46·45	——
3 equivs. of Chlorine	106·50	27·80	——
1 equiv. of Platinum	98·68	25·75	25·77
1 equiv. of Platinum-salt	383·18	100·00	

I have not examined any farther the compounds of this base; I have only ascertained that the sulphate, nitrate, oxalate and hydrochlorate crystallize, although with difficulty.

When oxide of phenylotriethylammonium is submitted to the action of heat, it yields, as might have been expected, water, olefiant gas and diethylophenylamine (diethylaniline).

$$\left\{\begin{matrix}C_4\ H_5\\C_4\ H_5\\C_4\ H_5\\C_{12}H_5\end{matrix}\right\}NO,HO=2HO+\left\{\begin{matrix}C_4\ H_5\\C_4\ H_5\\C_{12}H_5\end{matrix}\right\}N+C_4H_4.$$

The olefiant gas was identified by the conversion into the bromine-compound, the diethylaniline by the analysis of the characteristic platinum-salt.

0·2965 grm. of platinum-salt gave 0·0820 grm. of platinum.

This corresponds to 27·65 per cent. of platinum.

The formula

$$C_{20}H_{15}N,HCl,PtCl_2=\left\{\begin{matrix}C_4\ H_5\\C_4\ H_5\\C_{12}H_5\end{matrix}\right\}N,HCl,PtCl_2$$

requires 27·78 per cent. of platinum.

Action of Iodide of Methyl upon Ethylamylaniline.

Ethylamylaniline, which I formerly obtained* by the action of bromide of amyl upon ethylaniline, or of bromide of ethyl upon amylaniline, may be formed with the

* Philosophical Transactions, 1850, Part I. p. 117.

same, or greater facility, by exposing the latter base to iodide of ethyl. When submitted to the action of iodide of methyl at the temperature of boiling water for three or four days, ethylamylaniline exhibits all the phenomena described in the preceding paragraph. The lower layer, which solidifies, is a mixture of hydriodate of ethylamylaniline and iodide of methylethylamylophenylammonium. *Sit venia verbo*!

$$\left.\begin{matrix} C_4\ H_5 \\ C_{10}\ H_{11} \\ C_{12}\ H_5 \end{matrix}\right\} N + C_2\ H_3\ I = \left\{\begin{matrix} C_2\ H_3 \\ C_4\ H_5 \\ C_{10}\ H_{11} \\ C_{12}\ H_5 \end{matrix}\right\} N\ I.$$

The corresponding oxide is obtained as the preceding one, namely, by treating the clear solution of the two iodine-compounds, previously freed by ebullition from an excess of iodide of methyl, or ethylamylaniline, with protoxide of silver, when unchanged ethylamylaniline and the base with the long name separate. The latter remaining in solution (and forming a very bitter liquid of a strongly alkaline reaction), whilst the former is insoluble, there was no difficulty in obtaining a pure platinum-salt whereby to prove the composition of the compound in question. This salt is a pale-yellow amorphous precipitate, very slightly soluble in water.

On analysis—

0·4335 grm. of the platinum-salt gave 0·1045 grm. of the platinum.

I collate the percentage number corresponding to this result with the theoretical value of the formula—

$$C_{28}\ H_{24}\ N\ Cl,\ Pt\ Cl_2 = \left\{\begin{matrix} C_2\ H_3 \\ C_4\ H_5 \\ C_{10}\ H_{11} \\ C_{12}\ H_5 \end{matrix}\right\} N\ Cl,\ Pt\ Cl_2.$$

	Theory.		Experiment.
1 equiv. of Methylethylamylophenylammonium . .	206·00	50·01	——
3 equivs. of Chlorine	106·50	26·00	——
1 equiv. of Platinum	98·68	23·99	24·11
1 equiv. of Platinum-salt	411·18	100·00	

This compound is certainly remarkable for the diversity of its components, for it contains the radicals of not less than four different alcohols!

As to the action of heat upon the oxide under examination, there could be scarcely any doubt after the result of the preceding investigation. Still I have established by experiment that in this reaction olefiant gas is eliminated, whilst a new nitrile-base, containing methyl, amyl and phenyl, is generated.

$$\left.\begin{matrix} C_2\ H_3 \\ C_4\ H_5 \\ C_{10}\ H_{11} \\ C_{12}\ H_5 \end{matrix}\right\} N\ O,\ H\ O = 2H\ O + \left\{\begin{matrix} C_2\ H_3 \\ C_{10}\ H_{11} \\ C_{12}\ H_5 \end{matrix}\right\} N + C_4\ H_4.$$

Methylamylophenylamine is an oil of an odour similar to that of amylaniline; it is almost insoluble in water: I have not prepared enough of this compound to determine the boiling-point, which at all events is very high. It forms a crystalline platinum-salt, whose analysis was sufficient to establish its composition.

0·3795 grm. of platinum-salt gave 0·0866 grm. of platinum.

The formula

$$C_{24}H_{19}N, HCl, PtCl_2 = \left\{\begin{matrix} C_2H_3 \\ C_{10}H_{11} \\ C_{12}H_5 \end{matrix}\right\} N, HCl, PtCl_2,$$

requires the following values:—

	Theory.		Experiment.
1 equiv. of Hydrochlorate of Methylamylaniline . . .	213·5	55·72	——
2 equivs. of Chlorine	71·0	18·53	——
1 equiv. of Platinum	98·68	25·75	25·81
1 equiv. of Platinum-salt	383·18	100·00	

The paragraphs now following are devoted to a brief account of several bases belonging to the methyl- and amyl-series.

Bases of the Methyl-series.

The experiments described under this head were originally undertaken by my friend Captain Reynolds, who by many occupations unfortunately was prevented from continuing the investigation, so that I am obliged to take alone the responsibility of the following statements.

Action of Ammonia upon Iodide of Methyl.

If iodide of methyl be treated with a concentrated aqueous solution of ammonia, the former is rapidly dissolved, the completion of the reaction being indicated by the liquid assuming a yellowish tint. On opening the boiler-tube the liquid is found to be strongly acid, and to contain not less than five different iodides, namely—

Iodide of Ammonium . . . . . $\left\{\begin{matrix} H \\ H \\ H \\ H \end{matrix}\right\} NI.$

Iodide of Methylammonium . . . . . $\left\{\begin{matrix} H \\ H \\ H \\ C_2H_3 \end{matrix}\right\} NI.$

Iodide of Dimethylammonium . . . . . $\left\{\begin{matrix} H \\ H \\ C_2H_3 \\ C_2H_3 \end{matrix}\right\} NI.$

Iodide of Trimethylammonium . . . . . $\left\{\begin{matrix} H \\ C_2H_3 \\ C_2H_3 \\ C_2H_3 \end{matrix}\right\} NI.$

Iodide of Tetramethylammonium . . . . . $\left\{\begin{matrix} C_2H_3 \\ C_2H_3 \\ C_2H_3 \\ C_2H_3 \end{matrix}\right\} NI.$

Of these five compounds, the first and the last are formed in considerable excess. Of the remainder I have as yet only been able to establish the presence of the iodide of methylammonium by numbers. The two others I have not as yet obtained in sufficiently large quantity and of the requisite purity. The solution of this mixture deposits, on cooling, magnificent dazzling-white hard flat needles, sometimes of an inch in length, which are very difficultly soluble in cold water, and may be obtained in the state of purity by washing with cold and recrystallizing from boiling water. These crystals are the iodide of tetramethylammonium, all the other salts remaining in the mother-liquor, which rapidly turns brown on exposure to the atmosphere. The facility with which ammonia acts upon iodide of methyl, renders it unnecessary to perform the process at a high temperature. The reaction is accomplished after some days, even at the common temperature, if an alcoholic solution be employed, after some hours, a considerable amount of heat being disengaged.

Iodide of Tetramethylammonium.

This substance possesses the family-features of the group. It is soluble in water, but far less so than the corresponding ethyl-compound, forming a perfectly neutral solution of an extremely bitter taste. It is almost insoluble in absolute alcohol, insoluble in ether. Like the ethyl-compounds, this salt is far less soluble in potassa than in pure water.

On analysis the following results were obtained:—

I. 0·4753 grm. of iodide gave 0·5519 grm. of iodide of silver.
II. 0·4256 grm. of iodide gave 0·4973 grm. of iodide of silver.

	Percentage.	
	I.	II.
Iodine	62·75	63·14

The formula

$$C_8H_{12}NI = \left\{\begin{matrix} C_2H_3 \\ C_2H_3 \\ C_2H_3 \\ C_2H_3 \end{matrix}\right\} NI$$

requires the following values:—

	Theory.		Experiment.
1 equiv. of Tetramethylammonium . .	74·0	36·75	——
1 equiv. of Iodine	127·1	63·25	62·94
1 equiv. of Iodine-compound	201·1	100·00	

The action of protoxide of silver upon the iodine-compound is perfectly analogous to that which it exerts upon the ethyl-body. The oxide of tetramethylammonium thus liberated is endowed with scarcely less marked properties than the ethyl-base. When evaporated over sulphuric acid *in vacuo*, it likewise dries up to a crystalline mass, attracting moisture and carbonic acid with the greatest avidity; saturated with acids, it yields crystallizable salts. I have prepared the sulphate, oxalate, nitrate and chloride; the nitrate especially crystallizes well, in long brilliant needles.

The chloride yields with bichloride of platinum a magnificent double salt, which is less insoluble than the corresponding salt in the ethyl-series; it crystallizes in perfectly regular octohedrons of a deep orange colour. When recrystallized from boiling water, this salt exhibits the peculiarity to which I have alluded previously (see p. 374), of yielding a smaller percentage of platinum. Analysis I. was made with the precipitated, analysis II. and III. with the recrystallized salt.

I. 0·2695 grm. of platinum-salt gave 0·0950 grm. of platinum.
II. 0·1925 grm. of platinum-salt gave 0·0670 grm. of platinum.
III. 0·3420 grm. of platinum-salt gave 0·1190 grm. of platinum.

	Percentage.		
	I.	II.	III.
Platinum	35·21	34·86	34·79

Values corresponding to the formula:—

$$C_8\,H_{12}\,N\,Cl,\,Pt\,Cl_2 = \left\{\begin{matrix} C_2\,H_3 \\ C_2\,H_3 \\ C_2\,H_3 \\ C_2\,H_3 \end{matrix}\right\} N\,Cl,\,Pt\,Cl_2$$

	Theory.		Experiment.
1 equiv. of Tetramethylammonium . .	74·00	26·50	——
3 equivs. of Chlorine	106·50	38·15	——
1 equiv. of Platinum	98·68	35·35	35·21
1 equiv. of Platinum-salt	279·18	100·00	

I was exceedingly desirous to submit this oxide to the action of heat; for this process, provided the methyl-base followed the deportment of its ethyl-fellow, promised to give me the long-desired methylene $C_2\,H_2$. Oxide of tetramethylammonium, when heated violently, intumesces, and is completely volatilized; the product of the distil-

lation is an extremely volatile and powerfully alkaline liquid; but, strange to say, during the whole process not a trace of gas is disengaged. I have repeatedly performed the experiment with exactly the same result. Here then we meet with a new mode of decomposition, and unless I am much mistaken, with a decomposition of a very peculiar character. I have made a few experiments with the basic liquid disengaged, but as I have not as yet arrived at perfectly definite results, I refrain from entering into farther details.

The mother-liquor of iodide of tetramethylammonium, which contains chiefly iodide of ammonium, together with small quantities of the intermediate iodides, yields all the corresponding bases by distillation with potassa. However, the ammonia prevails in the distillate to such an extent, that the attempt to effect a separation would have been hopeless. It was only by precipitating the liquid with bichloride of platinum, and separating the more soluble from the less soluble salts, that I succeeded in obtaining evidence of the presence of the intermediate bases. As yet I have been able to prepare only the methylamine-salt in a state of tolerable purity, of which I subjoin the analysis.

0·1415 grm. of platinum-salt gave 0·0590 grm.=41·69 per cent. of platinum.

The formula

$$C_2 H_5 N, H Cl, Pt Cl_2 = \left\{ \begin{matrix} H \\ H \\ C_2 H_3 \end{matrix} \right\} N, H Cl, Pt Cl_2$$

requires 41·61 per cent. of platinum.

Bases of the Amyl-Series.

While I was engaged with the investigation of the lower ethyl-bases, Messrs. William and Alfred Bennett, who at that period were studying in the Laboratory of the College of Chemistry, undertook, at my request, the examination of the corresponding bodies in the amyl-series. Prevented from completing their researches by leaving London, these gentlemen have placed their results at my disposal, which I have verified in every respect, and which I subjoin, together with a series of experiments of my own, especially on the oxide of tetramylammonium.

Amylamine.

Ammonia acts but very slowly upon iodide of amyl, and the product of the reaction contains, as in the methyl-series, not less than five iodides. Among these the iodides of amylammonium and diamylammonium are present in the smallest quantity. The preparation of amylamine succeeds much better according to the process of M. Wurtz, namely, by submitting cyanate of amyl to the action of hydrate of potassa.

Action of Bromide of Amyl upon Amylamine.

Diamylamine.

The formation of diamylamine takes place even in the cold, although slowly. At the temperature of boiling water, a mixture of amylamine and bromide of amyl is rapidly converted into a beautiful white crystalline mass of hydrobromate of diamylamine, which is purified, and decomposed in the usual manner. When purified, diamylamine is a light oily substance, very little soluble in water, to which it imparts, however, an alkaline reaction. Its aromatic odour is peculiar, not unpleasant, and reminds us of amylamine; its taste hot and pungent; it boils at about 170°. From want of pure material, the boiling-point could not be determined with accuracy.

Diamylamine forms very beautiful crystalline salts with the acids, which are all rather insoluble in water. The hydrochlorate is almost insoluble in cold water, but may be recrystallized from boiling water. This solution yields with bichloride of platinum a double salt, which is rather soluble, and separates frequently, in the first instance, in the form of oily drops, which become gradually crystalline.

It was by the analysis of these two salts that the composition of diamylamine was fixed.

0·2545 grm. of hydrochlorate gave 0·1852 grm. of chloride of silver.

The formula

$$C_{20}\,H_{23}\,N,\,H\,Cl = \left\{\begin{matrix} H \\ C_{10}\,H_{11} \\ C_{10}\,H_{11} \end{matrix}\right\} N,\,H\,Cl$$

requires the following values:—

	Theory.		Experiment.
1 equiv. of Diamylamine	157·0	81·14	——
1 equiv. of Hydrochloric Acid	36·5	18·86	18·51
1 equiv. of Hydrochlorate of Diamylamine .	193·5	100·00	

These numbers show that the hydrochlorate employed was not perfectly pure; however, the two following platinum-salts leave no doubt respecting the base under examination.

I. 0·1634 grm. of platinum-salt gave 0·0444 grm. of platinum.

II. 0·1805 grm. of platinum-salt gave 0·0488 grm. of platinum.

	Percentage.	
	I.	II.
Platinum	27·17	27·04

The following theoretical values correspond to the formula

$$C_{20}\,H_{23}\,N,\,H\,Cl,\,Pt\,Cl_2 = \left\{\begin{matrix} H \\ C_{10}\,H_{11} \\ C_{10}\,H_{11} \end{matrix}\right\} N,\,H\,Cl,\,Pt\,Cl_2.$$

	Theory.		Experiment.
1 equiv. of Hydrochlorate of Diamylamine .	193·50	53·29	——
2 equivs. of Chlorine	71·00	19·54	——
1 equiv. of Platinum	98·68	27·17	27·10
1 equiv. of Platinum-salt	363·18	100·00	

Action of Bromide of Amyl upon Diamylamine.

Triamylamine.

This base arises from diamylamine in the same manner as the latter from amylamine. It is moreover obtained, and at once in a state of purity, by exposing oxide of tetramylammonium, which I shall describe presently, to a high temperature; prepared by either way, this substance resembles in most of its physical and chemical characters diamylamine; it boils however at a much higher temperature, namely, at 257°. Its composition was fixed as that of diamylamine, by the analysis of the hydrochlorate and of the platinum-salt, the former being a very characteristic salt, which separates at once in nacreous plates on adding hydrochloric acid to the base, the latter being a viscid salt gradually solidifying to a crystalline mass.

0·1022 grm. of the hydrochlorate gave 0·0551 grm. of chloride of silver.

The formula

$$C_{30}\,H_{33}\,N,\,H\,Cl = \left\{\begin{matrix} C_{10}\,H_{11} \\ C_{10}\,H_{11} \\ C_{10}\,H_{11} \end{matrix}\right\} N,\,H\,Cl$$

requires the following values:—

	Theory.		Experiment.
1 equiv. of Triamylamine	227·00	86·15	——
1 equiv. of Hydrochloric Acid	36·50	13·85	13·73
1 equiv. of Hydrochlorate of Triamylamine .	263·50	100·00	

The following platinum-determinations were made with three different salts: I, and II. were prepared with triamylamine, obtained by the action of bromide of amyl upon diamylamine; salt III. was made with the base resulting from the decomposition of oxide of tetramylammonium.

I. 0·3270 grm. of platinum-salt gave 0·0750 grm. of platinum.
II. 0·2182 grm. of platinum-salt gave 0·0500 grm. of platinum.
III. 0·3795 grm. of platinum-salt gave 0·0860 grm. of platinum.

	Percentage.		
	I.	II.	III.
Platinum	22·90	22·91	22·66

The theoretical values of the formula

$$C_{30}\,H_{33}\,N,\,H\,Cl,\,Pt\,Cl_2 = \left\{\begin{matrix} C_{10}\,H_{11} \\ C_{10}\,H_{11} \\ C_{10}\,H_{11} \end{matrix}\right\} N,\,H\,Cl,\,Pt\,Cl_2$$

are as follows:—

	Theory.		Experiment.
1 equiv. of Hydrochlorate of Triamylamine .	263·50	60·82	——
2 equivs. of Chlorine	71·00	16·40	——
1 equiv. of Platinum	98·68	22·78	22·82
1 equiv. of Platinum-salt	433·18	100·00	

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Action of Iodide of Amyl upon Amylamine and upon Ammonia.

Oxide of Tetramylammonium.

A mixture of triamylamine and iodide of amyl solidifies after two or three days' ebullition to a solid, somewhat fat-like mass of iodide of tetramylammonium. A similar effect is produced by boiling an excess of iodide of amyl with a concentrated solution of ammonia. This reaction, however, takes place very slowly; in fact, the diminution of affinities characterizing the amyl-compounds in general was in none of these cases more marked. After two or three days' ebullition the volume of the ether appeared scarcely diminished; but then a gradual change occurred, the heavy amyl-compound rising and floating upon the solution of the new iodide which was formed. After a fortnight's ebullition the aqueous layer began to solidify, when allowed to cool; but even after keeping the mixture for a period of three weeks in ebullition, on opening the boiler-tube the reaction was found to be far from complete.

The semi-solid mixture thus obtained was subjected to distillation, first alone, in order to recover the iodide of amyl not acted upon; then with potassa, to separate ammonia and the lower amyl-bases. The alkaline liquid contained a small quantity of iodide of tetramylammonium in solution, which crystallized on cooling in fat-like leaves; the larger quantity, however, of this substance collected at the bottom of the vessel in form of a heavy oil, which solidified on cooling to a hard mass of the appearance of stearin.

The new iodide resembles in its general properties the corresponding compounds I have previously described. Like them it is soluble, but only with difficulty, in water, forming a most intensely bitter liquid, and reprecipitated from this solution in the crystalline form by the addition of the alkalies. On drying, the iodide assumes a slightly yellowish tint.

0·3890 grm. of the iodide gave 0·2130 grm. of iodide of silver.

The theoretical values of the formula

$$C_{40}H_{44}NI = \left.\begin{matrix} C_{10}H_{11} \\ C_{10}H_{11} \\ C_{10}H_{11} \\ C_{10}H_{11} \end{matrix}\right\} NI$$

are as follows:—

	Theory.		Experiment.
1 equiv. of Tetramylammonium . . .	298·0	70·11	——
1 equiv. of Iodine	127·1	29·89	29·60
1 equiv. of Iodide	425·1	100·00	

On boiling the iodide with protoxide of silver, an alkaline, very bitter solution of the oxide of tetramylammonium is obtained. This substance is evidently far less soluble in water than the corresponding bases in the methyl- and ethyl-series. It would almost appear as if this compound stood to the methyl- and ethyl-bases as an alkaline earth does to potassa and soda.

On adding potassa to a solution of oxide of tetramylammonium, the base rises in the form of an oily layer on the surface of the liquid. The same separation is observed when the solution is much evaporated; in the latter case the oily layer gradually solidifies. On allowing a moderately concentrated solution of this base to stand for some time in a vessel in which it is protected from carbonic acid, magnificent perfectly definite crystals, sometimes an inch in length and in thickness, are deposited; they are but moderately deliquescent, and attract but slowly carbonic acid from the air. These crystals are the hydrate of tetramylammonium, containing a certain amount of water of crystallization. I have not yet been able to determine the number of equivalents. On heating these crystals, they fuse in their water of crystallization, and yield on evaporation in the water-bath a viscid semi-solid transparent mass, which is the hydrated oxide of tetramylammonium,

$$\left.\begin{matrix} C_{10}H_{11} \\ C_{10}H_{11} \\ C_{10}H_{11} \\ C_{10}H_{11} \end{matrix}\right\} NO, HO.$$

This substance is extremely deliquescent, and opposes to an ordinary analysis the same obstacles which I met with in the case of tetrethylammonium. I succeeded, however, in the following manner, in determining with tolerable accuracy the composition of this mass. Having observed that this oxide is far less easily decomposed by heat than the corresponding ethyl-compound, I endeavoured to determine the state of hydration in which the oxide remained, by decomposing a known quantity of the iodide with protoxide of silver, and evaporating the solution thus obtained in a glass vessel, through which a current of air previously dried and decarbonated by potassa

was passed at the temperature of boiling water. On weighing the residue, the amount of water retained by the oxide was given. The results obtained in this way, although not perfectly accurate on account of the oxide yielding a small quantity of triamylamine even at 100°, before the last traces of water of crystallization are expelled, nevertheless leave but little doubt respecting the nature of the residue. In an experiment of this kind

1·0610 grm. of iodide of tetramylammonium left 0·8000 grm. of residue.

This corresponds to 75·43 per cent.; if the viscid residue had been the pure monohydrated oxide I should have found 74·10 per cent.

I hope to repeat this experiment at a lower temperature, and also to determine the amount of water of crystallization in the crystallized compound; the latter is soon reproduced on adding again a moderate quantity of water to the viscid hydrate.

I have just now mentioned that the oxide, when heated in the water-bath, yields a small quantity of triamylamine. At a higher temperature this reaction proceeds with perfect regularity; an inflammable carbohydrogen being disengaged, together with the base. There can be scarcely any doubt that the former is amylene. This substance boils at a rather low temperature, 39°, and was therefore partly collected as gas. Another portion however dissolved in the triamylamine, from which it was expelled by ebullition after the base had been fixed by addition of hydrochloric acid. The analysis of the triamylamine obtained in this reaction has been mentioned at page 386. Hence the decomposition of oxide of tetramylammonium is perfectly analogous to that of its analogues in the ethyl-series, and is represented by the equation

$$\left.\begin{matrix}C_{10}H_{11}\\C_{10}H_{11}\\C_{10}H_{11}\\C_{10}H_{11}\end{matrix}\right\}NO,HO = 2HO + \left\{\begin{matrix}C_{10}H_{11}\\C_{10}H_{11}\\C_{10}H_{11}\end{matrix}\right\}N + C_{10}H_{10}.$$

Oxide of tetramylammonium is distinguished by forming crystalline salts of remarkable beauty. The sulphate crystallizes in long hair-like filaments, the nitrate in needles, the oxalate in splendid large perfectly defined plates, of a very bitter taste, and extremely deliquescent. The chloride crystallizes in leaves with palm-like ramifications; it is likewise deliquescent, but less so than the preceding salt. The solution of the chloride yields with bichloride of platinum a pale-yellow curdy precipitate, which gradually solidifies into beautiful orange-yellow needles.

On analysis the following results were obtained:—

0·2420 grm. of platinum-salt gave 0·0475 grm. of platinum.

The theoretical numbers of the formula

$$C_{40}H_{44}NCl,PtCl_2 = \left\{\begin{matrix}C_{10}H_{11}\\C_{10}H_{11}\\C_{10}H_{11}\\C_{10}H_{11}\end{matrix}\right\}NCl,PtCl_2$$

are as follows:—

	Theory.		Experiment.
1 equiv. of Tetramylammonium . . .	298·0	59·21	——
3 equivs. of Chlorine	106·5	21·19	——
1 equiv. of Platinum	98·68	19·60	19·63
1 equiv. of Platinum-salt	503·18	100·00	

The analysis of the tetramylammonium-salt concludes the experimental part of this paper; and here I cannot but make mention publicly of the valuable assistance which Mr. James S. Brazier, Senior Assistant in the Royal College of Chemistry, has given me during the prosecution of these inquiries. I thankfully acknowledge the aid of this able young chemist, whose experimental skill is equalled by his love of science.

Before passing on to some general considerations, which the experiments detailed in the preceding paper suggest, it appears to be desirable to condense the whole of the materials into a synoptical form. The appended Table requires no commentary; it contains the formulæ of the alkaloids described in this and the preceding memoir, exhibiting in a clear light the arrangement of the molecules composing them, and the manner in which the various compounds are interchained.

TYPE.	AMIDOGEN-BASES.	IMIDOGEN-BASES.	NITRILE-BASES.	ANALOGUES OF OXIDE OF AMMONIUM.
Ammonia (Amine) $\left\{\begin{matrix}H\\H\\H\end{matrix}\right\}$N.	Aniline (Phenylamine) $\left\{\begin{matrix}H\\H\\C_{12}H_5\end{matrix}\right\}$N.	Ethylaniline (Ethylophenylamine) $\left\{\begin{matrix}H\\C_4H_5\\C_{12}H_5\end{matrix}\right\}$N.	Diethylaniline (Diethylophenylamine) $\left\{\begin{matrix}C_4H_5\\C_4H_5\\C_{12}H_5\end{matrix}\right\}$N.	Oxide of Triethylophenylammonium $\left\{\begin{matrix}C_4H_5\\C_4H_5\\C_4H_5\\C_{12}H_5\end{matrix}\right\}$NO, HO.
		Methylaniline (Methylophenylamine) $\left\{\begin{matrix}H\\C_2H_3\\C_{12}H_5\end{matrix}\right\}$N.	Methylethylaniline (Methylethylophenylamine) $\left\{\begin{matrix}C_2H_3\\C_4H_5\\C_{12}H_5\end{matrix}\right\}$N.	
		Amylaniline (Amylophenylamine) $\left\{\begin{matrix}H\\C_{10}H_{11}\\C_{12}H_5\end{matrix}\right\}$N.	Diamylaniline (Diamylophenylamine) $\left\{\begin{matrix}C_{10}H_{11}\\C_{10}H_{11}\\C_{12}H_5\end{matrix}\right\}$N.	
			Ethylamylaniline (Ethylamylophenylamine) $\left\{\begin{matrix}C_4H_5\\C_{10}H_{11}\\C_{12}H_5\end{matrix}\right\}$N.	
			Methylamylaniline (Methylamylophenylamine) $\left\{\begin{matrix}C_2H_3\\C_{10}H_{11}\\C_{12}H_5\end{matrix}\right\}$N.	Oxide of Methylethylamylophenylammonium $\left\{\begin{matrix}C_2H_3\\C_4H_5\\C_{10}H_{11}\\C_{12}H_5\end{matrix}\right\}$NO, HO.
	Chloraniline (Chlorophenylamine) $\left\{\begin{matrix}H\\H\\C_{12}\begin{pmatrix}H_4\\Cl\end{pmatrix}\end{matrix}\right\}$N.	Ethylochloraniline (Ethylochlorophenylamine) $\left\{\begin{matrix}H\\C_4H_5\\C_{12}\begin{pmatrix}H_4\\Cl\end{pmatrix}\end{matrix}\right\}$N.	Diethylochloraniline (Diethylochlorophenylamine) $\left\{\begin{matrix}C_4H_5\\C_4H_5\\C_{12}\begin{pmatrix}H_4\\Cl\end{pmatrix}\end{matrix}\right\}$N.	
	Bromaniline (Bromophenylamine) $\left\{\begin{matrix}H\\H\\C_{12}\begin{pmatrix}H_4\\Br\end{pmatrix}\end{matrix}\right\}$N.	Ethylobromaniline (Ethylobromophenylamine) $\left\{\begin{matrix}H\\C_4H_5\\C_{12}\begin{pmatrix}H_4\\Br\end{pmatrix}\end{matrix}\right\}$N.		
	Nitraniline (Nitrophenylamine) $\left\{\begin{matrix}H\\H\\C_{12}\begin{pmatrix}H_4\\NO_4\end{pmatrix}\end{matrix}\right\}$N.	Ethylonitraniline (Ethylonitrophenylamine) $\left\{\begin{matrix}H\\C_4H_5\\C_{12}\begin{pmatrix}H_5\\NO_4\end{pmatrix}\end{matrix}\right\}$N.		
	Ethylamine (Ethylammonia) $\left\{\begin{matrix}H\\H\\C_4H_5\end{matrix}\right\}$N.	Diethylamine (Diethylammonia) $\left\{\begin{matrix}H\\C_4H_5\\C_4H_5\end{matrix}\right\}$N.	Triethylamine (Triethylammonia) $\left\{\begin{matrix}C_4H_5\\C_4H_5\\C_4H_5\end{matrix}\right\}$N.	Oxide of Tetrethylammonium $\left\{\begin{matrix}C_4H_5\\C_4H_5\\C_4H_5\\C_4H_5\end{matrix}\right\}$NO, HO.
			Diethylamylamine (Diethylamylammonia) $\left\{\begin{matrix}C_4H_5\\C_4H_5\\C_{10}H_{11}\end{matrix}\right\}$N.	Oxide of Triethylamylammonium $\left\{\begin{matrix}C_4H_5\\C_4H_5\\C_4H_5\\C_{10}H_{11}\end{matrix}\right\}$NO, HO.
				Oxide of Triethylomethylammonium $\left\{\begin{matrix}C_4H_5\\C_4H_5\\C_4H_5\\C_2H_3\end{matrix}\right\}$NO, HO.
			Methylethylamylamine (Methylethylamylammonia) $\left\{\begin{matrix}C_2H_3\\C_4H_5\\C_{10}H_{11}\end{matrix}\right\}$N.	Oxide of Diethylomethylamylammonium $\left\{\begin{matrix}C_2H_3\\C_4H_5\\C_4H_5\\C_{10}H_{11}\end{matrix}\right\}$NO, HO.
	Methylamine (Methylammonia) $\left\{\begin{matrix}H\\H\\C_2H_3\end{matrix}\right\}$N.	Dimethylamine (Dimethylammonia) $\left\{\begin{matrix}H\\C_2H_3\\C_2H_3\end{matrix}\right\}$N.	Trimethylamine (Trimethylammonia) $\left\{\begin{matrix}C_2H_3\\C_2H_3\\C_2H_3\end{matrix}\right\}$N.	Oxide of Tetramethylammonium $\left\{\begin{matrix}C_2H_3\\C_2H_3\\C_2H_3\\C_2H_3\end{matrix}\right\}$NO, HO.
	Amylamine (Amylammonia) $\left\{\begin{matrix}H\\H\\C_{10}H_{11}\end{matrix}\right\}$N.	Diamylamine (Diamylammonia) $\left\{\begin{matrix}H\\C_{10}H_{11}\\C_{10}H_{11}\end{matrix}\right\}$N.	Triamylamine (Triamylammonia) $\left\{\begin{matrix}C_{10}H_{11}\\C_{10}H_{11}\\C_{10}H_{11}\end{matrix}\right\}$N.	Oxide of Tetramylammonium $\left\{\begin{matrix}C_{10}H_{11}\\C_{10}H_{11}\\C_{10}H_{11}\\C_{10}H_{11}\end{matrix}\right\}$NO, HO.

The preceding researches show that the action of the bromides and iodides of the alcohol-radicals upon ammonia, gives rise to the formation of not less than four distinct groups of organic bases. Of these, the members of three groups, corresponding to ammonia ($H_3 N$), are volatile, while those of the fourth, corresponding to oxide of ammonium ($H_4 N O$), cannot be volatilized without decomposition. The facility with which the members of this last class arise from those of the preceding ones, and the readiness with which their reconversion is effected, renders the former group as it were the connecting link between the volatile and the non-volatile organic alkaloids. I am inclined to attach some importance to the latter point; for the acquisition of a general method, by means of which we may rise from the volatile to the non-volatile bases, is not unlikely to pave the way to the artificial production of the fixed native alkaloids, whose constitution is still shrouded in darkness. We may here remember that a very considerable number of these *fixed* native alkaloids yield *volatile* organic bases when submitted to the action of heat or of potassa,—that is under precisely those conditions under which the new ammonium-compounds, which I have described in this paper, are converted into conjugate ammonias. Thus quinine, cinchonine, strychnine, and pelosine yield among their products of decomposition leucoline (chinoline), while piperine gives picoline (?), and morphine, caffeine, codeine and narcotine have actually been converted into alcohol-bases, the former yielding methylamine, the latter two propylamine*.

I readily admit that those processes are by no means so simple as the passage of oxide of tetrethylammonium into triethylamine, several processes of transformation being in most of these cases accomplished side by side. A certain analogy, however, cannot possibly be denied, although, on account of the more complicated formulæ of the native bases, it cannot as yet be traced in simple equations. In some instances the connection even now appears almost palpable. If we compare the formulæ of quinine and of leucoline, the volatile base derived from its destruction, we are surprised to find that these substances exhibit the same elementary difference which we observe between oxide of tetramethylammonium and methylamine.

* M. Wertheim, the discoverer of this base, believes that it stands to the unknown propyl-alcohol in the same relation which exists between methylamine, ethylamine and amylamine on the one hand, and methylic, ethylic and amylic alcohol on the other. It deserves, however, to be noticed that the formula $C_6 H_9 N$ for this compound, as determined by experiment, expresses not only propylamine but also trimethylamine and even methylethylamine,

$$C_6 H_9 N = \left\{ \begin{matrix} H \\ H \\ C_6 H_7 \end{matrix} \right\} N = \left\{ \begin{matrix} C_2 H_3 \\ C_2 H_3 \\ C_2 H_3 \end{matrix} \right\} N = \left\{ \begin{matrix} H \\ C_2 H_3 \\ C_4 H_5 \end{matrix} \right\} N.$$

In the absence of decisive reactions, it remains doubtful which of these formulæ represents the base obtained by the action of soda-lime on narcotine; I may state here that propylamine, as prepared by Wertheim's process, and the liquid containing trimethylamine, for as yet I have not obtained this substance perfectly pure, exhibit the same remarkable fishy odour. It deserves moreover to be remarked that Dr. Anderson found that propyl-, amine occurs, associated with methylamine, among the basic products of the decomposition of codeine.

$$\underbrace{C_{20}\,H_{12}\,N\,O_2}_{\text{Quinine.}} - C_2\,H_4\,O_2 = \underbrace{C_{18}\,H_8\,N}_{\text{Leucoline.}}$$

$$\underbrace{C_5\,H_{13}\,N\,O_2}_{\substack{\text{Hydrated oxide}\\ \text{of Tetramethyl-}\\ \text{ammonium.}}} - C_2\,H_4\,O_2 = \underbrace{C_6\,H_9\,N}_{\substack{\text{Trimethyl-}\\ \text{amine.}}}$$

Accordingly, we might expect that the action of iodide of methyl upon leucoline, and the subsequent decomposition of the iodide produced by means of protoxide of silver, would enable us to reconvert leucoline in this manner into quinine. This metamorphosis would be the more interesting, as it would open a source for the artificial production of quinine; leucoline being, as is well known, contained in considerable quantity in the basic portion of coal-tar naphtha. It may be stated here that leucoline, which is a substance remarkable for its antipathy to the crystalline form, when submitted to the action of iodide of methyl, solidifies at once into a splendid mass of crystals of a new iodide*, which in fact resembles in its outward appearance hydriodate of quinine, and which is still under examination. I am, however, by no means sanguine as to the result of this experiment, for the above play of formulæ rests as yet on a very precarious foundation; neither the formula of quinine nor that of leucoline being established beyond a doubt. Nevertheless, I intend to complete this investigation, because even if no other result is obtained, yet by means of this process new data for ultimately fixing the formulæ of quinine and leucoline may be found.

In conclusion, it may not be out of place to consider how far the preceding researches affect the received views regarding the constitution of the ammonia-salts. Without reproducing all the arguments brought forward by the supporters of the various theories, we may remember that, irrespectively of the impossibility of isolating ammonium itself, the instability of its oxide has been adduced as one of the most important objections against the assumption of the ammonium-theory as originally suggested by Ampère, and subsequently elaborated by Berzelius. It deserves to be noticed that Berzelius expressly states that he considers the solution of ammonia-gas in water as a solution of the hydrated oxide of ammonium.

This idea, which is but a logical conclusion from the generalization of the facts, is discountenanced to a certain extent by the chemical and physical character of this solution. Everybody knows that, even at the common temperature, this liquid splits again into water and ammonia, while it still exhibits the character of the latter in so marked a manner, as almost to preclude the idea that it had undergone as essential a change as the transformation into oxide of ammonium necessarily must be. Under these circumstances, some interest is attached to the discovery of a series of compound bases, corresponding in their composition to hydrated oxide of ammonium,

* A similar result is obtained by the action of iodide of ethyl upon conine and nicotine.

from which they differ only by containing methyl, ethyl and amyl in the place of hydrogen, and exhibiting a deportment which agrees much better with the anticipated character of such compounds as suggested by analogy. Here we find a very marked difference between the properties of the compound ammonia, and those of the ammonium-oxide belonging to it; in the latter, we observe no longer any feature which could possibly betray the presence of the former; all their habits, volatility, odour, taste, &c. are totally changed; there is a difference between the two groups which is not inferior to that between ammonia and potassa. The solutions of the new oxides may be boiled for hours without the slightest quantity of the corresponding ammonia being disengaged; several of these oxides, containing more or less water of constitution or crystallization, may actually be obtained in the dry state. It is evident that the arguments mentioned above, as adduced in refutation of the ammonium-theory, cannot well be raised against the compound ammoniums. But who could deny the parallelism of these substances with the Berzelian type,—with the oxide of ammonium?

Again, many have found it difficult to conceive, that in the combination of ammonia with hydrochloric or hydrobromic acid, the hydrogen of the latter should leave the chlorine and bromine, for which it is known to possess so powerful an affinity, in order to unite with ammonia converting it into ammonium. And they were the less inclined to admit of such a disposition of the elements, as every day's experience showed that the alleged chloride or bromide of ammonium was incapable of exchanging oxygen for chlorine or bromine, without losing the additional equivalent of hydrogen again in the form of water. In other terms, the decomposition of sal-ammoniac, by lime, into chloride of calcium, ammonia-gas and water, induced them to consider this salt as a compound of ammonia and hydrochloric acid; for in the conception of the ammonium-theory we should have to assume in this decomposition *two* consecutive changes, the transformation of the chloride into oxide, and the subsequent splitting of the latter into ammoniacal gas and water. I readily admit that the latter view is less simple, but I am inclined to think that this slight inconvenience is altogether overruled by the general advantages of the ammonium-theory, especially for the purposes of instruction, by the facility with which it accounts for all phenomena of transposition and substitution, and by the simple explanation it gives of the isomorphism of the potassium- and ammonium-compounds, which will always be the firmest foundation of this theory. On the other hand, we have to inquire which of the two views comes nearest to truth, and here a comparative consideration of the deportment exhibited by the compound ammoniums may be of some interest. In many respects their properties are more clearly pronounced; and their behaviour is explicit and unequivocal in those very points in which the typical ammonium leaves room for speculation. In the combination of triethylamine with bromide or iodide of ethyl, it is no longer a matter of doubt whether the ethyl leaves the iodine in order to unite more intimately with the triethylamine, for we see that

the new iodide thus produced is capable of exchanging its iodine for oxygen without the newly-formed oxide suffering immediate decomposition, as is the case with oxide of ammonium. On the contrary, we find this new oxide endowed with remarkable stability; although under the influence of heat it is liable to the same change which befalls the oxide of ammonium, its corresponding ammonia being reproduced. Here then, in the decomposition of iodide of triethylammonium by metallic oxides, we are obliged by irresistible evidence to acknowledge those very two stages, the assumption of which in the analogous change of iodide of ammonium appeared to us deficient in simplicity and probability.

The conception of ammonium does not in any way imply the notion that the different hydrogen-atoms united with nitrogen in the molecule of the compound metal, retain their position in the molecular system with equal persistency. We are forced by unequivocal facts to admit that the fourth atom of hydrogen is in a peculiar state of mobility, and it is on the facility with which this fourth atom is dislodged from its position that one of the foundations of the ammonia-theory rests. In the compound ammoniums the mobility of the fourth atom of hydrogen, or the hydrocarbon replacing it, still prevails, although less so than in the type itself. The decomposition of the ammonium-bases under the influence of heat is particularly instructive in this respect; oxide of tetrethylammonium loses the fourth equivalent of ethyl in the form of olefiant gas and water; and this deportment might be graphically indicated by writing the formula of this compound in accordance with the ammonia-theory, namely, thus—

$$\left.\begin{matrix} C_4\,H_5 \\ C_4\,H_5 \\ C_4\,H_5 \end{matrix}\right\} N,\ C_4\,H_5\,O.$$

The iodide accordingly would be represented by the formula

$$\left.\begin{matrix} C_4\,H_5 \\ C_4\,H_5 \\ C_4\,H_5 \end{matrix}\right\} N,\ C_4\,H_5\,I,$$

an expression which is moreover in perfect harmony with the mode in which this compound is produced, namely, by the direct union of iodide of ethyl with triethylamine.

But now we combine the triethylamine with iodide of amyl, whereby the iodide

$$\left.\begin{matrix} C_4\,H_5 \\ C_4\,H_5 \\ C_4\,H_5 \end{matrix}\right\} N,\ C_{10}\,H_{11}\,I$$

is formed, which, as we have seen in the preceding pages, may be converted without difficulty into the corresponding oxide; this oxide however cannot possibly be considered as

$$\left.\begin{matrix} C_4\,H_5 \\ C_4\,H_5 \\ C_4\,H_5 \end{matrix}\right\} N,\ C_{10}\,H_{11}\,O,$$

for the disengagement of olefiant gas under the influence of heat proves to us that it is an ethyl-atom which in this compound occupies the supplemental position, if I may so call it, as represented in the formula

$$\left.\begin{matrix} C_4\ H_5 \\ C_4\ H_5 \\ C_{10}\ H_{11} \end{matrix}\right\} N, C_4\ H_5\ O,$$

and that the iodide, which is not likely to differ in its constitution from the oxide, has likewise to be represented by the formula

$$\left.\begin{matrix} C_4\ H_5 \\ C_4\ H_5 \\ C_{10}\ H_{11} \end{matrix}\right\} N, C_4\ H_5\ I.$$

The preceding considerations clearly show, that, whatever the actual disposition of the molecules in ammonium or its congeners may be, the atoms re-arrange themselves whenever the fourth equivalent of hydrogen, or of its substitute, joins the compound.

This re-arrangement, so evident in the ammonium-bases, containing various hydrocarbons, may be traced moreover in the lower ethyl-bases in a very obvious manner. For as long as there is any basic hydrogen present in the ammonia-skeleton, this hydrogen assumes what I have previously called the supplemental position, whenever the ammonia passes into the state of ammonium by the accession of a radical. Bromide of ethylammonium formed by the combination of ammonia with bromide of ethyl, when decomposed by a metallic oxide, yields ethylammonia, water and a metallic bromide, the oxide of ethylammonium formed in the first instance being decomposed like oxide of ammonium itself. It is this very transposition which we are in the habit of representing by the equation

$$H_3\ N + C_4\ H_5\ Br = C_4\ H_5,\ H_2\ N,\ H\ Br.$$

In the preceding pages I have stated some of the reasons which induced me to adopt the idea of an ammonium for the new class of compounds which I have had the honour to place before the Royal Society in the present memoir. I need scarcely mention, that such a step involves as a matter of necessity the assumption of a similar view for all the lower bases which form part of this investigation. It would be inconsistent to speak any longer of hydrochlorate of ethylamine, of hydrobromate of diethylamine, &c.; these salts have henceforward to be called chloride of ethylammonium, bromide of diethylammonium, &c., these compounds being nothing but intermediate substitution-terms between the type and the last derivative. On considering the various chlorides from this point of view, we arrive at the following series:—

Chloride of Ammonium	H_4	N Cl.
Chloride of Ethylammonium	$\left\{\begin{matrix} H_3 \\ C_4\ H_5 \end{matrix}\right\}$	N Cl.

Chloride of Diethylammonium . . . . $\left\{\begin{matrix}H_2\\(C_4H_5)_2\end{matrix}\right\}N\,Cl.$

Chloride of Triethylammonium . . . . $\left\{\begin{matrix}H\\(C_4H_5)_3\end{matrix}\right\}N\,Cl.$

Chloride of Tetrethylammonium . . . . $(C_4H_5)_4\,N\,Cl.$

The idea of an ammonium once adopted, we have to follow it out in as general a form as possible, and hence I perfectly agree with those chemists who consider the salts of REISET's second base as compounds of the Platammonium (Platosammonium)

$$\left\{\begin{matrix}H_3\\Pt\end{matrix}\right\}N\,X,$$

the ammoniosulphate of silver as sulphate of argentammonium,

$$\left\{\begin{matrix}H_3\\Ag\end{matrix}\right\}N\,SO_4,$$

and the ammoniochloride of copper as chloride of cuprammonium,

$$\left\{\begin{matrix}H_3\\Cu\end{matrix}\right\}N\,Cl.$$

But I would go farther and advocate an analogous constitution for a great number of mercury-compounds, which are now usually considered in a different way. Since the careful experiments of Sir ROBERT KANE have pointed out the true composition of the white precipitate, chemists are in the habit of viewing this substance as a compound of chloride and amide of mercury,

$$Hg\,H_2\,N,\,Hg\,Cl;$$

it may be viewed with equal justice as chloride of dimercurammonium,

$$\left\{\begin{matrix}H_2\\Hg_2\end{matrix}\right\}N\,Cl,$$

and would correspond in this form to the chloride of diethylammonium; dimercurammonium, intimately combined with protoxide of mercury, may be assumed in all the various salts of M. MILLON's mercury-base. It would be interesting experimentally to follow out in detail the analogy of the mercury-compounds with the series of ethyl bases. Even now the materials at our disposal appear to point to the existence of the other terms. In MITSCHERLICH's garnet-octohedrons we might perceive a chloride of mercurammonium,

$$\left\{\begin{matrix}H_3\\Hg\end{matrix}\right\}N\,Cl,$$

while PLANTAMOUR's nitride of mercury presents itself as trimercuramine, $Hg_3\,N$; and the red compound obtained by MITSCHERLICH on heating the white precipitate, might be viewed as a combination of chloride of mercury with chloride of tetramercurammonium,

$$Hg_4\,N\,Cl,\,Hg\,Cl.$$

The ammonium type may be traced even in a class of compounds which has hitherto been regarded in a perfectly different point of view. I refer to the substances produced by the combination of metallic salts with more than one equivalent of ammonia. Protochloride of platinum unites with two equivalents of ammonia, forming the chloride of Reiset's first base; in the same manner, sulphate of copper, nitrate of silver, absorb both two equivalents of ammonia, the latter salt even three equivalents. These substances, although containing 2 equivs. of nitrogen, may still be viewed as ammonium-compounds, if we apply to them an interesting observation made by Professor Graham, namely, that ammonia, whenever it joins a compound containing hydrogen, may be regarded as ammonium replacing 1 equiv. of hydrogen. When viewed in this light, the substances above quoted would assume the following formulæ:—

Reiset's first chloride	$\left.\begin{matrix} H_2 \\ Pt \\ (H_4N) \end{matrix}\right\}$ N Cl.	Chloride of ammonio-platammonium.
Diammoniacal sulphate of copper	$\left.\begin{matrix} H_2 \\ Cu \\ (H_4N)_2 \end{matrix}\right\}$ N SO_4	Sulphanide of ammonio-cuprammonium.
Diammoniacal nitrate of silver	$\left.\begin{matrix} H_2 \\ Ag \\ (H_4N) \end{matrix}\right\}$ N NO_6	Nitranide of ammonio-argentammonium.
Triammoniacal nitrate of silver	$\left.\begin{matrix} H \\ Ag \\ (H_4N)_2 \end{matrix}\right\}$ N NO_6	Nitranide of diammonio-argentammonium.

I readily admit that here speculation appears rather in advance of experiment. Nevertheless it deserves to be noticed, that among the substances produced by the action of ammonia upon metallic salts, of which so great a variety has been investigated by Henry Rose, there is, as far as I know, not a single one containing more than 4 equivs. of ammonia, which still admits of their being included in the idea to which I have alluded.

XV. *Contributions to Animal Chemistry.*—Paper V.

On the Oxidation of Ammonia in the Human Body, with some Remarks on Nitrification.

By Henry Bence Jones, *M.D., M.A. Cantab., F.R.S., Physician to St. George's Hospital.*

Received December 18, 1850,—Read January 30, 1851.

In a paper lately communicated to the Royal Society, I showed that the effect of tartrate of ammonia on the acidity of the urine was totally different from that of tartrate of potash; and that carbonate of ammonia taken in very large quantities did not produce any alkaline reaction of the urine; on the contrary, the acidity was rather increased than diminished by large doses of carbonate of ammonia. I repeated these experiments with carbonate of ammonia, hoping to obtain more decided results; but I could not determine the fact of any great increase in the acidity of the urine, although it was again apparent that no diminution of the acid reaction resulted from taking carbonate of ammonia.

I also repeated these experiments on many patients in St. George's Hospital suffering from chronic rheumatism, but though some took on an average 50 grs. of carbonate of ammonia daily for seven or more weeks, yet no alkaline state of the urine could be produced.

In consequence of these observations, I suggested in my paper that an inquiry into the occurrence of nitric acid in the urine would probably give the solution of this unexpected effect of carbonate of ammonia.

Hitherto but few observations have been made on the occurrence of nitric acid in the urine in health or disease. Dr. Prout, in the Medico-Chirurgical Transactions, vol. ix. p. 481, mentions that he found nitric acid in the pink sediment from the urine of those labouring under febrile and inflammatory diseases, but he says nothing of the urine containing it in solution; hence, possibly, it came from the baryta used in the process of testing the sediment. Wurtzer also obtained nitric acid; Lehmann thinks, that in this case also it came from the impure baryta.

Thus, then, regarding the presence of nitric or nitrous acids in the urine our knowledge is deficient. Moreover, the difficulty of recognizing very small quantities of nitric acid is considerable, and the accurate determination of the quantity present in organic liquids is almost impossible.

Through the kindness of Mr. Faraday I was allowed the use of the laboratory at the Royal Institution, and I obtained the assistance of Dr. Price to conduct my experiments without interruption.

The delicacy of the different tests for nitric acid was first ascertained, and then a

series of experiments was made to determine what was the smallest amount of nitrate of potash that could be detected in the urine when nitre was dissolved in the water after it had passed out of the body.

The indigo test for nitric acid being more delicate than the protosulphate of iron test, it was chiefly employed. But far more delicate than either test was a mixture of starch with one drop or two of a solution of hydriodate of potash, specific gravity 1052, and very dilute hydrochloric acid, specific gravity 1005. This test Dr. PRICE suggested to me, and the precautions necessary in its use and its advantages he will more fully describe elsewhere. Depending on the production of a deep colour, and not on the removal of colour, it gave evidence when the indigo test afforded none; and if the quantity of iodide of potassium was very small, and if the hydrochloric acid was very dilute, comparative experiments showed that reliance could be placed upon this test.

In the examination of the urine the method was this. From four to eight ounces were taken, and if a larger quantity was passed, it was evaporated to about this quantity in a water-bath, and then mixed with half an ounce of strong and pure sulphuric acid, perfectly free from every trace of nitric acid. Distillation was carried on in a retort until at least two thirds of the fluid had passed into the receiver, when the distillate was neutralized with pure carbonate of potash and evaporated to a very small bulk. From a drop to half the residue was added to the mixture of starch, hydriodate of potash and dilute hydrochloric acid. Another portion was placed in a bason; a very small quantity of indigo was added with an excess of sulphuric acid, and heat was applied for some minutes. The greenish colour of the indigo with the residue of the distillate arose from a small quantity of yellow colouring matter which came over in the distillation and grew darker by evaporation. If too much indigo was used, slight traces of nitric acid could not be detected. Occasionally the protosulphate of iron was also used as a test.

Beginning at first with 10 grs. of nitrate of potash added to 10 ounces of urine, it was found at last that as little as 1 gr. of nitrate to 10 ounces of urine could be detected by this process when the starch test was used, with the greatest certainty and clearness. This quantity could not be detected as surely by the indigo and sulphuric acid test.

Healthy urine was first examined to see if it contained nitric acid.

(1.) One hundred and twenty ounces, the produce of five persons, were evaporated to a small bulk; on distillation with sulphuric acid no nitric acid could be found.

(2.) Thirty ounces, the produce of two other persons, were examined, and no nitric acid was found.

(3.) Twenty ounces of urine, passed by a healthy man, were concentrated and distilled, but no nitric acid was found.

The same person was then made the subject of the following experiments:—

Experiments with Carbonate of Ammonia.

Breakfast was usually taken at 8 o'clock and dinner at 6 o'clock. Both meals consisted of mixed diet; cocoa was taken at breakfast, and water only at dinner.

(4.) At $11^h\ 40^m$ A.M. a single dose of 40 grs. of carbonate of ammonia was taken dissolved in 8 ounces of distilled water.

At 3 P.M. 8 ounces of urine were passed, which gave very decided evidence of nitric acid by the indigo test and by the starch test.

At 6 P.M. 15 ounces of water: no result was obtained in consequence of an accident.

At 11 P.M. 9 ounces. By the indigo test no nitric acid was found. By the starch test a slight trace of nitric acid was detectable.

At $7^h\ 25^m$ A.M. 14 ounces. Gave no evidence of nitric acid by either test.

Up to $7^h\ 25^m$ A.M. the following day 41 ounces of urine were passed. In this large quantity the indigo test gave no nitric acid; but the starch test still showed slight evidence of the presence of the acid.

Thus then after 40 grs. of carbonate of ammonia, nitric acid could be detected in the urine made three hours after the ammonia was taken. After twelve hours a trace was perceptible, and after twenty hours it had not altogether disappeared.

(5.) This experiment was again repeated on the same person.

At $7^h\ 20^m$ A.M. 16 ounces of urine were passed; on examination no trace of nitric acid could be detected by the indigo or starch tests. 40 grs. of carbonate of ammonia were taken, dissolved in distilled water; very active exercise followed for nearly an hour after the volatile alkali was taken.

At $8^h\ 40^m$ A.M. 4 ounces of urine were passed, and breakfast was then taken; nitric acid was found to be present in considerable quantity, both by the indigo and starch tests.

At $12^h\ 30^m$ A.M. 7 ounces of urine gave no proof of nitric acid by the indigo test, but distinct evidence by the starch test.

At $3^h\ 30^m$ P.M. 5 ounces. Gave no proof by indigo; less evident by the starch test.

At 11 P.M. 12 ounces. Gave no evidence of nitric acid by either test.

In this experiment, previous to any food being taken, nitric acid was found in the urine after a dose of carbonate of ammonia; although before the carbonate of ammonia was taken no nitric acid could be detected in four times the quantity of urine.

For eight hours the nitric acid was detectable by the starch test.

(6.) The same experiment was again repeated, but instead of active exercise there was perfect rest, after taking the carbonate of ammonia.

At $6^h\ 45^m$ A.M. 15 ounces, in which no trace of nitric acid could be detected by the indigo or starch test. At this hour 40 grs. of carbonate of ammonia were taken in distilled water.

At $8^h\ 15$ A.M. 5 ounces passed. The indigo test gave no proof of nitric acid; the starch test showed the presence of a small quantity.

At $11^h\ 45^m$ A.M. $5\frac{1}{2}$ ounces. Both the indigo and starch test gave proof of nitric acid.

At $2^h\ 45^m$ P.M. 4 ounces, in which nitric acid was proved to be present by both tests.

At $6^h\ 10^m$ P.M. $6\frac{1}{2}$ ounces. The indigo test gave no proof. The starch test gave it readily.

At $12^h\ 25^m$ P.M. 10 ounces. Still the starch test showed a trace of nitric acid.

At $7^h\ 40^m$ A.M. 14 ounces. Still the slightest trace by starch test.

Comparing this with the previous experiment, it appears that exercise after the carbonate of ammonia causes the nitric acid to appear in the urine more quickly and to pass out more rapidly. With perfect rest, after twenty-four hours a trace of nitric acid was perceptible. With active exercise, after eight hours it ceased to be detected.

Further experiments were then made to determine what was the smallest quantity of carbonate of ammonia that could cause nitric acid to appear in the urine.

(7.) At 1 P.M. 20 grs. of carbonate of ammonia were taken in 6 ounces of distilled water. Moderate exercise was taken until 6 P.M., when 8 ounces of urine were passed. In this nitric acid was detected by the indigo and starch tests.

At 11 P.M. 7 ounces of urine gave nitric acid by both tests. Urine secreted from $10^h\ 30^m$ P.M. the following night to $7^h\ 30^m$ A.M. the next morning, about 24 ounces, gave no proof of nitric acid by the indigo test, but gave a trace with the starch test.

Still smaller quantities of carbonate of ammonia were taken.

(8.) At $12^h\ 30^m$ A.M. about 7 ounces gave no evidence of nitric acid by the indigo or starch test. At this time 5 grs. of carbonate of ammonia were taken in an ounce and a half of water.

At $5^h\ 55^m$ P.M. $7\frac{1}{2}$ ounces gave no proof of nitric acid by the indigo test; slight evidence by starch.

At $12^h\ 10^m$ P.M. 15 ounces; slightest evidence by starch.

At $7^h\ 40^m$ A.M. 15 ounces; the same.

(9.) At $12^h\ 20^m$ A.M. 12 ounces; doubtful evidence of nitric acid. At this hour 10 grs. of carbonate of ammonia were taken in $2\frac{1}{2}$ ounces of distilled water.

At $6^h\ 50^m$ P.M. 9 ounces; gave proof of nitric acid by the indigo and starch tests.

At $12^h\ 20^m$ P.M. 15 ounces; indigo gave no proof; starch decided proof.

At $7^h\ 30^m$ A.M. 17 ounces; no decided evidence.

Hence it appears that 10 grs. of carbonate of ammonia was the smallest quantity that gave decided evidence of nitric acid by both tests. In the course of ten days, in all, 155 grs. of carbonate of ammonia were taken.

(10.) After this quantity of carbonate of ammonia, the water passed at $7^h\ 30^m$ A.M., as I have said, gave no decided evidence.

At $12^h\ 25^m$ A.M. 12 ounces of urine gave decided evidence with starch; none with indigo.

At $11^h\ 50^m$ P.M. 19 ounces; still evidence of nitric acid with the starch test.

(11.) At $12^h\ 20^m$ A.M. 17 ounces. No evidence of nitric acid; and for the five next days the total quantity of urine passed was collected and examined, and each day the slightest evidence of nitric acid was obtained by the starch test. The examination was continued for the four next days; each day the whole quantity passed was evaporated and distilled, but not a trace of nitric acid could be detected.

Thus seven days after the carbonate of ammonia was omitted still traces of nitric acid could be detected in the urine, if large quantities were evaporated and examined.

In order to be more certain regarding the effect of single doses of carbonate of ammonia, the urine of two children, the one aged seven, the other six, was examined. The total quantity passed by both in the day was 40 ounces. It was evaporated and examined, but no nitric acid could be detected by the indigo or starch tests.

(12.) The child aged seven took 5 grs. of carbonate of ammonia at 11 A.M. The urine passed during the evening and the following morning was in quantity 13 ounces. It was acid, and gave no evidence of nitric acid.

The same child took rather less than 10 grs. of carbonate of ammonia.

(13.) At 8 A.M. the urine made during the day up to the following morning, in quantity 16 ounces, contained a trace only of nitric acid by the starch test.

(14.) The other child, aged six, took rather less than 10 grs. of carbonate of ammonia.

At 8 A.M. the urine first passed after this dose was lost by an accident. The water passed from 5 P.M. to 7 A.M., in quantity 10 ounces, gave no trace of nitric acid.

(15.) The experiment was repeated, 10 grs. of carbonate of ammonia were taken at 8 A.M.

At 6 P.M. about 10 ounces of urine gave nitric acid with the indigo test and with the starch test.

The water from 6 P.M. to the next day at 1 P.M. was lost, but in the water passed during the afternoon and night of the next day no nitric acid could be detected.

(16.) A patient in St. George's Hospital was given 7 grs. of carbonate of ammonia every four hours for rheumatism, after he had taken smaller quantities for three days previously. Twenty-four ounces of urine evaporated gave distinct evidence of nitric acid by the indigo and starch tests.

Experiments with Liquor Ammoniæ.

(17.) A patient was given for ten days small quantities of Liquor Ammoniæ, in all about half a drachm of the Pharmacopœia liquid; twelve ounces of urine evaporated gave evidence of nitric acid with the starch test, but none with the indigo.

Having thus satisfied myself that when carbonate of ammonia was taken small quantities of nitric acid passed off in the urine, I tried whether tartrate of ammonia would give the same result.

Experiments with Tartrate of Ammonia.

(18.) Breakfast and dinner were at the same hours as before.

Water passed from $7\frac{1}{3}$ A.M. to $11\frac{1}{3}$, in quantity 8 ounces, gave slight evidence of nitric acid by the starch test, none by the indigo.

At $11\frac{1}{3}$ A.M. tartrate of ammonia, 60 grs., were taken in 4 ounces of distilled water.

At 3 P.M. 7 ounces of urine gave evidence of nitric acid by the indigo, iron, and starch tests.

At 6 P.M. 7 ounces gave nitric acid with the starch test, but not with the indigo.

At $12^h\ 30^m$ A.M. 10 ounces. Starch test gave evidence of nitric acid.

At $8^h\ 15^m$ A.M. 18 ounces. No evidence of nitric acid by either test.

(19.) Another day. Water at $6^h\ 45^m$ A.M. thrown away.

At $11^h\ 45^m$ A.M. about 8 ounces. Still gave a trace of nitric acid by the starch test. At this hour 40 grs. of tartrate of ammonia were taken in distilled water.

At $6^h\ 20^m$ P.M. 11 ounces of urine gave evidence of nitric acid with the starch test.

At $12^h\ 15^m$ P.M. 7 ounces. Nitric acid much more evidently.

At 7 A.M. 14 ounces. Still distinctly evident.

Up to $7^h\ 30^m$ A.M. the following morning, 54 ounces of water passed contained no trace of nitric acid.

(20.) A child, seven years old, took 20 grs. of tartrate of ammonia in some tea at 8 A.M. Urine secreted from this time to 6 A.M. the following day, in quantity 18 ounces, gave no trace of nitric acid.

As small quantities of nitric acid were detected in two of these experiments previous to the time when the tartrate of ammonia was taken, though the quantity of nitric acid was much increased afterwards, yet further experiments were made. Another healthy person took the same amount of tartrate of ammonia.

(21.) The urine made before the ammonia was taken contained no nitric acid.

At 12 A.M. 40 grs. of tartrate of ammonia were taken dissolved in 6 ounces of distilled water.

At 3 P.M. the urine was acid and contained a trace of nitric acid, by the starch test.

Up to 9 A.M., the following morning, the urine was collected, concentrated and distilled, but not the slightest trace of nitric acid was detected.

(22.) The experiment was repeated at 1 P.M.: 60 grs. of tartrate of ammonia were taken dissolved in 9 ounces of water. In three quarters of an hour it acted on the bowels. The water passed prior to the dose, in quantity $8\frac{1}{2}$ ounces, was examined for nitric acid, but not a trace could be detected.

At $4^h\ 15^m$ urine highly acid, $6\frac{1}{2}$ ounces. It gave a large quantity of nitric acid.

At 11 P.M. 6 ounces. Nitric acid found most readily.

At 8 A.M. nitric acid was not detected.

At $2^h\ 45^m$ P.M. 7 ounces. A considerable quantity of nitric acid was detected.

At 12 P.M. gave a trace of nitric acid.

Experiments with Muriate of Ammonia.

As in the first experiments on the same person, the dinner and breakfast were the same as before.

(23.) Water passed from $7^h\ 30^m$ A.M. to $11^h\ 20$ A.M. 7 ounces. When examined for nitric acid no trace was found.

At $11^h\ 20^m$ A.M. 120 grs. of muriate of ammonia were taken in 10 ounces of distilled water.

At $2^h\ 40^m$ P.M. $7\frac{1}{2}$ ounces of water passed contained so much nitric acid that the distillate required no condensation to give immediate evidence by all the tests.

At $5^h\ 50^m$ P.M. 8 ounces. No trace of nitric acid was found.

At $11^h\ 45^m$ P.M. 8 ounces. No nitric acid was detected.

At $7^h\ 35^m$ A.M. 19 ounces. In this larger quantity a trace of nitric acid was found.

At $7^h\ 35^m$ A.M. the day following, 42 ounces. Contained no trace of nitric acid; to $7^h\ 35^m$ A.M. the day following, 38 ounces. The slightest trace of nitric acid was detected.

(24.) The same person took 10 grs. of muriate of ammonia.

The water passed from $7^h\ 30^m$ A.M. to $11^h\ 10^m$ A.M., in quantity 4 ounces, previous to the muriate of ammonia, contained scarcely a trace of nitric acid.

At $11^h\ 10^m$ A.M. 10 grs. of muriate of ammonia were taken in distilled water.

At $2^h\ 30^m$ P.M. 5 ounces of water contained so much nitric acid that it was detectable without evaporating the distillate.

At $6^h\ 50^m$ P.M. 5 ounces. The evaporated distillate gave nitric acid most readily.

In this experiment a trace of nitric acid was found in the urine passed previous to the taking of the muriate of ammonia. The occurrence of minute traces of nitric acid after breakfast was also observed in (9.), (18.), (19.) and (25.); from these observations it is not improbable that the nitric acid came from minute quantities of the salts of ammonia occasionally present in the food.

(25.) By another person, 25 grs. of muriate of ammonia were taken.

The urine made just previous to the medicine, was examined at midday; it contained a small quantity of nitric acid by the starch test, perhaps resulting from the tartrate of ammonia previously taken.

At 12 o'clock 25 grs. of muriate of ammonia were taken in water.

At $1^h\ 30^m$ P.M. a very considerable quantity of nitric acid was detected by all the tests.

At 4 P.M. nitric acid was detected without concentrating the distillate.

At 12 P.M. no nitric acid could be detected.

At 8 A.M. no nitric acid was found.

(26.) A child, six years old, was given 10 grs. of muriate of ammonia in tea, at breakfast. The urine made during the day, up to 6 o'clock the following morning, in quantity 20 ounces, contained only a small quantity of nitric acid by the starch test.

(27.) A child, seven years old, was given 5 grs. of muriate of ammonia in tea, at breakfast. The urine made during the day, and up to 8 o'clock the following morning, in quantity 16 ounces, contained a considerable quantity of nitric acid, by indigo, starch and protosulphate of iron tests.

Finding that salts of ammonia *in transitu* through the body gave rise to nitric acid, I made the following experiment to see whether out of the body nitric acid could be produced by the direct combustion of ammonia.

Ammoniacal gas was passed into alcohol. This ammoniacal alcohol, specific gravity about ·861, was burnt in a spirit-lamp: the products of combustion were passed through a dilute solution of carbonate of potash by means of an aspirator. In all about 3 ounces of alcohol were burnt, the combustion lasting one hour. The solution of carbonate of potash was tested for nitric acid, without being evaporated to a smaller bulk; and the presence of nitric acid was most distinctly proved by the starch test, by the indigo test, and by the protosulphate of iron test. Thus by simple combustion out of the body, as well as in the body, nitric acid was produced.

The close relation of urea to carbonate of ammonia immediately gave rise to the idea that possibly it also would be changed *in transitu*.

Experiments with Urea.

(28.) At $11^h\ 30^m$ A.M. 20 grs. of urea were taken in 3 ounces of distilled water.

At 6 P.M. the slightest trace of nitric acid was found by the starch test in 9 ounces of urine.

At $12^h\ 30^m$ P.M. about 9 ounces of urine gave no nitric acid.

At $7^h\ 30^m$ A.M. 11 ounces gave no trace of nitric acid.

(29.) The experiment was repeated with 40 grs. of urea.

The urine passed from $7^h\ 30^m$ to $11^h\ 45^m$ A.M. was about 5 ounces; it contained no trace of nitric acid.

At $11^h\ 45^m$ A.M. 40 grs. of urea were taken in 4 ounces of distilled water.

At 6 P.M. about 12 ounces of urine gave evidence of plenty of nitric acid by the indigo test and the starch test.

At 12 P.M. about 7 ounces. No proof of nitric acid.

At $7^h\ 30^m$ A.M. about 12 ounces. Gave no proof.

To $7^h\ 30^m$ A.M., the following morning, the urine in quantity 48 ounces, gave no trace of nitric acid.

(30.) These experiments were also made on another person.

At 12 A.M. 20 grs. of urea were taken dissolved in 5 ounces of water.

At $1^h\ 30^m$ P.M. 6 ounces of urine passed, acid to test-paper. Gave with the starch decided proof of nitric acid.

At 4 P.M. 3 ounces gave slighter proof of nitric acid.

To 9 A.M. the water when evaporated and distilled gave no evidence of nitric acid.

(31.) On another day 40 grs. of urea were taken.

At $11^h\ 30^m$ A.M. 40 grs. of urea in 5 ounces of distilled water.

At $1^h\ 30^m$ P.M. 7 ounces of acid urine gave evidence of nitric acid by the starch test.

At $5^h\ 30^m$ P.M. nitric acid was present.

Up to $9^h\ 30^m$ A.M. In the total quantity passed nitric acid, in small quantity, was found by the starch test.

In order to determine whether the same effect would be produced out of the body by combustion of the urea, some urea was dissolved in alcohol, the specific gravity of the solution = ·851; it was burnt in a spirit-lamp; the products of combustion were collected for one hour. The carbonate of potash solution was examined for nitric acid, and without any concentration it was found by all the tests.

Thus, then, whether in the body or out of the body, urea, by oxidation, gives rise to nitric acid.

In St. George's Hospital the urine of two patients with dropsy and albuminous urine, in whose blood urea might be considered to be probably present, was examined for nitric acid. In each case a pint of the urine was boiled, filtered and distilled, but no trace of nitric acid was found. In one case the experiment was repeated with two pints of urine, but still only a negative result was obtained.

These experiments with urea showed that when small quantities were taken no nitric acid could be detected; this led me to suppose that a large dose of the alkaloids would be necessary to produce any decided effect on the urine, and on this account no alkaloid was taken.

Caffein, however, was dissolved in alcohol, and when burnt for one hour in a spirit-lamp, it gave rise to nitric acid.

As it appeared from these experiments that nitric acid was produced more readily and frequently than has been supposed to be the case, I was led to try whether combustions in the atmosphere, without ammonia, could not give nitric acid.

I first tried the products of the combustion of alcohol. The spirit-lamp was used as before, and the products of combustion were passed through a solution of pure carbonate of potash. The alcohol had not the slightest alkaline reaction, but after an hour's combustion nitric acid was proved to exist in the carbonate of potash, by the starch test, and by the indigo test also.

As it was possible that a small quantity of ammonia might have been present in the alcohol, I determined to collect the product of hydrogen burnt in the air. The gas was generated by pure sulphuric acid and distilled water, and it was burnt for one hour; nitric acid was found to be present in the carbonate of potash in small quantity*.

The combustion of coal in a small furnace was then made, so that the products

* After this paper was written, I found that Saussure had described a similar experiment in the Annales de Chemie for 1809, vol. lxxi. p. 285. Nitric acid and ammonia were detected by him after the combustion of hydrogen gas in atmospheric air.

were passed through dilute carbonate of potash; after an hour no trace of nitric acid could be detected.

The experiment was repeated with a brisker fire, and a trace of nitric acid was found.

By the combustion of a wax candle, a slight trace of nitric acid was detected after it had burnt one hour.

The product of the combustion of ordinary coal-gas gave plenty of nitric acid. To purify it from ammonia, it was passed through dilute sulphuric acid; nitric acid in much smaller quantity than before was then detected.

This experiment was repeated, passing the coal-gas through a tube 3 feet long, full of asbestos and pumice-stone moistened with strong sulphuric acid, and then through a thin layer of strong sulphuric acid. After combustion for an hour, nitric acid was detected by all the tests. When the laboratory air was passed for an hour through the same solution of carbonate of potash, no trace of nitric acid could be detected. When the atmospheric air, as well as the coal-gas, were first passed through tubes containing sulphuric acid, and afterwards burnt for one hour, nitric acid was detected among the products of combustion, by the starch and by the indigo test.

If instead of depriving the gas of ammonia it was made to pass over the vapour of ammonia and then burnt, nitric acid could be detected by the starch test with the greatest certainty after two minutes' combustion; but with the same flame, after six hours' combustion, I was unable to obtain any very considerable quantity of nitrate of potash.

These experiments on the production of nitric acid in all cases of combustion in the air, render it very probable that small quantities of nitric acid exist always in the atmosphere, and that nitric acid will be constantly detected in rain and snow, not only after a thunder-storm, but at all other times also. At least, by adding pure carbonate of potash to rain-water, and evaporating it to a very small bulk, I have succeeded in detecting nitric acid in the water which fell in London, on a continuously rainy day in December. Fourteen pints, when evaporated to two ounces, furnished positive proof of the presence of nitric acid by the indigo, starch and iron tests.

On another day ten pints, evaporated to a very small bulk, gave evidence of nitric acid.

Should nitric acid be found to be present in the air at all times and in all places, its importance to the growth of plants will not be less than that of the ammonia which has been detected there.

Thus, whether in the body or out of the body, the oxidation of ammonia gives rise to a combination of oxygen with nitrogen. Nitrous and nitric acids are produced. It is very difficult to determine the quantity of these acids formed in the body, but the experiments mentioned above prove that some portion of the nitrogen is oxidized in the passage of ammonia through the system.

The importance of this oxidizing action in effecting a change in the injurious substances which happen to be present in the blood, cannot be overlooked, when experiment proves that urea *in transitu* is partly converted into nitric acid. Whether nitric acid is thus made to appear in the urine in any diseases, is a question of great interest.

The conclusions I come to from my experiments are,—

1st. That the action of oxygen takes place in the body, not only on hydrogen, carbon, sulphur and phosphorus, but also on nitrogen.

2ndly. That in all cases of combustion, out of the body and in the body, if ammonia is present, it is converted partly into nitric acid.

3rdly. That the nitrogen of the air is not indifferent in ordinary cases of combustion, but that it gives rise to minute quantities of nitric acid.

The general result is, that the production of nitric acid from ammonia in the body adds another to the many instances of the action of oxygen in Man.

In the formation of water and carbonic acid, oxygenation has long been recognized as the great cause of the animal heat.

In the Philosophical Transactions for 1846 and 1850, I have endeavoured to show that the excretion of phosphoric and sulphuric acids in excess may be the means of ascertaining in what tissue the most energetic action of oxygen is taking place.

In the present paper, it appears that the detection of nitric acid in the urine may lead to the conclusion, that the blood is being freed from ammonia, or from substances closely related to it, as urea, or possibly caffein and other alkaloids.

The same action then takes place in the fluids of the body as occurs in wells and streams when tainted with sulphuretted hydrogen and ammoniacal animal matter. In the body, as well as out of it, from these substances, sulphuric and nitric acids are produced by the purifying action of the oxygen of the atmosphere.

APPENDIX.

On Nitric Acid in Rain-water.

During January, on different wet days, rain-water was collected in London and decreasing quantities were evaporated with perfectly pure carbonate of potash; and I found that nitric acid was always present and could be detected even in a pint of rain-water by the starch test.

Moreover, in rain-water collected about the same time at Kingston in Surrey; at Melbury in Dorsetshire, many miles from any town; and near Clonakilty in the county of Cork, when a south-west wind was blowing, I found distinct evidence of nitric acid.

XVI. *On the Relation of the Direction of the Wind to the Age of the Moon, as inferred from Observations made at the Royal Observatory, Greenwich, from* 1840 *November to* 1847 *December.* *By* G. B. Airy, *Esq., Astronomer Royal.*

Received December 27, 1850,—Read March 6, 1851.

In the year 1849, in a voyage to Shetland, I heard allusions to the belief entertained generally by Norwegian seamen, that a northerly wind may always be expected about the time of new moon. The expression of this belief was so positive, and the implication of the interests of the persons entertaining it was so distinct, that it appeared to me extremely probable that there was some physical foundation for it. At the first convenient opportunity, therefore, I took measures for discussing, with reference to this question, the directions of the wind at the Royal Observatory, during a period of rather more than seven years, as ascertained from the records of Osler's Self-registering Anemometer. I extended the research so far as to enable every reader to judge whether there is any probable relation between any Direction of Wind and any Age of the Moon.

The collection and summation of the numbers was effected under the immediate superintendence of Mr. Glaisher. Great pains were taken to establish such checks on the operation that error is almost impossible.

The general result is contained in the Table subjoined to this paper. And, while it shows that there is great uncertainty in the verification of an empirical law, even from nearly ninety lunations, it seems very distinctly to negative the asserted law which gave rise to the inquiry.

In explanation of the Table, it is only necessary to remark that the civil day on which the new moon occurred is taken as the day of new moon (at whatever hour the conjunction occurred), and that the other days are counted in succession from it. The moon's synodic period being nearly $29\frac{1}{2}$ days, the month sometimes consists of twenty-nine days, sometimes of thirty, in almost equal proportions in the long run; and thus the numbers in the last horizontal row of figures must, to make them comparable with the others, be very nearly doubled. Although the sums of numbers in the other horizontal lines are not absolutely equal, they are so nearly equal that no remarkable error will be produced by assuming them as equal.

TABLE exhibiting the number of hours during which the wind blew in each of sixteen equal divisions of the azimuthal circle, and also the number of hours of sensible calm, in the period extending (with very small interruptions) from 1840 November to 1847 December; from the records of OSLER's Self-registering Anemometer at the Royal Observatory, Greenwich: arranged in reference to the days of the Moon's Age.

Days of the Moon's Age.	Number of hours of wind in each direction.																Number of hours of calm.	Total number of hours for each day.
	N.	N.N.E.	N.E.	E.N.E.	E.	E.S.E.	S.E.	S.S.E.	S.	S.S.W.	S.W.	W.S.W.	W.	W.N.W.	N.W.	N.N.W.		
1	146	52	84	42	80	6	...	4	194	260	234	282	128	32	40	38	386	2008
2	140	58	86	66	54	4	6	26	200	242	264	174	78	34	74	90	454	2050
3	164	80	52	52	36	32	12	32	166	242	254	180	84	32	36	84	502	2040
4	112	88	48	18	56	10	18	44	198	244	238	220	100	22	14	74	560	2064
5	126	58	62	34	104	22	10	26	126	210	254	250	78	44	44	52	508	2008
6	132	46	110	76	66	20	6	30	132	192	268	214	98	46	46	86	414	1982
7	108	66	84	72	102	12	26	20	120	206	288	148	176	28	44	54	488	2042
8	116	72	96	58	66	18	4	14	172	174	208	278	124	46	56	56	484	2042
9	150	78	64	48	68	30	10	28	130	160	246	244	96	56	48	80	540	2076
10	190	62	106	32	98	18	18	36	170	184	188	162	138	44	36	74	530	2086
11	184	76	88	52	66	16	10	22	144	210	204	212	120	20	54	76	534	2088
12	100	92	98	66	50	12	6	22	168	116	204	264	146	44	26	48	576	2038
13	194	72	68	58	74	18	12	32	172	110	166	170	104	44	46	76	608	2024
14	190	72	60	80	78	24	20	34	156	144	206	164	82	24	66	86	568	2054
15	278	86	68	60	86	30	34	30	140	190	206	148	104	30	32	64	478	2064
16	150	84	50	104	86	30	52	26	84	158	252	194	100	28	62	80	524	2064
17	208	70	64	46	78	16	20	16	136	166	244	188	96	14	52	60	590	2064
18	212	82	60	70	98	12	14	42	128	194	188	184	100	20	40	50	558	2052
19	228	68	114	80	104	16	26	44	64	86	166	202	126	36	52	88	510	2010
20	174	80	116	76	138	50	20	24	92	114	176	190	122	38	26	40	544	2020
21	168	56	72	98	168	36	16	40	88	140	216	222	122	38	52	44	496	2072
22	206	68	102	76	118	36	26	32	74	150	170	176	90	42	50	80	562	2058
23	224	70	110	56	62	30	24	48	180	126	196	176	86	32	32	74	506	2032
24	236	72	80	38	42	34	20	24	140	156	242	246	100	28	26	176	378	2038
25	206	64	58	76	52	30	8	22	124	202	238	196	136	28	40	90	476	2046
26	126	46	72	100	70	18	16	14	114	178	230	164	164	30	56	78	532	2008
27	98	58	100	64	26	8	10	24	192	210	286	230	88	28	22	26	518	1988
28	88	68	48	68	74	28	32	50	154	190	322	286	92	22	30	50	402	2004
29	82	64	74	60	106	18	4	10	180	198	220	226	170	44	32	50	454	1992
30	80	26	32	30	20	...	2	16	96	124	110	154	48	18	12	26	262	1056
Sum for each direction	4816	2034	2326	1956	2226	634	482	832	4234	5276	6684	6144	3296	992	1246	2050	14942	60170

Royal Observatory, Greenwich,
December 24, 1850.

XVII. *On the Action of Nitric Acid on various Vegetables, with a more particular Examination of* Spartium scoparium, *Linn., or Common Broom.*

By JOHN STENHOUSE, *Esq., Ph.D., F.R.S.*

Received November 18,—Read December 12, 1850.

THE last paper which I had the honour of presenting to the Royal Society contained an account of the effects produced by the action of sulphuric and hydrochloric acids on the *matière incrustante* of several plants belonging to the different great classes of vegetables. In the present paper I intend to describe the effects of nitric acid upon a variety of vegetable groups; these researches having been undertaken in the hope that, by means of this powerful reagent, some light might perhaps be thrown on peculiarities in their respective constitutions.

Populus balsamifera.

The first plant experimented on with this view was the *Populus balsamifera*, as representing the numerous family of the Poplars. A quantity of the branches of this, in Great Britain, very common tree, was therefore cut into small pieces and was boiled with water till it was completely exhausted. The dark-coloured bitter-tasted liquid which it yielded was evaporated to the state of an extract. This extract was then digested with dilute nitric acid for about four and twenty hours. A great deal of nitrous fumes were given off, together with a small quantity of an agreeably smelling aromatic oil. The strongly acid solution was evaporated to dryness on the water-bath, so as to drive off as much of the adhering nitric acid as possible. The dried residue was next dissolved in a considerable quantity of hot water, and after being allowed to cool, was carefully filtered through a cloth. The clear liquid was then concentrated by cautious evaporation, and exactly saturated in the cold with carbonate of potash, great care being taken not to add an excess of the alkali. A yellow sediment soon began to appear, and its amount was greatly increased in the course of a few hours. It consisted of nitropicrate of potash and of the potash salt of a new acid, to which I have given the name of nitropopulic acid. The mother-liquor contained a great deal of oxalate and nitrate of potash. The mixed precipitates were then collected on a filter and dried by pressure, so as to remove as much of the adhering mother-liquor as possible. They were then washed with a little cold water and were treated with a cold solution of dilute carbonate of potash, which readily dissolved out the nitropopulate of potash, while it left the nitropicrate of potash unacted on. The two salts were then separated by filtration, the nitropicrate of potash remaining on the filter, while the nitropopulate of potash was dissolved in the alkaline liquid. The mother-liquor was again slightly supersaturated with muriatic

acid, when the nitropopulate of potash precipitated as a compact crystalline powder, which was collected on a filter and washed with a little cold water. It was dissolved a second time in a dilute solution of carbonate of potash, and again precipitated by muriatic acid. The object of this second treatment with an alkaline carbonate, was to remove any trace of nitropicrate of potash which might adhere to it. The nitropopulate of potash was still further purified by being digested with animal charcoal, and repeatedly crystallized out of hot water. No potash was added in these latter crystallizations, because an excess of potash, when assisted by heat, has the effect of decomposing nitropopulic acid. When, through these operations, the nitropopulate of potash has been sufficiently purified, it has merely a pale lemon-yellow colour, and may be employed for preparing nitropopulic acid. It is therefore boiled with a very considerable excess of muriatic acid, and on the cooling of the solution, the nitropopulic acid is deposited in silky needles, forming concentric groups of a pale yellow colour. Nitric acid cannot be advantageously employed for this purpose instead of muriatic acid, for, as will subsequently appear, nitropopulic acid is destroyed when it is boiled with strong nitric acid. The nitropopulic acid in this stage of its preparation is not altogether free from a small quantity of a yellowish colouring matter, which adheres to it with great tenacity. Its crystals require, therefore, to be again dissolved in dilute muriatic acid; and they are subsequently still further purified by being repeatedly digested with animal charcoal, and crystallized out of hot aqueous solutions. The crystals of nitropopulic acid, when quite pure, are perfectly colourless, and the solution from which they have crystallized has only a slightly yellowish shade. The pure nitropopulic acid requires to be pretty quickly separated from the mother-liquor by being pressed between folds of blotting-paper and immediately dried *in vacuo*, as when it is exposed to the air, even for a short time, it is very apt to become yellow. When it is deposited by slow cooling out of dilute aqueous solutions, it forms hard prismatic crystals arranged in concentric groups; but when a hot concentrated solution of the acid is rapidly cooled, it becomes filled with a mass of long slender silky needles. Muriatic acid has the effect of preventing the oxidation of nitropopulic acid, which may be readily obtained from a dilute solution of that acid in colourless crystals.

Nitropopulic acid is very soluble in water, and still more so in weak and strong spirits of wine. An addition of sulphuric or muriatic acid to its aqueous solution has the effect of considerably diminishing its solubility, and the colour of the solution becomes fainter in proportion as the amount of the mineral acid is increased. But when the acid solution is diluted with water, the yellow colour again reappears. When an excess of nitropopulic acid is boiled with dilute muriatic acid part of it dissolves, while the portion which remains undissolved melts and forms a yellowish oil, which solidifies and crystallizes when the liquid cools. Nitropopulic acid may likewise be gently heated in sulphuric acid without being decomposed, and it crystallizes again when the solution cools. When nitropopulic acid, however, is digested with pretty concentrated nitric acid, it is decomposed, being converted into nitropi-

cric acid. If nitropopulic acid is boiled with a mixture of hydrochloric acid and chlorate of potash, it is also decomposed, being changed into chloranil. When nitropopulic acid is treated with a cold solution of hypochlorite of lime, no effect is produced; and even when the mixture is treated to nearly the boiling-point, no chloropicrine is evolved; but when it has been boiled for a few minutes, a sudden effervescence takes place, and the nitropopulic acid gives off abundance of chloropicrine. This reaction is highly characteristic of nitropopulic, indigotic and chrysammic acids, and furnishes an easy test by which any of these acids may be immediately distinguished from either nitropicric or oxypicric acids, both of which evolve chloropicrine, either in the cold, or when very gently heated with hypochlorite of lime.

I must here rectify a slight error which I committed, in a short notice upon indigotic acid, which I published about two years ago. I then stated that indigotic acid might be heated with a solution of hypochlorite of lime without the production of chloropicrine. This is so far quite true; but if, as has just been stated, the mixture is boiled for some minutes, effervescence ensues and chloropicrine is abundantly evolved. Nitropopulic acid may also be easily distinguished from the above-mentioned acids by the peculiar character of its potash salt. For though, as already stated, nitropopulate of potash is very slightly soluble in pure water, it dissolves very readily in an alkaline solution, and is precipitated again when the solution is neutralized with an acid. The potash salts of nitropicric, chrysammic and oxypicric acids, on the contrary, are less soluble in alkaline lyes than they are in pure water. And indigotate of potash, though it resembles the nitropopulate of that base, in being very soluble in alkaline solutions, differs from it in being much more soluble in cold water, and in being much more easily decomposed by any of the stronger acids. Thus, when a concentrated solution of indigotate of potash is treated in the cold with a slight excess of muriatic acid, the salt is decomposed and a considerable portion of the indigotic acid is precipitated. Nitropopulate of potash, on the contrary, can only be decomposed when it is boiled with a great excess of muriatic acid. The small amount of solubility in cold water which the neutral potash, soda and ammonia salts of nitropopulic acid exhibit, is quite characteristic, and distinguishes them from the corresponding salts of indigotic acid. The taste of nitropopulic acid is very peculiar, being at first very strongly acid like oxalic acid, then astringent, and finally exceedingly bitter. Nitropopulic acid stains the skin permanently yellow. When gently heated in a retort, it sublimes and crystallizes on cooling. When strongly heated in the open air, it burns with a bright yellow flame, and when any of its salts are heated on platinum foil, they explode with considerable violence. Nitropopulic acid strikes a deep red colour with a solution of perchloride of iron. With the protosulphate of iron it merely yields a deep yellow.

The chief points which require to be attended to in preparing nitropopulic acid, may be shortly stated as follows:—The mixture of dilute nitric acid and the extract of the *Populus balsamifera*, must never be so highly heated as to boil; for if the heat is allowed to rise very high and the acid is concentrated, almost the whole of the nitro-

populic acid will be decomposed, being changed into nitropicric acid. In this respect nitropopulic acid resembles indigotic acid, which, by digestion with strong nitric acid, is also converted into nitropicric acid. When the mixture of the crude acids is dissolved in water, it must be filtered only when perfectly cold, so as to separate the resinous matter of the extract and retain it upon the filter. This resinous matter, which has usually a deep green colour when it is treated a second time with nitric acid, yields a second quantity of the mixed nitrogenated acids. It is also necessary to precipitate the mixture of the nitropicric and nitropopulic acids both together by exactly saturating them with potash, and to withdraw them from the mother-liquor before we attempt to separate the mixture of the two salts from each other; for if much resinous matter and any considerable amount of inorganic salts are contained in the solution, neither the nitropicrate nor the nitropopulate of potash readily crystallizes out of it. The mother-liquor always contains a large quantity of quadroxalate of potash and free oxalic acid, the removal of which latter substance cannot be effected by means of lime, as the basic lime-salt of nitropopulic acid is scarcely more soluble in water than the neutral oxalate of lime. It is likewise very necessary to exclude the air from nitropopulic acid as m ich as possible during its purification; and the dried acid should be preserved in well-stoppered bottles, as it rapidly becomes yellow on exposure to the air.

Nitropopulic acid, when dried first in the air and then *in vacuo*, loses 7·21 per cent. water. Thus 0·957 grm. lost *in vacuo* 0·069=7·21 per cent.

The nitropopulic acid dried *in vacuo*, when burned with oxide of copper and copper turnings, gave the following results:—

I. 0·323 grm. of acid gave 0·4355 CO_2 and 0·05 water.

II. 0·339 grm. of acid gave 0·455 carbonic acid and 0·058 water.

The nitrogen was determined qualitatively by Liebig's method.

Tubes.	Vol. mixed gases.	Vol. after absorption, or N.	Vol. of carbonic acid.
1	33	4	29
2	25	3	22
3	32	4	28
4	39	5	34
5	24	3	21
6	27	3½	23½
7	33	4	29
8	35	4½	30½
9	29	3½	25½
10	33	4	29
11	32	4	28
12	43	5½	37½
13	37	4½	32½
14	28	3½	24½
15	36	4½	31½
16	33	4	29
17	27	3½	23½
18	32	4	28
	578	72	506

It is plain from these determinations that the nitrogen in the acid is to the carbon almost exactly in the proportions of 2 to 14 or of 1 to 7, and when calculated accordingly, they have given the subjoined result, 12·23 per cent. nitrogen.

			I.	II.
C 14	1050·00	36·84	36·77	36·64
H 4	50·00	1·75	1·71	1·79
N 2	350·50	12·30	12·23	12·23
O 14	1400·00	49·11	49·29	49·34
	2850·50	100·00	100·00	100·00

1 equivalent of nitropopulic acid dried first in the air and then

		Calculated.	Found.
in vacuo	= 2850·50	=92·69	92·79
2 equivs. water	225·00	7·31	7·21
	3075·50	100·00	100·00

The formula of the acid dried *in vacuo*, as confirmed by the analysis of the potash and silver salts subsequently given, is therefore $C_{14}\,H_3\,N_2\,O_{13}+HO$, and the formula of the acid dried in the air $C_{14}\,H_3\,N_2\,O_{13}+HO+2Aq$.

Nitropopulate of Potash.

One portion of this salt was prepared by digesting crude nitropopulate of potash with animal charcoal and repeatedly crystallizing it out of water. The pure salt had a lemon-yellow colour and consisted of small prisms. It is very slightly soluble in cold water, but exceedingly soluble in alkaline solutions. A second portion of the purified salt was crystallized out of a solution which had been rendered slightly alkaline by the addition of a few drops of potash. Both portions of the salt were dried first *in vacuo*, and then in the water-bath at 212° Fahr., when they lost no water.

Their analysis was conducted as follows:—A weighed portion of each salt was gently heated in a covered platinum crucible with a few drops of sulphuric acid, and after the mixture of acids had been cautiously volatilized, the heat was considerably increased. A little more sulphuric acid was then added, and the excess of the acid removed by heating the sulphate with a little carbonate of ammonia.

I. 0·300 grm. of salt gave 0·0987 $KO\,SO_3$=0·0532 potash=17·73 per cent.

II. 0·299 grm. of salt gave 0·097 $KO\,SO_3$=0·047 KO=17·42 per cent.

0·2723 grm. of salt gave 0·317 carbonic acid and 0·029 water.

	Calculated numbers.		I.	II.
C 14	1050·00	31·57	31·74	
H 3	37·50	1·12	1·17	
N 2	350·50			
O 13	1300·00			
KO	588·00	17·70	17·73	17·42
	3326·00			

Nitropopulate of Silver.

The silver salt was prepared by dissolving oxide of silver in a hot solution of pure nitropopulic acid. On the cooling of the liquid the salt was deposited in small hard granular crystals, which were dried at 212°, and subjected to analysis.

0·448 grm. of salt gave 0·190 chloride of silver=0·1536 oxide of silver.

I. 0·6205 grm. of salt gave 0·562 carbonic acid and 0·055 water.

II. 0·379 grm. of salt gave 0·347 carbonic acid and 0·031 water.

	Calculated numbers.			I.	II.
C	14	1050·00	25·07	24·70	24·97
H	3	37·50	00·89	00·98	00·90
N	2	350·50			
O	13	1300·00			
AgO		1449·01	34·60	34·28	
		4187·01			

Nitropopulate of Soda.

This salt was prepared by exactly saturating a hot solution of nitropopulic acid with carbonate of soda. On the cooling of the liquid the soda salt was deposited in small acicular crystals, which, like the potash salt, were very slightly soluble in cold water.

0·1700 grm. of salt dried *in vacuo* gave 0·048 sulphate of soda=0·0209. NaO=12·29 per cent.

		Calculated.		Found.
C	14	1050·00		
H	3	37·50		
N	2	350·50		
O	13	1300·00		
NaO		387·00	12·39	12·29
		3125		

The nitropopulate of ammonia is prepared in exactly the same way as the potash and soda salts, and its solubility and reactions are very similar to theirs.

The neutral baryta salt is readily formed by adding a hot solution of baryta to a hot solution of nitropopulic acid. The addition of the baryta is cautiously continued so long as the precipitate which falls is redissolved on shaking the flask. The filtered liquid deposits, on cooling, small granular crystals, which, after being dried *in vacuo* and afterwards in the water-bath, appear to have the following composition:—

0·231 grm. salt, when treated with sulphuric acid, gave 0·091 BaO SO_3=0·0597 BaO=25·84 per cent.

		Calculated numbers.		Found.
C	14	1050·		
H	3	37·50		
N	2	350·50		
O	13	1300·00		
BaO		954·85	25·85	25·84
		3692·00		

Basic Nitropopulate of Baryta.

This salt is obtained by adding a hot solution of caustic baryta to a hot solution of nitropopulic acid, so long as a precipitate is formed. This precipitate, which consists of very minute crystals, must be immediately collected on a filter and thoroughly washed with boiling distilled water, in which it is nearly insoluble. It was also first dried *in vacuo*. When it was afterwards heated in the water-bath to 212° FAHR., it did not lose any weight.

0·241 grm. salt gave 0·151 BaO SO_3=0·0991 BaO=41·12 per cent.

	Calculated numbers.			Found.
C	14	1050·00		
H	3	37·50		
N	2	350·50		
O	13	1300·00		
BaO	2	1909·00	41·08	41·12
		4647·		

The formula of this salt therefore is $C_{14}H_3N_2O_{13}(BaO)_2$.

Nitropopulate of Lead.

Both the neutral and basic lead salts of nitropopulic acid may be readily obtained by treating a hot solution of the potash salt with neutral or basic acetate of lead. Both of these lead salts are uncrystallizable and insoluble in water.

Populus nigra.

As I was anxious to ascertain if nitropopulic acid could likewise be obtained from the various other species of Poplar as well as from the *Populus balsamifera*, a quantity of the small branches of the *Populus nigra* was cut into pieces, boiled with water, and the solution concentrated to the state of an extract, exactly in the way already described. This extract was also digested with dilute nitric acid. The products were found to consist chiefly of an acid, having all the external characters of nitropopulic acid, together with a considerable amount of nitropicric acid. In order to ascertain, however, that the supposed nitropopulic acid was really what it appeared to be, a quantity of it was purified in the way already detailed, and subjected to analysis.

0·2325 grm. of the acid, dried *in vacuo*, gave 0·314 CO_2 and 0·040 HO.

	Calculated numbers.	Found numbers.
C	36·84	36·81
H	1·75	1·90

Two quantities of the potash salt, prepared at different times, were also subjected to analysis.

I. Prep. 0·2355 grm. salt gave 0·0378 KO SO_3=0·0421 KO=17·45 per cent.

II. Prep. 0·214 grm. salt gave 0·07 KO SO_3=0·0378 KO=17·65 per cent.

The calculated quantity of potash in the nitropopulate of that base is 17·70 per

cent. From these determinations, therefore, there can be no doubt that the *Populus nigra* also yields nitropopulic acid. I think there is good reason for expecting, therefore, that the other species of Poplars will also yield nitropopulic acid, which I believe will be found characteristic of the Poplar tribe.

It has long been known, through the researches of Braconnot and other chemists, that the family of Poplars contains two crystallizable, very analogous principles, viz. salicine and populine. Now salicine, as was first observed by Piria, when it is distilled with a mixture of bichromate of potash and sulphuric acid, yields salicylous acid. On subjecting, therefore, separate quantities of the extracts of *Populus balsamifera* and *P. nigra* to distillation with bichromate of potash and sulphuric acid, I succeeded in procuring a considerable amount of salicylous acid from both of them. There is every reason to believe, therefore, that both of these Poplars contain salicine, which is probably the source of much of the nitropicric acid which they yield when digested with nitric acid. The nitropopulic acid, which appears characteristic of the Poplar tribe, I strongly suspect will be found to result from the action of nitric acid upon the populine they contain; though, for the present, I only throw out this idea as a probable conjecture.

In order to be quite certain that the other nitrogenous organic acid obtained from both these Poplars, and accompanying the nitropopulic acid, was really the nitropicric acid, as its external characters seemed to indicate, a quantity of its purified potash salt was subjected to analysis.

0·555 grm. salt gave 0·182 sulphate of potash = 0·0984 potash.

0·4075 grm. salt gave 0·401 CO_2 and 0·028 water.

	Calculated numbers.		Found numbers.
12 C	900·00	26·94	26·83
2 H	25·00	0·75	0·76
3 N	525·75	15·74	
13 O	1300·00	38·94	
KO	588·94	17·63	17·73
	3339·69	100·00	

It is clear from these results that the acid in question is really the nitropicric, just as I had previously supposed.

In conclusion, I subjoin a tabular view of the formulæ of nitropopulic acid and its salts:—

Nitropopulic, dried *in vacuo*	$HO, C_{14} H_3 N_2 O_{13}$
Nitropopulic, dried in the air	$HO, C_{14} H_3 N_2 O_{13} + 2$ Aq
Potash salt	$KO, C_{14} H_3 N_2 O_{13}$
Soda salt	$NaO, C_{14} H_3 N_2 O_{13}$
Neutral baryta salt	$BaO, C_{14} H_3 N_2 O_{13}$
Basic baryta salt	$2BaO, C_{14} H_3 N_2 O_{13}$
Silver salt	$AgO, C_{14} H_3 N_2 O_{13}$

The following are the formulæ of a few of the other nitrogenated acids which most closely resemble nitropopulic acid:—

Indigotic acid	HO, C_{14} H_4 N O_9
Nitropopulic acid	HO, C_{14} H_5 N_2 O_{13}
Nitrophenesic acid	HO, C_{12} H_3 N_2 O_{13}
Nitropicric acid.	HO, C_{12} H_3 N_3 O_{13}
Oxypicric acid	HO, C_{12} H_3 N_3 O_{15}

It is plain, therefore, that nitropopulic acid only differs from nitrophenesic acid by two equivalents of carbon.

Salix Russelliana, or Bedford Willow.

An extract was prepared from the wood and branches of the *Salix Russelliana*, exactly in the way already described in the two preceding instances. When digested with dilute nitric acid, it yielded a great deal of oxalic and nitropicric, but no nitropopulic acid. The following pretty numerous list of trees and shrubs, when treated with nitric acid, also yielded oxalic and nitropicric acids:—

1. *Cytisus laburnum*, or the Laburnum-tree.
2. *Swietenia mahogani*, or Mahogany Wood.
3. *Pyrus malus*, or the Apple-tree.
4. *Cratægus oxyacántha*, or the Hawthorn.
5. *Ribes nigrum*, or the Black Currant-bush.
6. *Betula alnus*, or the Alder.
7. *Ulex europæus*, or the Furze.
8. *Calluna vulgaris*, or Common Heather.
9. The root of the *Curcuma longa*, or Turmeric.
10. Extract of seeds of the *Bixa orellana*, or Annotto.
11. *Sambucus nigra*, or Common Elder.
12. *Cytisus scoparius*, the *Spartium scoparium* of Linn., or Common Broom.

The extracts of *Quercus robur*, or Common Oak, and of *Betula alba*, the Beech-tree, when digested with nitric acid, only yielded oxalic acid, but no nitropicric or any analogous nitrogenated acid. It is somewhat remarkable, therefore, that of the extracts of seventeen vegetable substances no fewer than fifteen should, when treated with nitric acid, yield nitropicric acid; while two of their number, in addition to nitropicric, should also yield nitropopulic acid, and only two out of the seventeen should produce oxalic acid, unaccompanied by a nitrogenous organic acid.

The results of these experiments seem to indicate therefore that a far greater number of plants are capable of yielding nitropicric acid than has generally been supposed, those which fail to do so apparently constituting a very small minority. I fully expected to have met with a greater variety of nitrogenated acids, and am rather surprised that I did not find either the indigotic or oxypicric acids.

As the extract of broom, *Spartium scoparium*, besides yielding nitropicric acid, exhibited some interesting peculiarities, I was induced to subject it to a more minute examination, the results of which I now briefly subjoin.

Spartium scoparium, LINN.

The first portion of broom operated on was grown on low-lying sandy ground in an open, sunny situation, about two miles to the east of Glasgow. I mention this circumstance from a reason which will subsequently appear. The broom plants, though full-grown, were only from two to two and a half feet in height, and had an exceedingly bitter taste. They were cut into small pieces and then boiled for six or eight hours with a considerable quantity of water, till they were thoroughly exhausted. The decoction was evaporated down to about one-tenth of its bulk, and set aside in a cool place for four-and-twenty-hours. It was then found to have gelatinized into a greenish-brown, coherent mass, which was thrown upon a cloth filter and washed with a little cold water. This very impure jelly consisted chiefly of a crystalline yellow colouring matter (scoparine), which was contaminated with a considerable amount of chlorophyl and oxide of iron, derived from the iron vessel in which the decoction of the broom had been made. It also contained a small quantity of a volatile organic base (sparteine), which gave the jelly its extremely bitter taste. In order to obtain the scoparine in a pure crystalline state, the first step taken was to dissolve the crude jelly by boiling it with a considerable quantity of water, acidulated with a little hydrochloric acid. The hot aqueous solution was then poured through a funnel, the neck of which was imperfectly stopped with a little cotton wool. The clear liquid which passed through coagulated on cooling again into a greenish-yellow jelly, which was collected on a filtering cloth and again washed with a little cold water. The jelly was then gently compressed to remove the greater portion of the adhering mother-liquor, and was evaporated to dryness on the water-bath. The impure scoparine, when thoroughly dried, was reduced to powder and again dissolved in boiling water, while the greater portion of the chlorophyl having been rendered insoluble remained behind. Instead of evaporating the crude gelatinous scoparine to dryness, in the way just described, the greater portion of the chlorophyl adhering to it may likewise be separated either by long-continued boiling or by filtering its solutions after they have cooled to a certain degree, the impure portion of the scoparine being that which is first deposited. The gelatinous scoparine obtained by either of these methods, when dried at a moderate temperature, either in the open air or *in vacuo*, forms a pale yellow, or sometimes greenish-yellow, brittle mass, which is perfectly amorphous. It is very slightly soluble in cold water, a little more so in cold spirits of wine, but pretty soluble in boiling water and in hot spirits. The colour of its aqueous solution is pale yellow with a greenish shade, that of its alcoholic is pale yellow. It dissolves readily in the caustic and carbonated alkalies, forming deep greenish-yellow solutions. When scoparine has been dissolved in a cold solution of ammonia or carbonate of soda, and the

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alkali is slightly supersaturated with either hydrochloric or acetic acids, the scoparine is thrown down as a white precipitate, which appears however to be a little more compact and denser than before its solution in the alkali. When this precipitate, after being washed to remove any adhering sal-ammoniac, is again dissolved in boiling water, it forms a pale yellow solution, which, when it is allowed to cool very slowly, deposits a quantity of pale yellow prisms arranged in stars, which attach themselves to the bottom and sides of the containing vessel, while some gelatinous scoparine remains floating in the centre of the liquid.

As scoparine appeared to crystallize out of its aqueous solutions with considerable difficulty, a quantity of it, while still but imperfectly purified, was evaporated to dryness on the water-bath and then treated with boiling spirits of wine. A small portion of it remained undissolved, and the yellowish solution, which was formed after it had remained for a day in a covered glass vessel, did not crystallize; but when a portion of the alcohol was slowly evaporated, the scoparine precipitated in the gelatinous state. The solution was therefore again heated, when the gelatinous matter dissolved; and on leaving the solution to spontaneous evaporation, in the course of two or three days the bottom and sides of the glass became covered with short prisms of scoparine, arranged in little stars, precisely similar to those obtained by slow cooling from the aqueous solution. These crystals were then dried by pressure between folds of bibulous paper and were again dissolved in hot spirits of wine, and after twelve hours they crystallized in short prisms of a pale yellow colour and possessing considerable lustre. If these crystals are boiled with a quantity of strong or absolute alcohol, insufficient for their entire solution, a portion dissolves, while that which remains appears to undergo a kind of molecular change, and becomes more difficultly soluble, both in hot alcohol and in water. It may be easily restored to its original state again (from this apparently allotropic condition) by dissolving it in ammonia and neutralizing with acetic acid, when the scoparine precipitates as a jelly and becomes as soluble as before. If it is then treated therefore with boiling water it readily dissolves, and on allowing the solution to cool very slowly, the scoparine crystallizes out in small stars, possessing all its original properties.

Scoparine, when prepared in any of the ways above described, has, as we shall presently see, invariably the same composition. It is tasteless and inodorous, and does not affect either litmus or turmeric paper. It dissolves very readily in caustic and carbonated alkalies, and it is also soluble, to some extent, in concentrated acids, the colour of its solutions being greenish yellow. When the alkaline or acid solutions of scoparine are boiled it is slowly decomposed, being changed into a greenish-brown resinous matter. Scoparine dissolves pretty readily in lime and baryta water, but not nearly to the same extent as in ammonia and the fixed alkalies. When its ammoniacal solution is left to spontaneous evaporation in the open air, almost the whole of the ammonia volatilizes and the residue forms a greenish gelatinous mass. A solution of hypochlorite of lime changes scoparine to a deep green colour, probably

by oxidizing it, and when it is digested with nitric acid it is converted into nitropicric acid. Bromine changes the colour of gelatinous scoparine into brownish-yellow, but yields no crystalline product. A solution of scoparine produces no precipitate, either with corrosive sublimate or nitrate of silver. With neutral and basic acetate of lead it yields greenish yellow, flocculent precipitates, which are neither copious nor stable. Scoparine, when dried *in vacuo*, did not lose any weight when afterwards heated to 212° Fahr. When sufficiently heated in a glass tube, it melts into a dark brownish resin, but yields no sublimate. When strongly heated in the open air, it catches fire and burns with a bright yellow flame.

I. Of scoparine, crystallized several times out of ordinary spirit of wine, 0·2740 grm., dried *in vacuo*, gave 0·578 carbonic acid and 0·134 water.

II. 0·286 grm. scoparine, crystallized out of absolute alcohol, gave 0·606 carbonic acid and 0·135 water.

III. 0·2725 grm. gelatinous scoparine, out of absolute alcohol, gave 0·578 carbonic acid and 0·133 water.

IV. 0·210 grm. residue of scoparine, left from the treatment with alcohol, dissolved in ammonia, precipitated with acetic acid and crystallized out of hot water, gave 0·444 carbonic acid and 0·099 water.

V. 0·2775 grm. gelatinous scoparine, purified only with water, gave 0·586 carbonic acid and 0·131 water.

	Calculated numbers.		Found numbers.				
			I.	II.	III.	IV.	V.
21 C	1575·0	58·09	57·53	57·76	57·83	57·66	57·60
11 H	137·5	5·06	5·43	5·24	5·41	5·23	5·42
10 O	1000·0	36·88	37·04	37·00	36·76	37·11	36·98
	2712·5	100·00	100·00	100·00	100·00	100·00	100·00

The formula $C_{21}\,H_{11}\,O_{10}$, which has been deduced from these analyses of scoparine, is merely empirical, as I have not as yet been enabled to obtain any definite combination from which its atomic weight might be ascertained. From what has been stated above respecting scoparine, it evidently belongs to the pretty numerous class of colouring matters, which, if they can be said to possess any determinate chemical character, are merely extremely feeble acids. Indeterminate, however, as the chemical characters of scoparine certainly are, and notwithstanding its seeming inertness, it appears to possess important medical virtues; for from a pretty extensive series of experiments, made both on animals and men, I am induced to conclude that scoparine is the diuretic principle of broom. Broom was highly prized and recommended as a diuretic both by Mead and Cullen, who found it highly effective in the reduction of dropsical affections; and in more modern times, Drs. Pearson and Pereira recommend it as more certain than any other diuretic in dropsies. I shall advert to this subject again, however, before the close of the paper.

Sparteine.

The acid mother-liquor from the crude scoparine, after having been concentrated to a very moderate bulk, was introduced along with a considerable excess of soda into a capacious distilling apparatus. As the somewhat viscid liquor was very apt to froth up and come over, only a very moderate heat was applied at first; but as the distillation proceeded the fire was increased, and the boiling was kept up pretty briskly, fresh quantities of water being added so long as the liquid which distilled over had a strongly bitter taste. These liquids were then mixed, and were repeatedly rectified with an excess of common salt. During the concluding rectification, the first portion of the liquid which came over contained a considerable amount of ammonia, and as the distillation proceeded a small quantity of colourless heavy oil (sparteine) began to appear, and soon sunk to the bottom of the receiver. The amount of the oil obtained from any one portion of the distilled liquid was never great, but it continued to come over for a very long time, so that by pouring back into the still the clear liquid which had deposited its oil and redistilling it, new quantities of oil continued to be obtained for a very considerable length of time.

The sparteine procured in the way just described is easily freed from any adhering ammonia by being washed with successive quantities of cold water, in which this oily base is but very slightly soluble. Sparteine is a somewhat viscid colourless oil, which when first distilled is quite transparent, but on remaining for some hours in contact with water, it becomes slightly opalescent. In the course of a week or two a thin white coating forms at the line of junction with the water, and if but a small quantity of water is present, it is absorbed by the base, a hydrate being probably produced. When sparteine is exposed to the air for some days, it gradually acquires a brownish yellow colour. It is considerably heavier than water and has a feeble smell, somewhat resembling that of aniline. It has a distinctly alkaline reaction, neutralizing the strongest acids perfectly. Sparteine and its salts have an extremely bitter taste. When brought in contact with hydrochloric acid sparteine immediately combines with it, though owing to the slight amount of volatility in the base scarcely any white vapours are evolved. Neither its acid or its neutral solution in hydrochloric acid, though evaporated *in vacuo*, could be made to crystallize, and I was not more successful with the nitrate. When sparteine was boiled for a considerable length of time with an excess of fuming nitric acid, it was slowly decomposed. The solution was concentrated to drive off the excess of acid, and on adding a little water a slight flocculent precipitate fell (probably a resin), while a clear yellow solution was obtained. On treating one portion of this solution with hypochlorite of lime, chloropicrine was evolved; and on saturating a second portion of it with potash and distilling, a few drops of an apparently new base were produced. When sparteine is boiled for a considerable time with an excess of strong hydrochloric acid, it seems also to be partially decomposed, as it acquires an odour resembling that of mice. When a few drops of bromine are poured upon a little sparteine much heat is pro-

duced, and the sparteine is changed into a brownish resin. Bleaching powder exhibits no peculiar reaction with sparteine; and when this base is distilled with fused potash no aniline is produced.

A quantity of sparteine was dried by contact with fused chloride of calcium, and when poured off from the latter substance was rectified in a small glass retort. The sparteine, owing to its extremely high boiling-point, 550° Fahr., distilled over very slowly and had a yellowish shade. A second quantity of it was rectified in a current of dry carbonic acid gas with a much better result, as it then came over nearly colourless. Its odour was however slightly altered, having become somewhat less agreeable. It will be presently seen from its analysis, and from those of its salts, that the sparteine had undergone no essential alteration. Another quantity of perfectly colourless sparteine which had been rectified along with the vapour of water, and consequently at a much lower temperature, was dried *in vacuo* over sulphuric acid, till it ceased to lose weight. This required about ten days. The oil had also acquired a faintly yellowish shade, but remained quite transparent, and its odour was unaltered.

I. 0·247 grm. sparteine, dried by chloride of calcium, gave 0·6945 carbonic acid and 0·246 water.

II. 0·2545 sparteine, dried *in vacuo*, gave 0·716 carbonic acid and 0·256 water.

	Calculated numbers.		I.	II.
15 C	1125·0	76·91	76·68	76·70
13 H	162·5	11·10	11·02	11·17
N	175·2	11·99		
	1462·7	100·00		

Nitropicrate of Sparteine.

As the more common simple salts of sparteine, such as the hydrochlorate, nitrate, &c., were found to be so very soluble and difficultly crystallizable, it seemed desirable to try whether perhaps some organic salt might prove more suitable in this respect. Accordingly nitropicric acid, on account of its sparing solubility, was selected for this purpose. A saturated solution of nitropicric acid, made with cold spirits of wine, was therefore heated to nearly 212° Fahr., and was then added to a hot alcoholic solution of sparteine. At first the nitropicric acid threw down a yellow milky precipitate, which redissolved when the solution was agitated. But when a sufficiency of the nitropicric acid had been employed, a permanent bulky crystalline precipitate was produced, which was collected on a filter and washed with hot water. It was then digested with a considerable quantity of boiling spirits of wine and filtered. The clear solution, on cooling, deposited the nitropicrate of sparteine in long shining needles, from 1 to 2 inches in length. These crystals are very brittle, and so closely resemble nitropicrate of potash in appearance, that they cannot be distinguished from that salt by the eye. Nitropicrate of sparteine is very slightly soluble, either in cold water or alcohol; and it is by no means very soluble even in boiling water or spirits of wine,

so that it can be readily crystallized out of either of these liquids. It is also a very stable salt, so that it is not altered by exposure to the air; and even when it is treated with a dilute solution of potash in the cold it is not decomposed. With the assistance of heat, however, nitropicrate of potash is formed while the sparteine is eliminated. When nitropicrate of sparteine is sharply heated it detonates pretty strongly. The portion of the salt subjected to analysis was dried *in vacuo*, and when afterwards heated in the water-bath it lost no additional weight.

I. 0·272 grm. salt gave 0·464 carbonic acid and 0·1165 water.

II. 0·2615 grm. salt gave 0·449 carbonic acid and 0·1145 water.

The nitrogen was determined qualitatively according to LIEBIG's method; 571 measures of mixed gases yielded 72 measures of nitrogen and 499 of carbonic acid; being nitrogen in the proportion of 8 to 55½ carbonic acid, or nearly as 4 to 27.

	Calculated numbers.		Found numbers.	
			I.	II.
27 C	2025·0	46·81	46·51	46·63
16 H	200·0	4·62	4·75	4·86
4 N	700·8	16·20	15·68	15·68
14 O	1400·0	32·37	33·06	32·83
	4325·8	100·00	100·00	100·00

The formula of this salt therefore is $C_{15}\ H_{13}\ N, C_{12}\ H_2\ N, O_{13} + HO$.

Chloride of Platinum and Sparteine.

When an excess of bichloride of platinum is added to a cold solution of sparteine in hydrochloric acid, a bulky yellow precipitate immediately falls, which is but slightly soluble in either water or alcohol, and when boiled with these liquids it is partially decomposed. It dissolves, however, without decomposition either in hot concentrated hydrochloric acid, or in that acid when diluted with an equal weight of water. The double chloride is deposited from the latter solution in very regularly formed orange-coloured crystals of considerable size and great lustre. These crystals may be generally described as a modification of the rectangular prism, with elongated triangular facets at either extremity, being precisely similar to that form which the ammonio-phosphate of magnesia has when spontaneously deposited from neutral or alkaline urine. The double chloride does not suffer the slightest change by exposure to the air. It was washed with a little cold alcohol, and when dried *in vacuo* was submitted to analysis.

0·224 grm. salt gave 0·065 Pt=29·02 per cent. platinum.
0·709 grm. salt gave 0·203 Pt=28·63 per cent. platinum.
0·369 grm. salt gave 0·106 Pt=28·73 per cent. platinum.
0·466 grm. salt gave 0·134 =28·75 per cent. platinum.
0·4395 grm. salt gave 0·1272 =28·94 per cent. platinum.
0·7204 grm. salt gave 0·2054 =28·51 per cent. platinum.

Mean . . 28·76 Pt.

The preceding determinations of the platinum were made with the salt prepared at several different times.

I. 0·4879 grm. salt gave 0·475 carbonic acid and 0·217 water.

II. 0·3573 grm. salt gave 0·345 carbonic acid and 0·1528 water.

0·8531 grm. salt gave 1·06 Cl Ag=0·2621 Cl=30·72 per cent. chlorine.

	Calculated numbers.		I.	II.
13 C	1125·00	26·38	26·55	26·33
16 H	200·00	4·68	4·99	4·74
N	175·20	4·10		
2 O	200·00	4·73		
Pt	1233·50	28·93	28·76	28·76
3 Cl	1329·60	31·18	30·72	30·72
	4263·30	100·00		

The double chloride of sparteine and platinum, when dried *in vacuo*, has therefore the following formula,

$$C_{15}\,H_{13}\,N,\,Cl\,H+Pt\,Cl_2+2HO.$$

As it was plain from the result of these analyses that the double chloride of platinum and sparteine, when dried *in vacuo*, contained 2 equivs. of water, it was transferred to the water-bath and heated to 212° Fahr.; and as it continued to give off its water very slowly, it was exposed for twelve hours to a temperature of 266° Fahr., when it lost 5·54 per cent. water. The salt then ceased to lose any additional weight, and was not decomposed, though the heat was raised to 300° Fahr.

1·892 grm. salt lost 266° Fahr., 0·105 grm. =5·54 per cent. water; the calculated quantity for 2 equivs. of water is 5·57 per cent.

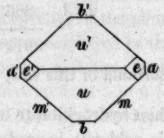
Double Chloride of Mercury and Sparteine.

When a solution of hydrochlorate of sparteine was added to a tolerably concentrated solution of corrosive sublimate, a white crystalline precipitate was immediately produced. This was collected on a filter, washed, and then dissolved in dilute hydrochloric acid. On the cooling of the solution, the double chloride crystallized in right rhombic prisms of considerable size and very high lustre. Through the kindness of Professor W. H. Miller of Cambridge, to whom I have been often indebted for similar obligations, I am enabled to subjoin an exact figure of these crystals, together with the measurements of their angles.

Bichloride of Platinum and Sparteine.

Prismatic:—Symbols of the simple forms,—*a* 100, *b* 010, *u* 011, *e* 101, *m* 110. Angles between normals to the faces,—

	°	′
ab	90	0
ub	37	18
uu'	105	24
ea	41	6
ee'	97	48
ma	48	52
mb	41	8
mm'	82	16



Double Mercurial Chloride of Sparteine.

Prismatic:—Symbols of the simple forms,—*a* 100, *b* 001, *e* 101, *x* 210, *z* 120, *r* 111. The angles between normals to the faces are,—

	°	′
ba	90	0
za	65	39
xa	28	55
zz	48	42
ea	62	35
ee'	54	50
ra	65	46
rb	63	3
re	26	57
rr'	48	28
rr''	53	54
r'r''	75	24

Cleavage:—*a*, very perfect and easily obtained.

The faces *b*, *e*, *x* are very narrow, and are usually altogether wanting.

This salt was nearly insoluble both in water and in spirits of wine, but its solubility in these liquids was greatly increased by the addition of hydrochloric acid. The salt was first dried *in vacuo*, and did not lose any additional weight though heated afterwards to 212° Fahr. The amount of hydrogen, it will be seen, is slightly in excess; this arises from a small quantity of metallic mercury having sublimed into the chloride of calcium tube.

I. 0·785 grm. salt gave 0·3105 sulphuret of mercury =0·2676 Hg.

II. 0·806 grm. salt gave 0·2750 sulphuret of mercury =0·275 Hg.

0·553 grm. salt gave 0·624 carbonic acid and 0·262 water.

0·7145 grm. salt gave 0·7000 Ag Cl=0·1731 Cl=24·22 per cent. chlorine.

	Calculated numbers.		I.	II.
15 C	1125·0	31·14	30·77	
14 H	175·0	4·84	5·26	
N	175·2	4·87		
Hg	1250·9	34·62	34·08	34·11
2 Cl	886·4	24·53	24·22	
	3612·5			

The formula of this salt, therefore, is C_{15} H_{13} N, H Cl, Hg Cl.

When hydrochlorate of sparteine is added to the solution of bichloride of gold, a double gold salt is produced. It is but slightly soluble either in water or spirits of wine, but it readily dissolves in hot hydrochloric acid, out of which it crystallizes in shining yellow scales. When sparteine is added to a solution of chloride of copper, a green coloured precipitate is produced, which contains sparteine, and is probably a double salt. Similar combinations are also formed with both neutral and basic acetate of lead.

Sparteine possesses pretty powerful narcotic properties. When a single drop of it dissolved in acetic acid was administered to a rabbit, it immediately became exceedingly excited, and was thrown into a state resembling intoxication. This lasted five or six hours, and during a considerable portion of the time the rabbit remained in a profound sleep, from which it could with difficulty be roused. A similar dose produced the same effects upon a small dog. When four grains of the base were administered to a full-grown rabbit, violent excitement was at first produced; the animal, however, soon fell into a comatose state and expired in the course of three hours, but without exhibiting any violent convulsions. Sparteine appears therefore to be a tolerably strong narcotic poison, though certainly very inferior in this respect to either nicotine or coniine. Though the narcotic effects of broom are not noticed, so far as I am aware, by any medical writer, they have long been familiar to the peasantry. Thus shepherds have observed that, during snow storms, when their sheep were compelled to eat the tops of broom for subsistence, they were apt to become excited and stupified, so that they not unfrequently left their shelter, and, wandering over the snow, they soon fell asleep, and if neglected not unfrequently perished.

Scoparine, as I already mentioned, acts as a powerful diuretic; the dose for a grown-up person being from five to six grains repeated three times successively at intervals of four hours. It begins to operate in about twelve hours, and the amount of the urine is considerably more than doubled*. It is plain therefore that in employing a decoction of broom tops, as has hitherto been the practice, the patient is subjected to the narcotic influence of the sparteine as well as to the diuretic effects of the scoparine, a result which in general is not likely to be desirable.

* I am indebted for the preceding experiments on the physiological effects of sparteine and scoparine, to my friend Dr. Arthur Mitchell of Glasgow, who kindly undertook them at my request.

I do not however think it is at all necessary to employ chemically pure scoparine for medical purposes. If a decoction of broom is evaporated to dryness on the water-bath, then treated with a little dilute hydrochloric acid, the mixture thrown upon a filter and washed with a small quantity of cold water, almost the whole of the sparteine will be removed, and the dark green gelatinous mass remaining on the filter will be found to possess the diuretic, without the narcotic properties of the plant.

I have already mentioned that the first quantity of the broom employed in this investigation was grown on sandy ground, freely exposed to the sun and air. A decoction was also made from a second quantity of very tall broom from 5 to 6 feet in height, which grew in a shady copse in the neighbourhood of Lanark. Its decoction had only a slightly bitter taste, and it did not yield more than one-fourth so much either scoparine or sparteine as the first quantity of broom had done; though the botanical properties of both plants were precisely the same.

This observation may perhaps help to explain the somewhat discordant results which eminent medical practitioners have experienced in regard to the diuretic effects of broom,—sometimes it having acted powerfully, while at others it appeared to be ineffective. Now, may not this have arisen from the circumstance that they had sometimes employed broom which had grown in a favourable, and sometimes that which had grown in a disadvantageous situation, and which therefore varied exceedingly in its medical properties?

Glasgow, November 16, 1850.

XVIII. *On Rubian and its Products of Decomposition.* *By* EDWARD SCHUNCK, *F.R.S.*

Received January 9,—Read February 13, 1851.

PART I.

AMONG the many discussions to which the subject of madder has given rise among chemists, there is none which is calculated to excite so much interest as that concerning the state in which the colouring matter originally exists in this root, and there is no part of this extensive subject which is at the same time involved in such obscurity. It is a well-known fact that the madder root is not well adapted for the purposes of dyeing until it has attained a growth of from eighteen months to three years, and that after being gathered and dried it gradually improves for several years, after which it again deteriorates. During the time when left to itself, especially if in a state of powder, it increases in weight and bulk, in consequence probably of absorption of moisture from the air, and some chemical change is effected, which, though not attended by any striking phenomena, is sufficiently well indicated by its results. There are few chemical investigations that have thrown any light on the nature of the process which takes place during this lapse of time, and in fact most of the attempts to do so have merely consisted of arguments based on analogy. It has been surmised that the process is one of oxidation, and that the access of atmospheric air is consequently necessary. We are indeed acquainted with cases, in which substances of well-defined character and perfectly colourless, as for instance orcine and hematoxyline, are converted by the action of oxygen, or oxygen and alkalies combined, into true colouring matters. A more general supposition is, that the process is one of fermentation, attended perhaps by oxidation, and in confirmation of this view the formation of indigo-blue from a colourless plant, by a process which has all the characters of one of fermentation, may be adduced. What the substance is however on which this process of oxidation or fermentation takes effect, what the products are which are formed by it, whether indeed the change is completed as soon as the madder has reached the point when it is best adapted for dyeing, or whether further changes take place when it is mixed with water and the temperature raised during the process of dyeing, are questions which have never been satisfactorily answered, if answered at all. It has indeed been suspected by several chemists, that there exists originally some substance in madder, which by the action of fermentation or oxidation is decomposed and gives rise by its decomposition to the various substances endowed either with a red or yellow colour, which have been discovered during the chemical investigations of this root. That several of these substances are merely

mixtures, and some of them in the main identical, has been satisfactorily proved by late investigators. But there still remain a number, which, though extremely similar, have properties sufficiently marked to entitle them to be considered as distinct.

In my papers on the colouring matters of madder*, I have described four substances derived from madder, only one of which is a true colouring matter, but all of them capable, under certain circumstances, as for instance in combination with alkalies, of developing red or purple colours of various intensity. To seek for a common origin for these various bodies so similar to one another and yet distinct, is very natural, and the discovery of it no improbable achievement.

Persoz† asserts the probability of this view in the following words:—"We may hence venture to conclude that the colouring matters which we extract from fabrics dyed with madder, as well as the alizarine which is obtained by submitting the products derived from madder to sublimation, do not exist ready formed in this root, and are only products derived from another substance which has not yet been isolated..... From numerous experiments which I have made on this subject, it follows that the colouring matter of madder may be compared, in respect to the derivatives to which it gives rise, to tannin, so that I do not despair of being able, as far as regards their metamorphoses, to establish a parallel between the products derived from madder and those obtained from tannin. If it should be possible to confer on the former that tendency to assume regular forms with which the latter are endowed, the separation of the proximate colouring or colour-giving (*colorable*) matters of madder will be easy, and it will thus be possible to establish their elementary composition and thence their relation to one another."

To Mr. J. Higgin is due the merit of having first called attention to the fact, that important changes take place during the process of dyeing with madder, which can only be explained by supposing that an actual formation of colouring matter takes place during the process. In his paper 'On the Colouring Matters of Madder‡,' Mr. Higgin has detailed his experiments on that peculiar substance discovered in madder by Kuhlmann and called by him *Xanthine*. I have shown, on a former occasion, that the xanthine of Kuhlmann and other investigators is not a pure substance, but a mixture of two distinct substances. This fact however does not affect the correctness of Mr. Higgin's conclusions, the general accuracy of which I shall have great pleasure in confirming in the course of this paper. The presence of xanthine is easily ascertained by the deep yellow colour and intensely bitter taste which it communicates to cold water. Guided by these two tests, Mr. Higgin arrived at the conclusion, that in an infusion of madder, made with cold or tepid water, when left to itself, or more rapidly when heated to 120° or 130° Fahr., the xanthine gradually disappears and there is formed a gelatinous or flocculent substance, which possesses all the tinctorial power originally belonging to the infusion, while the liquid has lost

* See Reports of the British Association for 1847 and 1848.

† Traité de l'Impression des Tissus, t. i. p. 501.

‡ Philosophical Magazine for Oct. 1848.

all trace of any such power, and that as alizarine is the only substance contained in madder capable of dyeing, the xanthine must, during this process, have been changed into alizarine. Mr. HIGGIN found that this process is completely arrested by heating the infusion to the boiling-point, or by adding alcohol, acids or acid salts to it, and hence he concludes that the decomposition of the xanthine is caused by the action of a peculiar ferment contained in madder, and which is extracted together with xanthine by cold water. Mr. HIGGIN did not however succeed in converting his xanthine into alizarine or effecting any change in it by means of fermentation, in consequence, as he supposed, of not being able to obtain the exciting substance in a soluble and consequently active condition. His inferences were all derived from experiments made by either removing from an extract of madder the xanthine contained in it, or by adding to it an additional quantity of that substance, and then ascertaining the effects produced by dyeing in the usual way with liquids thus artificially prepared. By the action of sulphuric acid on xanthine, Mr. HIGGIN obtained a dark brown powder, which he seems to consider as devoid of any tinctorial power.

A very simple experiment suffices to prove that madder, in its dry state, contains very little, if any alizarine ready-formed. If an extract of madder be made with cold water, it will be found that the brownish-yellow liquid thus obtained when gradually heated will dye as well and as strongly as the madder from which it has been prepared. Now if the colouring matter were originally present in the form of alizarine, this could not take place, since alizarine is almost insoluble in cold water; and in employing it for the purpose of dyeing, it is necessary to dissolve it in warm or boiling water before it begins to exert any effect, as is plainly seen in the case of garancine, which contains alizarine ready-formed. Nor is there much colouring matter left behind in the madder after extraction with cold water, for after converting the residue in the usual manner into garancine by means of sulphuric acid, it is found to be capable of dyeing only very slightly. Nay more, if madder be extracted with hot water instead of cold, I have found the residue to give a garancine which dyed darker colours than that obtained from the residue of an equal weight of madder extracted with cold water, so that it appears that the colour-producing substance is more completely removed by cold than by hot water. If an extract of madder with cold water be left to stand, there is formed in it, as Mr. HIGGIN has shown, a flocculent substance, while the liquid loses its bitter taste, part of its yellow colour and the whole of its power of dyeing, which is now found to reside in its whole extent in the flocculent substance. This change takes place equally well with or without the access of atmospheric air.

By adding a variety of substances to an extract of madder with cold water, I was enabled to ascertain under what circumstances and by what means the tinctorial power of the liquid is destroyed, and consequently what is the general character of the substance or substances to which it is due. I found that by adding sulphuric or muriatic acid to the extract and heating, the liquid, after neutralization of the acid,

was no longer capable of dyeing. The tinctorial power was also destroyed by the addition of hydrate of alumina, magnesia, protoxide of tin and various metallic oxides, but not by carbonate of lime or carbonate of lead. In all cases in which the property of dyeing in the extract was destroyed, I invariably found that its bitter taste and bright yellow colour were lost. Now in my former papers on this subject, I have shown that the intensely bitter taste of madder and its extracts is due to a peculiar substance, to which I have given the name of *Rubian*; and as it appeared from these preliminary experiments that this substance, though itself no colouring matter, is in some way concerned in the changes whereby a formation of colouring matter is induced in aqueous extracts of madder, I proposed to myself to examine its properties and products of decomposition more in detail than I had hitherto done.

The first step necessary to be taken for the attainment of this object, was of course to find a method of procuring this substance in quantities sufficiently large for the purposes of examination. I was at the commencement however far from appreciating the difficulties with which its preparation in a state of purity is attended. The process which I had formerly described, by precipitation with sulphuric acid, is not well adapted to the purpose, since rubian in a state of perfect purity is not precipitated by sulphuric acid, besides which it is easily decomposed by an excess of that acid. Neither is it precipitated by any metallic salt, with the exception of basic acetate of lead, which, from the circumstance of its precipitating also other substances from the extract, is not applicable to the purpose. It is decomposed by alkalies and alkaline earths. Even bicarbonate of lime exerts a decomposing effect on it in conjunction with the oxygen of the atmosphere. These substances must therefore be discarded in its preparation. Besides its great tendency to decomposition, there is another circumstance which presents obstacles to almost all attempts to prepare rubian in a state of purity. There is no investigation of madder which does not make mention of a substance, which when its solution in water is mixed with sulphuric or muriatic acid and boiled, gives rise to the formation of a dark green powder. To this substance, which possesses no bitter taste, and is in fact devoid of any characteristic property except the one mentioned, I have restricted the name of xanthine. The xanthine of most other chemists is however a mixture of rubian with this substance, and possesses therefore the bitter taste of the former, while showing the characteristic behaviour of the latter towards acids. To avoid confusion, I shall no longer employ the name of xanthine, and I shall call the substance which gives the green powder with acids *Chlorogenine*. Now these two substances, though of very different nature, behave similarly towards many reagents. If, for instance, basic acetate of lead be added to a watery extract of madder, according to the method proposed by Berzelius for the preparation of xanthine, and adopted with a slight modification by Mr. Higgin, there is produced a red precipitate, which after being washed and decomposed with sulphuretted hydrogen or sulphuric acid, gives a solution con-

taining rubian; but the presence of chlorogenine is also indicated by its turning dark green when boiled with the addition of sulphuric or muriatic acid. Hence it follows that chlorogenine, though it is not thrown down by basic acetate of lead when present alone in a solution, is still in part precipitated thereby when rubian is present at the same time. The same circumstance takes place with other precipitants.

After numerous experiments I discovered a property of rubian, which is perhaps more characteristic of it than any other, and that is the remarkable attraction which is manifested by it towards all substances of a porous or finely-divided nature, and it was this property by means of which I was at length enabled to obtain it in a state of purity. If to a watery extract of madder a quantity of protochloride of tin be added, a light purple lake is precipitated. Most of the rubian remains in the solution, which still retains its yellow colour and bitter taste. If, however, after filtering, sulphuretted hydrogen be passed through it, then, provided the quantity of tin still in solution be sufficiently large, the sulphuret of tin, at the moment of precipitation, carries down the whole of the rubian, and the solution loses its bitter taste and the greater part of its yellow colour. The whole of the chlorogenine remains in solution, and may easily be detected in the filtered liquid by means of acids. If the sulphuret of tin, after being collected on a filter and well washed with cold water until the percolating liquid no longer gives a green colour on being mixed with acid and boiled, be treated with boiling alcohol, a yellow solution is obtained, which on evaporation gives pure rubian, without any admixture of chlorogenine, in the shape of a dark yellow, brittle substance. The same effect is produced by sulphuret of lead. If sugar of lead be added to an extract of madder, a dark reddish-brown precipitate falls, the liquid still containing the rubian of the extract, as seen by its deep yellow colour and bitter taste. If sulphuretted hydrogen be now passed through the filtered liquid, a great part of the rubian goes down with the sulphuret of lead, and may again be separated from it by means of boiling alcohol. That this action of the sulphurets on rubian depends very much on their state of division, and is therefore mainly of a mechanical, and not chemical nature, is proved by the fact, that the sulphurets of tin and lead, if prepared by precipitation from solutions of salts in water, and then allowed to settle and repose for some time before being added to a watery extract of madder, remove far less rubian from it than they do, if they are formed in the extract itself, whence it follows that it is only in the minute state of division, in which they exist at the moment of precipitation, before the particles have time to cohere, that these sulphurets exert any great attraction for rubian. That they do however combine with some portion of the rubian is proved by the fact, that the power of dyeing in an extract of madder is very much diminished by adding to it sulphuret of tin or lead, previously precipitated. Of the two sulphurets, the sulphuret of tin, which is always precipitated in much finer particles than the other, is by far the most powerful absorbent of rubian. If equivalent quantities of protochloride of tin and acetate of lead be added to equal measures of watery extract of

madder, the sulphuret of tin from the former absorbs at least twice as much rubian as the sulphuret of lead from the latter. Sulphuret of copper acts differently. If sulphate of copper be added to the extract of madder, a precipitate is produced, as in the case of almost all metallic salts. On passing sulphuretted hydrogen through the filtered liquid, the latter becomes dark brown, but no sulphuret of copper is precipitated.

This attraction of surface exerted towards rubian by bodies in a state of minute division is not confined to metallic sulphurets. There are few bodies which exceed animal charcoal in porosity, or which, in other words, possess for the same bulk a greater extent of surface. I found accordingly that animal charcoal exhibits a still greater attraction for rubian than even sulphuret of tin. A very small quantity of animal charcoal is sufficient to deprive an aqueous extract of madder of its bitter taste and of its tinctorial power. Lamp-black acts in the same manner, though much less powerfully. Wood charcoal however has no absorbent effect whatever on rubian. All these substances attract rubian alone, leaving the other substances contained in the extract, such as chlorogenine, sugar and pectine, untouched. By means of boiling alcohol part of the rubian in combination with them is again removed, and thus an easy and efficient means is given of obtaining rubian in a state of purity. Of these substances none is so well adapted in all respects as animal charcoal. In employing sulphuret of tin, which is the only one that at all approaches it in efficiency, much time is consumed in the process of filtration and washing. Besides this, I found that on operating with it on a large scale, the rubian obtained was in great part decomposed on evaporating the alcoholic solution, just as if it contained a quantity of acid; and even on treating a portion of the solution with carbonate of lime, for the purpose of neutralizing any free acid that might be present, and evaporating over sulphuric acid at the ordinary temperature, there was obtained a deliquescent mass, which, as further experiments showed, could not be considered as pure rubian. After many trials I at length adopted the following method of preparation, which surpasses all others in facility and certainty of execution.

A weighed quantity of madder* being placed on a piece of calico or fine canvas stretched on a wooden frame, boiling water, which is preferable to cold water, as all decomposition of the rubian by means of fermentation is thereby arrested, is poured on it, four quarts of the latter being sufficient for every pound of madder. A dark yellowish-brown liquor is obtained, to which there is added, while hot, for every pound of madder taken 1 ounce of animal charcoal, prepared in the usual way from bones. This proportion of charcoal should not be exceeded, for if an excess of it be taken, as for instance $1\frac{1}{2}$ ounce for every pound of madder, the whole of the rubian is certainly removed from the solution; but on afterwards treating the charcoal with alcohol very little rubian is dissolved, from which it appears that the solvent power of the alcohol only overcomes the attraction of the charcoal for rubian in part. In

* Avignon madder, of the variety called *Rosa*, was the kind used.

using the first proportion, part of the bitter taste of the extract remains, showing that the rubian is in excess. The liquid being well stirred with the charcoal, the latter is allowed to settle, which it does in a very short time, and the liquid, which still retains a brown colour, is decanted. The charcoal is then placed on a piece of calico or on a paper filter and washed with cold water, until the percolating liquid, when mixed with muriatic acid and boiled, no longer acquires a green colour, which is a sign that the chlorogenine is removed. These operations occupy a very short time, in consequence of the rapidity with which the animal charcoal may be washed. The animal charcoal is now treated with boiling alcohol, which is filtered boiling hot, and the treatment is repeated until it no longer communicates to the alcohol any yellow colour. The rubian obtained by evaporating the alcoholic liquid is however impure; it contains a considerable quantity of chlorogenine, however carefully the charcoal may have been washed with water, and consequently gives a green powder when treated with boiling sulphuric or muriatic acid. This proceeds from the circumstance that fresh animal charcoal, when used in the preparation of rubian, invariably takes up, besides rubian, a quantity of chlorogenine, which is not removable by cold water, but which afterwards dissolves together with the rubian in boiling alcohol. Nevertheless, on using the charcoal which has been once employed, after treatment with alcohol, a second time for the same purpose, it seems to take up rubian alone and no chlorogenine, notwithstanding its being, as might be supposed, in the same condition for again absorbing the latter as it was in the first instance. At all events, the alcohol dissolves only rubian out of the charcoal, when it is used a second time; and if the alcohol should still contain chlorogenine, there will certainly not be a trace of the latter in the alcoholic solution, when the charcoal is used for the third time. That the attraction of the charcoal for rubian is not diminished after it has been once used and then exhausted with alcohol, however indifferent it then becomes towards chlorogenine, is proved by the fact that far more rubian is obtained when the charcoal is employed for the second time than in the first instance. If the animal charcoal, after being once used and exhausted, be heated red-hot so as to destroy all organic matter contained in it, it again behaves towards the two substances in the same manner as in the first instance, that is, it absorbs a mixture of rubian and chlorogenine. It is therefore advisable to reject the rubian which is obtained from the charcoal that has been used for the first time*. If a small portion of the alcohol with which the charcoal has been treated no longer gives a green colour when mixed with acid and boiled, but remains of a pure yellow, it is distilled or evaporated. During evaporation a small quantity of a dark brown flocculent substance is deposited, which is separated by filtration. The solution now contains, besides rubian, another substance in small quantity, which is a product of decomposition of rubian itself, and is

* This impure rubian cannot be purified by means of basic acetate of lead, since when rubian is present in a solution together with chlorogenine, the latter is, though not entirely, still in great part precipitated together with the rubian by that salt.

probably formed by the application of too great a heat in the process of drying the madder. There are two ways in which this substance may be removed. The first consists in adding to the solution sugar of lead, which precipitates it in dark reddish-brown flocks. These being separated by filtration, the rubian is precipitated by means of basic acetate of lead, and the light red compound or lake, after being washed with alcohol to remove all excess of lead salt, is decomposed either with sulphuretted hydrogen, or better still with sulphuric acid, the excess of the latter being removed by carbonate of lead. The other method, which is more expeditious, consists in adding sulphuric acid to the cold solution, after the greatest part of the alcohol has been evaporated. The sulphuric acid completely decomposes the foreign substance, provided a sufficient quantity is employed, and converts it into a substance which renders the solution milky, and then falls in the shape of brown resin-like drops. The sulphuric acid being neutralized with carbonate of lead, the filtered solution, which is yellow and now contains pure rubian, is evaporated to dryness. It is necessary to employ carbonate of lead, and not carbonate of baryta, for the neutralization of the sulphuric acid in both cases; for if carbonate of baryta be used, the bicarbonate of baryta which is usually formed, even if present only in small quantity, causes part of the rubian to undergo decomposition. In evaporating the solution of rubian, care must be taken not to employ too great a heat when the evaporation approaches to a conclusion. The ordinary heat of a sand-bath is sufficient to decompose rubian in great part, especially if a large quantity of the substance be present. It is therefore advisable, when the solution is nearly evaporated, to complete the evaporation either in a water-bath or in a moderately warm place. The free access of atmospheric air need not be feared, as rubian is not thereby decomposed, unless some other substance be present at the same time. The quantity of rubian which I have obtained, according to this method of preparation, amounts to about 1000 grs. from 1 cwt. of madder. It may be mentioned that the method of preparing rubian, as above described, by means of animal charcoal and alcohol, is not new in principle. Lebourdais* has proposed the same method for the preparation of several vegetable substances, such as colocynthine, strychnine, quinine, &c.

Properties of Rubian.—When prepared according to the method just described, rubian is obtained as a hard, dry, brittle, shining, perfectly uncrystalline substance, similar in appearance to gum or dried varnish. It is not in the least deliquescent, as xanthine is described to be. In thin layers it is perfectly transparent and of a beautiful dark yellow colour. In large masses it appears dark brown. It is very soluble in water and alcohol, more so in the former than the latter, but insoluble in ether, which precipitates it from its alcoholic solution in brown drops. Its solutions have an intensely bitter taste. When it is pure, its solution in water gives no precipitates with the mineral or organic acids, nor with salts of the alkalies or alkaline earths.

* On the Nature and Preparation of the Active Principles of Plants, Annal. de Chim. et de Phys. 3me ser. t. xxiv. p. 58.

Acetate of alumina, alum, protacetate and peracetate of iron, acetate of zinc, neutral and basic acetate of copper, acetate of lead, nitrate of silver, perchloride of tin, protonitrate of mercury, perchloride of mercury and chloride of gold produce no precipitate whatever in a watery solution of pure rubian, nor does any reaction take place, except a darkening of the solution in the case of some of these salts. If the rubian be impure, which is always the case when the solution has been incautiously evaporated and the rubian has been exposed to too great a heat after evaporation, then its solution, though it does not differ in appearance from one of pure rubian, when mixed with any mineral or organic acid, even acetic acid, or the salts of the alkalies or alkaline earths, is rendered milky, and a quantity of dark brown transparent resinous drops, mixed with yellow flocks, are deposited. These drops, in the case of the salts, consist merely of a substance insoluble in saline liquids, which dissolves again in pure water; but in the case of acids, they are, though similar in appearance, a product of decomposition of the latter substance, and do not redissolve in pure water. Sugar of lead gives, in a solution of impure rubian, a dark reddish-brown precipitate. Most metallic salts also give precipitates, consisting either of the substance itself which accompanies the rubian, or of compounds of this substance, with the respective metallic oxides. I shall return to these reactions when I come to treat of the action of heat on rubian. Basic acetate of lead gives a copious light red precipitate in a solution of pure rubian, the solution becoming colourless. This is the only definite compound of rubian with a base that I am acquainted with. Concentrated sulphuric acid dissolves rubian with a blood-red colour; on boiling the solution it becomes black and disengages sulphurous acid gas in abundance, after which water precipitates a black carbonaceous mass. If sulphuric acid be added to a watery solution of rubian, and the mixture be boiled, the solution, if dilute, becomes opalescent, and on cooling a quantity of light yellow flocks are deposited; and if the solution was concentrated, these are formed in such abundance on cooling as to render the liquid thick. If these flocks exhibit the least tinge of green, the presence of chlorogenine is indicated. Muriatic acid acts in precisely the same manner. Nitric acid produces in the cold no effect in a solution of rubian, but on boiling a disengagement of nitrous acid takes place, the liquid becomes light yellow, and now contains the acid which I called in my former papers alizaric acid, and which Laurent and Gerhardt consider as identical with naphthalic acid. Phosphoric, oxalic, tartaric and acetic acids produce no effect on the solution, even on boiling for some time. When a stream of chlorine gas is passed through a watery solution of rubian, the solution immediately becomes milky and begins to deposit a lemon-yellow powder, into which, on continuing the action, the whole of the rubian is converted, the liquid becoming colourless. Caustic soda turns the colour of the solution from yellow to blood-red, and on neutralizing the alkali with acid, a clear yellow solution is again obtained. By boiling the solution to which the soda has been added, the colour changes from blood-red to purple; and on now supersaturating the alkali with acid, a reddish yellow precipitate falls, while the

supernatant liquid becomes almost colourless. Ammonia changes the colour of a solution of rubian to blood-red; the colour is not changed by boiling; and by supersaturating the ammonia with acid either before or after boiling, no precipitate is formed. Lime and baryta water give dark red precipitates in a solution of rubian, which are soluble in pure water, forming dark red solutions. Magnesia turns the solution dark red; the solution contains magnesia. The carbonates of lime and baryta produce no perceptible effect on a solution of rubian; they do not change its colour, nor do they take up any rubian. Hydrate of alumina, when placed in a solution of rubian, acquires a brownish-yellow colour. If sufficient alumina be taken, the liquid is rendered almost colourless. Hydrated peroxide of iron acts in a similar manner. Oxide of copper also removes most of the rubian from its solution. Alkaline solutions of rubian do not reduce the oxides of silver and copper on the addition of salts of these oxides, but they reduce salts of gold to the metallic state. When heated on platinum foil, rubian melts, swells up very much, burns with a flame and gives a carbonaceous residue, which does not entirely disappear on being further heated, but leaves a quantity of ash. When heated gradually in a tube it begins to undergo decomposition, accompanied by loss of water at a temperature of about 130° C., and is converted into another substance, which I shall describe further on. When heated to a still higher degree in a tube or retort, it gives fumes of an orange colour, which condense on the colder parts of the vessel to a crystalline mass, consisting chiefly of alizarine.

Rubian cannot be considered as a colouring matter in the ordinary sense of the word. It imparts hardly any colour to mordanted cloth, when an attempt is made to dye with it in the usual way, the alumina mordant only acquiring a slight orange, the iron mordant a light brown colour.

Composition of Rubian.—In determining the composition of rubian, I found it necessary to take into consideration the fact of its leaving when burnt a considerable quantity of ash. This ash consists almost entirely of carbonate of lime. The amount of ash is not uniform in different specimens; it is greatest when the rubian has been purified by means of sulphuric acid, but I have never been able to obtain it in a state in which it burns without any residue. Even after being precipitated with basic acetate of lead and again separated from the oxide of lead, rubian leaves some ash on being burnt, so that it appears as if the lime which it contains were an essential constituent, or at all events, that it follows it into the lead compound, from which it cannot be removed by means of water or alcohol.

The following results were obtained on analysis:—

I. 0·3880 grm. rubian, which had been purified by means of sulphuric acid, dried at 100° C., gave, when burnt with oxide of copper, 0·7210 carbonic acid and 0·1745 water.

II. 0·4780 grm. of the same preparation, burnt with oxide of copper, gave 0·8865 carbonic acid and 0·2180 water.

III. 0·4755 grm. of the same preparation, burnt with oxide of copper, gave 0·8835 carbonic acid and 0·2180 water.

0·1690 grm., on being incinerated, left 0·0130 grm. of ash=7·69 per cent.

IV. 0·3910 grm. rubian, purified by means of acetate and basic acetate of lead, burnt with chromate of lead, gave 0·7455 carbonic acid and 0·1880 water.

0·4050 grm. of this preparation left 0·0215 grm. of ash=5·30 per cent.

V. 0·4235 grm. rubian, purified in the same way as I. and burnt with chromate of lead, gave 0·7890 carbonic acid and 0·2020 water.

0·6400 grm. of this preparation left 0·0465 ash=7·26 per cent.

VI. 0·4390 grm. rubian, purified in the same way as IV., burnt with chromate of lead, gave 0·8370 carbonic acid and 0·2120 water.

0·8400 grm. of this preparation left 0·0440 ash=5·23 per cent.

After making the necessary corrections for the ash, these numbers correspond in 100 parts to—

	I.	II.	III.	IV.	V.	VI.
Carbon	54·89	54·79	54·89	54·90	54·78	54·84
Hydrogen . . .	5·41	5·48	5·51	5·64	5·71	5·66
Oxygen	39·70	39·73	39·60	39·46	39·51	39·50

Rubian contains no nitrogen. On burning it with oxide of copper and collecting the gas over mercury, I found the latter to be entirely absorbed by caustic alkali. When burnt with lime and soda, only a minute trace of chloride of platinum and ammonium was obtained. The statement contained in my former paper, which was made at a time when I had not obtained rubian in a state of absolute purity, that nitrogen is one of its constituents, must therefore be corrected.

From the above analyses the following composition may be deduced:—

	Eqs.		Calculated.
Carbon	56	336	55·08
Hydrogen	34	34	5·57
Oxygen	30	240	39·35
		610	100·00

The compound with oxide of lead, which was the only one that could be employed for the determination of the atomic weight, was prepared by dissolving rubian in alcohol, adding acetate of lead, precipitating with a little ammonia, taking care to leave an excess of rubian, and washing with alcohol. If it be prepared by precipitation from a watery solution by means of basic acetate of lead, great difficulties are experienced in the course of filtration; the liquid begins to run through slowly, the precipitate becomes somewhat mucilaginous and adheres to the paper, and sometimes even it seems to be decomposed and no longer gives unchanged rubian, but a dark brown viscid substance. Its analysis gave the following results:—

I. 0·3670 grm., dried at 100° C. and burnt with chromate of lead, gave 0·3520 carbonic acid and 0·0875 water.

0·3360 grm. gave 0·2390 sulphate of lead.

II. 0·4440 grm. of another preparation, burnt with chromate of lead, gave 0·4190 carbonic acid and 0·1115 water.

0·4320 grm. gave 0·3100 sulphate of lead.

III. 0·4635 grm. of the same preparation as the last gave 0·4450 carbonic acid and 0·1050 water.

0·5405 grm. gave 0·3880 sulphate of lead.

These numbers lead to the following composition:—

	Eqs.		Calculated.	I.	II.	III.
Carbon	56	336	26·25	26·15	25·73	26·18
Hydrogen	34	34	2·65	2·64	2·87	2·51
Oxygen	30	240	18·76	18·89	18·62	18·51
Oxide of lead . . .	6	670	52·34	52·32	52·78	52·80
		1280	100·00	100·00	100·00	100·00

Hence it appears that oxide of lead in combining with rubian does not replace any basic water, as is usually the case.

It may easily be conceived that a body so readily decomposed as rubian gives a number of different products of decomposition. It is decomposed by acids, alkalies, chlorine, heat and ferments; and I shall now proceed to describe the products of decomposition to which these various reagents give rise.

Action of Sulphuric and Muriatic Acid on Rubian.—The action of these two acids is precisely the same; but for the purpose of studying it, it is better to employ sulphuric acid, as it is more easily removed again afterwards. On adding sulphuric acid in considerable quantity to a watery solution of rubian and boiling the liquid, no perceptible change takes place at first, except that the solution loses a little of its transparency and becomes slightly opalescent. If the solution was not very dilute, there begin to appear very soon a number of orange-coloured flocks. After boiling for some time and allowing to cool, these flocks are deposited in large quantities, and the liquid is now found to be much lighter in colour than before. After allowing to cool and filtering, the liquid, on being mixed with fresh acid and boiled again, often deposits on cooling a fresh quantity of these flocks. When after repeated boilings no more flocks separate on cooling, the process is completed. The last portions of rubian are usually more difficult to decompose than the first, and an additional quantity of acid is therefore necessary to effect their decomposition. The liquid retains to the last a light yellow colour. I shall return to it presently. The orange-coloured flocks are washed on the filter with cold water until all the acid is removed. They now consist of four different substances, three of which are bodies previously known,

the fourth one which has not hitherto been observed. The three former are,—1st, *Alizarine*; 2ndly, the substance which in my former papers I have called *alpha-resin*, but to which I prefer giving the name of *Rubiretine*; 3rdly, the substance which I formerly termed *beta-resin*, but I shall now call *Verantine* from *Verantia*, the name applied to madder in the middle ages. The fourth substance I shall denominate *Rubianine*.

The presence of alizarine in this mixture is indicated by the dark and beautiful colours which are produced when it is employed for dyeing a piece of mordanted cloth, and which contrast forcibly with the faint and dull tints produced by rubian. It may also easily be separated from the other substances by dissolving the mixture in alcohol, adding hydrate of alumina to the solution, filtering, treating the alumina compound repeatedly with a solution of carbonate of potash or soda, until nothing more is dissolved by the latter, decomposing the alumina compound with acid, and dissolving the residue in alcohol, when on evaporating the latter crystals of alizarine with its usual characters are obtained. In order however to obviate all objections which might arise from the use of alkalies in regard to the effect which the latter might be supposed to have in causing the formation of the alizarine, I determined if possible to use acids and salts only in the separation of the substances mentioned above. Of the four substances contained in the orange-coloured flocks, two, viz. alizarine and rubianine, are soluble in boiling water, and may thereby be separated from the two others which are insoluble in water. This method of separation is however tedious, on account of the sparing solubility of alizarine and rubianine in boiling water. I therefore prefer using the following method. The orange-coloured flocks containing the four substances are treated with boiling alcohol, in which they dissolve with a dark reddish-yellow colour. The alcohol is filtered boiling hot, and deposits on cooling a small quantity of yellow crystalline particles, consisting chiefly of rubianine. The treatment with alcohol is repeated as long as the latter acquires a dark yellow colour. The greatest part of the rubianine remains behind as a yellow or brownish-yellow crystalline mass, which is treated repeatedly with boiling alcohol, in which the whole at last dissolves, the greatest part again separating on the solution cooling, either in yellow needles or as a brownish-yellow crystalline mass. If its colour is not a pure yellow, or if it is imperfectly crystallized, it contains verantine and must be purified. For this purpose the whole of the mass which has been deposited on the alcohol cooling, after being collected on a filter, is again dissolved in boiling alcohol, and sugar of lead is added to the solution, by which means the verantine is precipitated in combination with oxide of lead, while the rubianine remains in solution and is again deposited, when the solution, after being filtered boiling hot, is allowed to cool, in long, lemon-yellow silky needles, which may be rendered perfectly pure by recrystallization. The compound of verantine and oxide of lead may be decomposed with sulphuric acid, and the verantine separated from the sulphate of lead by boiling alcohol. The alcoholic liquid from which the rubianine has been deposited contains

the three other substances besides a portion of the rubianine. By adding acetate of alumina to it, the whole of the alizarine as well as a part of the verantine are precipitated, in combination with alumina, in the shape of a dark red powder, while the liquid retains a dark brownish-red colour. This precipitate, after being collected on a filter and washed with alcohol until the latter runs through colourless, is decomposed with muriatic acid, which dissolves the alumina, leaving behind red flocks consisting of alizarine and verantine. These flocks, after being filtered off and washed with water, are again dissolved in alcohol, to which is then added a solution of neutral acetate of copper. This instantly changes the colour of the liquid to a beautiful dark purple. The copper compound of alizarine remains dissolved, while the verantine is entirely precipitated, in combination with oxide of copper, as a dark reddish-brown powder. The dark purple liquid, after filtration and evaporation, leaves a purple mass of alizarine-oxide-of-copper, which is decomposed with muriatic acid. Yellow flocks, consisting of alizarine, remain behind, which after being washed with water are dissolved in alcohol. The alcoholic solution on evaporation gives crystals of alizarine, which may be purified by recrystallization. The compound of verantine with oxide of copper is decomposed with muriatic acid. The liquid filtered from the alumina compound of alizarine and verantine is evaporated to dryness, muriatic acid is added to the residue, which is placed on a filter and washed with cold water until all the acid and salts of alumina are removed. On being now treated with boiling water, a quantity of dark brown resinous drops sink to the bottom of the vessel and cohere into a semi-fused mass, while brownish-yellow flocks float in the water. The water is decanted from the mass at the bottom, carrying with it the flocks. This process is repeated with fresh quantities of water until no more flocks are carried away by it. The resinous mass at the bottom now consists principally of rubiretine. It may be purified by dissolving in cold alcohol, which leaves behind a quantity of verantine. The brownish-yellow flocks consist chiefly of verantine and rubianine; they are treated with boiling water, in which the rubianine dissolves, and from which it is again deposited, on filtering the water boiling hot and allowing to cool, in orange-coloured flocks. The process is repeated until the water dissolves nothing more. The orange-coloured flocks of rubianine are collected on a filter and dissolved in boiling alcohol, out of which the rubianine crystallizes on cooling in yellow needles. The mother-liquor is somewhat darker than a mere solution of rubianine would be. It contains a little alizarine and rubiretine, which may be separated by means of acetate of alumina, as before described. The verantine which is left behind by the boiling water is mixed with the other portions obtained from the lead and copper compounds, and the whole is dissolved in a small quantity of boiling alcohol, out of which the verantine is deposited on cooling as a dark reddish-brown or yellowish-brown powder, which may be purified by a second solution in alcohol.

These substances can, as may be supposed, be obtained without any difference in properties by adding sulphuric or muriatic acid to an extract of madder made with

boiling water, boiling the liquid, and treating the dark green precipitate obtained in the same way as the orange-coloured flocks, from the decomposition of rubian. The dark green colour of the precipitate in this case proceeds from the decomposition of chlorogenine by the acid; the product of decomposition does not however in any way interfere, as it is insoluble in alcohol. It may be remarked, however, that very little rubianine is obtained in this manner, its place being supplied, from a cause which I shall mention hereafter, by rubiacine.

There still remains in the acid liquid filtered from the orange-coloured flocks, a substance which is an essential product of the action of acids on rubian. This liquid has, as I mentioned before, a light yellow colour. After neutralizing the acid with carbonate of lead it becomes almost colourless, while the carbonate of lead acquires a pink tinge. After filtration it is found to contain neither sulphuric acid nor lead; nor does it give any precipitate with neutral or basic acetate of lead, nor with alkalies, either before or after neutralization, unless it be boiled with an excess of the latter. This absence of reaction proves that no substance of a basic nature has been formed during the process. The liquid however contains a considerable quantity of an organic substance, which is obtained by carefully evaporating at the ordinary temperature over sulphuric acid. It is not advisable to evaporate with the assistance of heat, as the solution then becomes dark brown from the action of the air. After evaporation over sulphuric acid there is left at last a brownish-yellow, transparent syrup, having a sweetish taste, which I shall prove by its properties and composition to be a species of sugar.

I shall now describe more in detail the properties of the substances just mentioned.

Alizarine.—The alizarine obtained from the decomposition of rubian exhibits all the usual properties of this well-known substance. Its colour is dark yellow without any tinge of brown or red. The crystals possess a lustre which I have never seen equalled in this substance. Its analysis gave the following results:—

0·3200 grm. of the crystals, on being heated in the water-bath, lost 0·0580 grm. of water=18·12 per cent. According to the formula $C_{14}H_5O_4+3HO$, they should lose 18·24 per cent.

0·2575 grm. of the dry substance, burnt with chromate of lead, gave 0·6550 carbonic acid and 0·0945 water.

These numbers lead to the following composition:—

	Eqs.		Calculated.	Found.
Carbon	14	84	69·42	69·37
Hydrogen	5	5	4·13	4·07
Oxygen	4	32	26·45	26·56
		121	100·00	100·00

0·1050 grm. of the lead compound, prepared by precipitating the alcoholic solution with sugar of lead, gave 0·0720 sulphate of lead, equivalent to 0·0529 oxide of lead

=50·44 per cent. The formula $C_{14}H_4O_3+PbO$ requires 49·90 per cent. oxide of lead.

The formula here given is the same to which I was led by my former experiments, and it now receives a new confirmation from the relation in which it stands to that of rubian.

The formation of alizarine from rubian admits of a very easy explanation. By simply losing 14 equivs. of water, 1 equiv. of rubian is converted into 4 equivs. of alizarine, as the following equation shows:—

$$C_{56}H_{34}O_{30}=4(C_{14}H_5O_4)+14HO.$$

The action of sulphuric acid in the preparation of garancine from madder now becomes more intelligible. It consists simply, as far as the practical effect is concerned, in the conversion of rubian into alizarine.

MM. Wolff and Strecker, in a late paper 'On the Red Colouring Matters of Madder*,' have given another formula for alizarine, which they prefer on account of the relation in which they suppose this substance to stand to naphthaline. This formula is $C_{20}H_6O_6$, which requires in 100 parts—

Carbon	68·96
Hydrogen	3·45
Oxygen	27·59

In confirmation of this formula they adduce one analysis, in which they obtained from 0·0650 grm. alizarine 0·163 carbonic acid, equivalent to 68·4 per cent. If it be permitted to deduce any safe inference from the analysis of so small a quantity of substance, I should be inclined to say that the substance analysed was impure. Even when perfectly well crystallized, alizarine may contain an amount of impurity sufficient to affect its composition. This impurity generally consists of verantine. A large admixture of the latter substance entirely prevents alizarine from crystallizing, but a small quantity merely gives the crystals a brownish or reddish tinge. Alizarine can never be considered as perfectly pure unless it exhibits a pure dark yellow colour without admixture of red. In proof of this statement I may adduce the following experiments. In the course of my investigation I obtained from madder a specimen of alizarine in perfectly well-defined crystals, containing apparently no foreign substance, but having a brownish-red colour instead of the dark yellow characteristic of pure alizarine.

0·3530 grm. of these crystals, dried at 100° C., gave 0·8855 carbonic acid, equivalent to 68·41 per cent. of carbon.

The remainder of the substance was recrystallized from alcohol, and

0·2940 grm. now gave 0·7360 carbonic acid, equivalent to 68·27 per cent. of carbon.

On dissolving the rest in alcohol and adding to the solution acetate of copper, a dark reddish-brown precipitate of verantine-oxide-of-copper fell. From the dark

* Ann. der Chem. u. Pharm., vol. lxxv. p. 1.

purple solution, after being treated in the manner before described, there were obtained some beautiful dark yellow crystals of alizarine, which on analysis yielded the following result:—

0·1270 grm., dried at 100° C., gave 0·3245 carbonic acid, equivalent to 69·68 per cent. of carbon.

It appears therefore that alizarine cannot be separated from the last portions of impurity by recrystallization merely.

Were the above formula the correct one, it would be difficult to account for the circumstance that the purer the substance the greater is the excess, not only of hydrogen, but also of carbon. An excess of 0·6 per cent. of hydrogen, as this formula would pre-suppose, is unusually large; an excess of 0·4 per cent. of carbon seldom or never obtained in the analysis of a pure substance. The most common impurity of alizarine is verantine; and since the latter contains, as I shall presently show, more oxygen for the same amount of carbon and hydrogen, it follows that if a portion of it be mixed with the alizarine, the amount of carbon and hydrogen in the latter will be reduced, and the composition will approximate to that given by Wolff and Strecker.

Again, if the correct formula for alizarine be $C_{20}H_6O_6$, the formula of rubian must necessarily be $C_{20}H_{11}O_{11}$, which requires in 100 parts—

Carbon	54·79
Hydrogen	5·02
Oxygen	40·19

These numbers, as will be perceived, do not agree so well with those of the analyses as those corresponding to the formula which I have given above. The lead compound of rubian can, under this supposition, only be represented by the formula $6(C_{20}H_{11}O_{11})+13PbO$, which requires in 100 parts—

Carbon	26·03
Hydrogen	2·38
Oxygen	19·10
Oxide of lead	52·49

Here also it will be seen there is less accordance with the numbers found by experiment than in the case of the other formula. But besides this, the latter formula is of too complicated a nature to be received; I therefore consider the formula $C_{14}H_5O_4$, or perhaps $C_{28}H_{10}O_8$, for alizarine to be as firmly established as it can be with the means at present at our command.

Verantine.—This substance coincides in most of its properties with the substance to which I formerly gave the name of the beta-resin of madder. When prepared according to the method above described, it is obtained in the shape of a reddish-brown powder, similar in colour to snuff or roasted coffee. It has the following properties. When heated on platinum-foil it melts, and then burns away without leaving any residue. When heated in a glass tube it gives a small quantity of an oily sub-

limate without a trace of anything crystalline. When however it contains alizarine, as it very often does, it gives on being heated a crystalline sublimate consisting of the latter substance. It dissolves in concentrated sulphuric acid with a brown colour, and is reprecipitated by water in brown flocks. On heating the solution in concentrated sulphuric acid it becomes black, sulphurous acid is disengaged, and the substance is decomposed. Concentrated nitric acid dissolves it on boiling with a disengagement of nitrous acid, forming a yellow liquid, from which nothing separates on cooling. Dilute nitric acid does not affect it sensibly on boiling. It is almost insoluble in boiling water, but readily soluble in boiling alcohol with a dark brownish-yellow colour, and is again deposited, on the alcohol cooling, as a brown powder, which is its most characteristic property. It is soluble in alkaline liquids with a dirty brownish-red colour, and is reprecipitated by acids in brown flocks. If it be mixed with alizarine, then its solutions in alkalies have a reddish-purple colour. The ammoniacal solution loses its ammonia on evaporation, and leaves the substance behind as a brown transparent pellicle. The ammoniacal solution gives precipitates with the chlorides of barium and calcium. The alcoholic solution gives dark brown precipitates with the acetates of lead and copper, as I mentioned before. When it is free from alizarine, it does not communicate any colour to mordanted cloth, and is therefore no colouring matter in the usual sense.

In the opinion of most chemists who have examined madder, this root contains two distinct colouring matters, viz. alizarine and another, to which the names of *purpurine*, *oxylizaric acid* and *madder-purple* have been applied by different chemists. This opinion has been advocated with considerable ability by MM. Wolff and Strecker. I have however reason to suppose that purpurine is in fact no distinct substance, but a mixture of alizarine and verantine. The latter substance accompanies almost all the products which are obtained from madder, and it is this body which renders them so difficult to purify. It adheres so pertinaciously to alizarine, as to induce the belief that the two actually form a chemical compound. The mixtures of the two vary in appearance from that of dark red crystals to that of a red crystalline powder. In these mixtures the verantine may easily be detected by dissolving in alcohol and adding acetate of copper, which precipitates the verantine, as before described. It also accompanies rubianine and renders it difficult to crystallize, as I mentioned above, and I have never been able to obtain rubiretine without some trace of it. As a characteristic of purpurine is mentioned its property of giving a cherry-red solution with alkalies, having none of the violet appearance belonging to alkaline solutions of alizarine; and also its forming, when treated with boiling alum-liquor, a red opalescent solution, from which it separates again in orange-coloured flocks on the solution cooling. Now by adding to a solution of alizarine in caustic alkali a little verantine, the beautiful violet colour of the solution may be instantly changed to reddish-purple; and by dissolving in it still more of that substance the colour may be rendered cherry-red, these colours being evidently mixtures of the violet due to aliza-

rine and the brownish-red produced by verantine. Pure alizarine is not more soluble in boiling alum-liquor than in water, as has been repeatedly shown; it only communicates to the liquor a yellow colour, and crystallizes out again on the liquid cooling. Verantine is still less soluble in alum-liquor. If this substance be dissolved in caustic alkali and be then precipitated with a solution of alum, the precipitate does not dissolve in the least degree, however much alum be added; it only communicates a slight yellow tinge to the liquid. If, however, a mixture of alizarine and verantine be dissolved in caustic alkali and they be then precipitated together by means of a solution of alum added in excess, then on boiling the precipitate with the liquid, a bright red solution is obtained, and on filtering and allowing to cool, orange-coloured flocks are deposited, while the liquid still remains red, but gives a yellow precipitate on the addition of acid. By treating the residue with additional quantities of alum-liquor more is dissolved with the same colour, and this continues until either the alizarine or the verantine, whichever of the two was present in the smallest quantity, is removed. From this experiment I am inclined to conclude that alizarine and verantine are capable of forming a double compound with alumina soluble in boiling water, and that a mixture of the two in the proportion in which they exist in this compound, constitutes what has been called purpurine. At all events, it follows that alum is not adapted as a means of separating the substances derived from madder. The fact of rubianine also dissolving in boiling alum-liquor and crystallizing out again on cooling, is an additional objection to its use.

The difficulty of obtaining pure verantine in sufficient quantity for the purposes of analysis, has prevented me from determining its composition with the requisite accuracy. I have however obtained approximations sufficiently near to remove almost all doubts on the question.

I. 0·3280 grm. verantine, dried at 100° C. and burnt with chromate of lead, gave 0·7865 carbonic acid and 0·1215 water.

II. 0·3220 grm. gave 0·7740 carbonic acid and 0·1205 water.

III. 0·2890 grm. gave 0·6995 carbonic acid and 0·1040 water.

IV. 0·1255 grm. gave 0·3010 carbonic acid.

These numbers agree best with the following composition:—

	Eqs.		Calculated.	I.	II.	III.	IV.
Carbon	14	84	65·11	65·39	65·55	66·01	65·41
Hydrogen . . .	5	5	3·87	4·11	4·15	3·99	
Oxygen	5	40	31·02	30·50	30·30	30·00	
		129	100·00	100·00	100·00	100·00	

The composition here given approaches that of the oxylizaric acid of Debus, who obtained in analysing that substance as a mean of his experiments in 100 parts,—

Carbon	66·40
Hydrogen	3·82
Oxygen	29·78

The baryta compound, prepared by precipitating the ammoniacal solution with chloride of barium, gave on analysis the following results:—

0·2605 grm., dried at 100° C. and burnt with chromate of lead, gave 0·4640 carbonic acid and 0·0740 water.

0·4055 grm. gave 0·1835 sulphate of baryta.

In 100 parts:—

Carbon	48·57
Hydrogen	3·15
Oxygen	18·60
Baryta	29·69

The formula $C_{42}H_{13}O_{13}+2BaO=2(C_{14}H_4O_4+BaO)+C_{14}H_5O_5$ requires in 100 parts—

Carbon	48·27
Hydrogen	2·49
Oxygen	19·93
Baryta	29·31

The compound with oxide of copper gave the following results:—

0·3680 grm., dried at 100° C. and burnt with chromate of lead, gave 0·7050 carbonic acid and 0·1030 water.

0·4710 grm. gave 0·1200 oxide of copper.

These numbers correspond very nearly to the following composition:—

	Eqs.		Calculated.	Found.
Carbon	14	84	52·50	52·24
Hydrogen	4	4	2·50	3·10
Oxygen	4	32	20·00	19·19
Oxide of copper	1	40	25·00	25·47
		160	100·00	100·00

Another specimen, prepared in exactly the same manner and having the same appearance, gave a different composition.

0·4375 grm. gave 0·8910 carbonic acid and 0·1345 water.

0·5530 grm. gave 0·1080 oxide of copper.

This gives the following composition:—

	Eqs.		Calculated.	Found.
Carbon	56	336	55·17	55·54
Hydrogen	17	17	2·79	3·41
Oxygen	17	136	22·34	21·53
Oxide of copper	3	120	19·70	19·52
		609	100·00	100·00

The formula of this compound must be expressed in the following manner:

$$3(C_{14}H_4O_4+CuO)+C_{14}H_5O_5.$$

These analyses show that verantine, like many substances, the acid character of which is not well-marked, combines with bases in various and complicated proportions.

It appears therefore that verantine differs from alizarine by containing 1 equiv. more of oxygen. According to Debus, the same relation exists between alizarine and his oxylizaric acid. He gives for alizarine the formula $C_{30}H_{10}O_8$, and for oxylizaric acid $C_{14}H_6O_6$, so that 2 quivs. of the latter contain 1 equiv. more oxygen than 1 equiv. of the former. If my view of the composition of these substances be the correct one, the relation subsisting between the two is still more simple. Nevertheless I have some hesitation in asserting that verantine is to be considered as a higher oxide of the same radical as alizarine, or in supposing that it may be formed by oxidation from the latter. Its formation is due, as I shall presently show, not to any process of oxidation, but rather to the splitting up of an atom of rubian into two bodies.

Rubiretine.—This substance is identical with that which I formerly called the alpha-resin of madder, from which it does not differ in properties. It is obtained as a dark brown, opake, resinous mass, brittle when cold, but becoming soft and almost melting in boiling water. On being heated to a higher degree, it melts completely without being decomposed. It is generally found to be mixed with a small quantity of verantine, from which it may be separated by solution in cold alcohol, which leaves the greatest part of the verantine behind; I have however found it impossible to remove the last traces of that substance. It is almost insoluble in boiling water. Its solution in alcohol is dark yellow. It dissolves in concentrated sulphuric acid with a yellowish-brown colour, and is decomposed on boiling the solution with blackening and disengagement of sulphurous acid. Boiling nitric acid changes it into a yellow substance, which no longer softens at the temperature of boiling water, and is very little soluble in alcohol. It dissolves in alkaline liquids with a brownish-red colour, and is reprecipitated by acids in brown flocks, which on boiling the liquid cohere into dark-brown semifluid masses. When heated in a glass tube, it usually gives a small quantity of sublimed alizarine mixed with a brown oil. It is not capable of dyeing when quite free from alizarine. Its analysis yielded the following results:—

I. 0·5300 grm. from the decomposition of rubian, dried at 100° C. and burnt with chromate of lead, gave 1·3190 carbonic acid and 0·2405 water.

II. 0·3785 grm. of the same preparation as the last, heated to the melting-point, gave 0·9465 carbonic acid and 0·1730 water.

III. 0·4815 grm. obtained directly from madder, dried at 100° C., gave 1·2130 carbonic acid and 0·2335 water.

IV. 0·4290 grm. of the same preparation as the last, heated to the melting-point, gave 1·0735 carbonic acid and 0·2050 water.

V. 0·3120 grm. of another preparation obtained directly from madder, gave 0·7855 carbonic acid and 0·1460 water.

VI. 0·2350 grm. of the same preparation as the last, gave 0·5865 carbonic acid and 0·1065 water.

In 100 parts it therefore contains—

	I.	II.	III.	IV.	V.	VI.
Carbon	67·87	68·19	68·70	68·24	68·66	68·06
Hydrogen . . .	5·04	5·07	5·38	5·30	5·20	5·03
Oxygen . . .	27·09	26·74	25·92	26·46	26·14	26·91

I endeavoured in vain to determine the atomic weight of this substance. Neither the lead nor the baryta compound gave results which harmonized either with one another or with the analyses of the substance itself. There is however only one formula which is in accordance with the analyses, and at the same time satisfactorily explains its formation. This formula is $C_{14} H_8 O_4$, which requires in 100 parts—

Carbon	68·85
Hydrogen	4·91
Oxygen	26·24

It may be remarked that this is also the composition of benzoic acid; and even if the formula of rubiretine should not be exactly that given above, but perhaps the double or triple of it, it still remains remarkable that two such very different substances should have the same percentary composition.

The formation of rubiretine from rubian can only be explained in connection with that of verantine. If 2 equivs. of verantine, 2 equivs. of rubiretine and 12 equivs. of water be added together, the sum will be equal to 1 equiv. of rubian, as follows:—

2 equivs. of Verantine	$= C_{28} H_{10} O_{10}$
2 equivs. of Rubiretine	$= C_{28} H_{10} O_8$
12 equivs. of Water	$= H_{12} O_{12}$
1 equiv. of Rubian	$= C_{56} H_{34} O_{30}$

If this be the correct representation, it follows that verantine and rubiretine stand in an intimate relation to one another, that the formation of one always indicates that of the other. In confirmation of this view, I may state that I have never seen the formation of one of these substances taking place without it being possible to detect the presence of the other.

Rubianine.—This substance, as I mentioned before, has not hitherto been observed among the bodies derived from madder. It greatly resembles rubiacine in its appearance and many of its properties; it may however easily be distinguished by several characteristics, and above all by its composition. It is obtained from a solution in boiling alcohol in the form of bright lemon-yellow, silky needles, which, when dry, form an interwoven mass. It is soluble in boiling water, more so in fact than any of the products of decomposition hitherto mentioned. It crystallizes out again on the solution cooling in yellow silky needles. It is less soluble in alcohol than the preceding substances. Its colour is lighter than that of rubiacine. When heated on platinum-foil it melts to a brown liquid, then burns, leaving a carbonaceous residue,

which on further heating disappears entirely. When heated in a glass tube it gives a small quantity of a yellow crystalline sublimate, but not by far so large a quantity as is obtained under the same circumstances from rubiacine, which, when carefully heated, may be almost entirely volatilized. It is soluble in concentrated sulphuric acid with a yellow colour; the solution on boiling becomes black and gives off sulphurous acid. A solution of rubiacine in concentrated sulphuric acid, remains quite unchanged on boiling. It is not affected either by dilute or concentrated nitric acid, even on boiling; it merely dissolves in them, and crystallizes out again on the acid cooling, just as from boiling water. When treated in the cold with a solution of carbonate of potash or soda, or liquid ammonia, it does not dissolve, nor is its colour at all changed. When the liquid is boiled, it dissolves however with a blood-red colour. Nevertheless it cannot be said to combine with the alkali, but merely to be dissolved by it; for on allowing these solutions to stand for some time, a yellow crystalline mass again separates, which is nothing but the substance itself. The ammoniacal solution gives red precipitates with the chlorides of barium and calcium. The alcoholic solution gives no precipitate with sugar of lead, whereas a solution of rubiacine gives a dark red precipitate with sugar of lead. It dissolves in a concentrated solution of perchloride of iron with a dark brown colour, but is not thereby converted into rubiacic acid. It communicates to mordanted cloth only a slight tinge of colour, similar to that produced by rubiacine.

Its analysis gave the following results:—

I. 0·3520 grm. substance, dried at 100° C. and burnt with chromate of lead, gave 0·7400 carbonic acid and 0·1750 water.

II. 0·3805 grm. of the same preparation gave 0·7990 carbonic acid and 0·1890 water.

III. 0·3965 grm. of another preparation gave 0·8330 carbonic acid and 0·1890 water.

IV. 0·2480 grm. of the same preparation as the last, recrystallized from alcohol, gave 0·5290 carbonic acid and 0·1280 water.

V. 0·3735 grm. of a third preparation gave 0·7925 carbonic acid and 0·1785 water.

VI. 0·3995 grm. of the same preparation gave 0·8450 carbonic acid and 0·1855 water.

These numbers correspond in 100 parts to—

	I.	II.	III.	IV.	V.	VI.
Carbon	57·33	57·26	57·29	58·17	57·86	57·68
Hydrogen . . .	5·52	5·51	5·29	5·73	5·31	5·15
Oxygen	37·15	37·23	37·42	36·10	36·83	37·17

I have as yet been unsuccessful in my attempts to determine the atomic weight of rubianine. The little affinity which it has for bases is proved by the fact above men-

tioned, of its crystallizing unchanged out of its alkaline solutions. The baryta compound, which is obtained by adding chloride of barium to its ammoniacal solution, is easily decomposed when it comes to be washed with pure water, the baryta being dissolved by the water, a yellow residue of rubianine being left at last. It is not capable of separating oxide of lead from acetic acid. In fact it nearly approaches the character of a perfectly neutral body, a circumstance which might be *à priori* foreseen from its containing more carbon and less oxygen than rubian itself, the properties of which are not far removed from those of an indifferent substance.

There are three formulæ which all of them give for 100 parts, numbers not widely differing from those found by experiment, viz. $C_{28}H_{17}O_{13}$, $C_{32}H_{19}O_{15}$ and $C_{44}H_{24}O_{20}$. These formulæ require for 100 parts of substance the following amount of constituents:—

	$C_{28}H_{17}O_{13}$.	$C_{32}H_{19}O_{15}$.	$C_{44}H_{24}O_{20}$.
Carbon	58·13	58·00	57·99
Hydrogen	5·88	5·74	5·47
Oxygen	35·99	36·26	36·54

It will be seen that the last formula is that with which the analyses agree best.

If the first formula be the true one, then the formation of this substance from rubian is easily explained. It would then differ by 5 equivs. of water from 2 equivs. of rubiretine; and 1 equiv. of rubianine, 2 equivs. of verantine and 7 equivs. of water added together would be equal to 1 equiv. of rubian, as seen by the following equation:

$$C_{28}H_{17}O_{13}+2(C_{14}H_5O_5)+7HO=C_{56}H_{34}O_{30}.$$

I shall presently show, however, that there is more probability in favour of one of the two latter formulæ.

Sugar.—That the substance obtained from the acid liquid after the complete decomposition of rubian is a species of sugar, will, I think, be apparent from an enumeration of its properties. It is always obtained in the form of a transparent yellow syrup, which neither crystallizes, however long its solution may be left to stand, nor becomes dry, unless heated to 100° C. Its taste is sweetish, accompanied by a bitter after-taste, like that of burnt sugar. When heated for some time at 100° C. it loses a portion of its water, but remains soft and viscid. On allowing it however to cool, it becomes brittle and capable of pulverization. After a few moments' exposure to the air it again begins to attract moisture, which it does as rapidly as chloride of calcium, and is soon reconverted into syrup. This is a character which it has, in common with ordinary syrup, obtained by boiling a solution of cane-sugar in water. It is soluble in alcohol. It is not affected by dilute sulphuric acid, even on boiling; but on evaporating a solution to which sulphuric acid has been added, it is decomposed in proportion as the acid becomes concentrated, and is changed into a black powder like humus. Concentrated sulphuric acid destroys it immediately with disengagement of sulphurous acid. It is destroyed by nitric acid. By operating on a moderately large quantity of it, I was enabled to ascertain that the sole product of the

action of nitric acid is oxalic acid. It is not precipitated from its watery solution by any metallic or other salt, not even by basic acetate of lead. On the addition of caustic potash or soda to its solution and boiling, the solution immediately becomes brown, and a brown powder falls, just as in the case of grape-sugar. It is capable of fermentation. The watery solution when mixed with yeast soon begins to ferment, though the process is not so lively as in the case of an equal quantity of common sugar; and by distilling the liquid and boiling the distillate with dry carbonate of soda, alcohol may be obtained.

The analysis was attended with some difficulty on account of the great affinity which it has for water. By heating it however for some time at 100° C., then allowing to cool and pulverizing while in its brittle state as quickly as possible, it was obtained in a condition fit for analysis. Even then however the state of hydration was not uniform, so that the analyses differed considerably from one another. The following results were obtained:—

I. 0·4765 grm., burnt with chromate of lead, gave 0·6860 carbonic acid and 0·2905 water.

II. 0·3050 grm. gave 0·4450 carbonic acid and 0·1815 water.

III. 0·3820 grm. gave 0·5650 carbonic acid and 0·2205 water.

These numbers give in 100 parts—

	I.	II.	III.
Carbon	39·26	39·79	40·33
Hydrogen . . .	6·77	6·61	6·41
Oxygen	53·97	53·60	53·26

As this substance does not combine with bases, its atomic weight could not be determined by direct experiment. There are, however, two formulæ, both of which agree with the analyses and explain its formation, viz. $C_{14}H_{14}O_{14}$ and $C_{12}H_{12}O_{12}$. Both of these formulæ require in 100 parts—

Carbon	40·00
Hydrogen	6·66
Oxygen	53·34

If the formula $C_{14}H_{14}O_{14}$ be the true one, then its formation from rubian admits of an easy explanation. It would then differ from verantine by 9 equivs. of water; and by adding together 2 equivs. of it and 2 equivs. of rubiretine, the sum would be equal to 1 equiv. of rubian plus 6 equivs. of water, as the following equation shows:—

$$\left.\begin{array}{l} \text{2 equivs. of Sugar} \;=C_{28}H_{28}O_{28} \\ \text{2 equivs. of Rubiretine} \;=C_{28}H_{12}O_{8} \end{array}\right\} = \left\{\begin{array}{l} C_{56}H_{34}O_{30}=1 \text{ equiv. of Rubian.} \\ H_{6}O_{6}=6 \text{ equivs. of Water.} \end{array}\right.$$
$$C_{56}H_{40}O_{36} \qquad C_{56}H_{40}O_{36}$$

If the formula of rubianine be $C_{28}H_{17}O_{13}$, it may replace rubiretine in the above equation; 11 equivs. of water instead of 6 being added to the rubian, as follows,—

$$\left.\begin{array}{l} \text{2 equivs. of Sugar} \;=C_{28}H_{28}O_{28} \\ \text{1 equiv. of Rubianine} \;=C_{28}H_{17}O_{13} \end{array}\right\} = \left\{\begin{array}{l} C_{56}H_{34}O_{30}=1 \text{ equiv. of Rubian.} \\ H_{11}O_{11}=11 \text{ equivs. of Water.} \end{array}\right.$$
$$C_{56}H_{45}O_{41} \qquad C_{56}H_{45}O_{41}$$

To this view however it may be objected that all the species of sugar capable of fermentation with which we are acquainted contain 12 equivs. of carbon. Indeed it is difficult to conceive how a body of the formula $C_{14} H_{14} O_{14}$ can be decomposed into alcohol and carbonic acid. It is therefore far more probable that the formula of this substance is $C_{12} H_{12} O_{12}$, which is also that of grape-sugar when dried at 100° C. In fact, were it capable of being crystallized, it would no doubt be considered as identical with grape-sugar. If this be granted, then it follows that the formula of rubianine must be either $C_{32} H_{19} O_{15}$ or $C_{44} H_{24} O_{20}$, as will be seen by the following equations:—

2 equivs. of Sugar . .	$=C_{24} H_{24} O_{24}$	$C_{56} H_{34} O_{30}$ = 1 equiv. of Rubian.
1 equiv. of Rubianine .	$=C_{32} H_{19} O_{15}$	$H_9 O_9$ = 9 equivs. of Water.
	$C_{56} H_{43} O_{39}$	$C_{56} H_{43} O_{39}$

or

1 equiv. of Sugar . .	$=C_{12} H_{12} O_{12}$	$C_{56} H_{34} O_{30}$ = 1 equiv. of Rubian.
1 equiv. of Rubianine .	$=C_{44} H_{24} O_{20}$	$H_2 O_2$ = 2 equivs. of Water.
	$C_{56} H_{36} O_{32}$	$C_{56} H_{36} O_{32}$

The formula $C_{44} H_{24} O_{20}$ seems to me the more probable of the two. It agrees best with the results of analysis, and the high atomic weight of rubianine which follows from it explains the very neutral character of that substance. Hence it appears that rubianine stands in the same relation to the sugar as rubiretine does to verantine. When added together they contain the elements of rubian plus the elements of water, while rubiretine and verantine added together contain the elements of rubian minus the elements of water.

On the whole, it appears that the action of acids on rubian is not of so complicated a nature as might at first sight be supposed. The number of substances produced by this action is five. Nevertheless it does not follow that these five substances are all formed together, or in other words, that one atom of rubian by its decomposition gives rise to all five at the same time. From the composition of these substances, as compared with that of rubian, it follows that the latter by the action of acids undergoes decomposition in three different directions, or more correctly speaking, that the decomposition affects three separate atoms of rubian. One of these atoms loses 14 atoms of water, and is converted into alizarine. The second loses 12 atoms of water, and then splits up into verantine and rubiretine. The third takes up the elements of water, and then splits up into rubianine and sugar. What the circumstances are under which either one or the other of these three processes takes place I am unable to say. That the loss of a greater or smaller proportion of water or the addition, on the contrary, of the elements of water to those of rubian, are the immediate efficient causes of one or the other of the three processes taking place is very probable; but what again determines the elimination of more or less water from rubian, or, on the other hand, its combination with more water, remains uncertain. It is not

unlikely however that the degree of temperature at which the decomposition is effected may have something to do with it. It is probable that the lower the temperature at which the acid acts on the rubian, the more rubianine and sugar are formed, and that at a higher temperature more alizarine, verantine and rubiretine are produced. In all experiments hitherto mentioned I have always obtained all five products of decomposition, though by no means in equal proportions, the alizarine being formed in the smallest quantity, the amount of rubiretine and verantine being somewhat greater, and the rubianine and sugar being produced in the largest quantity. In the course of this paper I shall have occasion to mention circumstances in which a still greater preponderance takes place in the amount of several of these substances formed over that of the others. Whether it would be possible to confine the decomposition of rubian entirely to one of these processes, or whether all three are essential, is a question of the highest importance, not so much in a theoretical, as a practical point of view. That beautiful substance, alizarine, is the only one of these products which is capable of yielding dyes. It is this body which in my opinion gives rise to all the beautiful colours for the production of which madder is employed. The others are not only useless, they are positively injurious, as I have shown on a former occasion. Though experimentally alizarine is formed in the smallest proportion, it is nevertheless theoretically possible to convert rubian entirely into alizarine, without the least quantity of the other substances being produced. From this point of view the other substances may be considered as formed at the expense of alizarine. In fact, by adding together 1 equiv. of verantine and 1 equiv. of rubiretine, and subtracting 1 equiv. of water, we obtain the elements of 2 equivs. of alizarine, for

$$C_{14}H_5O_5+C_{14}H_6O_4=2(C_{14}H_5O_4)+HO.$$

Also by adding together 1 equiv. of rubianine and 1 equiv. of sugar, and subtracting 16 equivs. of water, we obtain the elements of 4 equivs. of alizarine, for

$$C_{44}H_{24}O_{20}+C_{12}H_{12}O_{12}=4(C_{14}H_5O_4)+16HO.$$

If any chemist should succeed in changing rubian entirely into alizarine, an undertaking in which there is no occasion to despair of success, he would be the means of giving a great stimulus to many branches of manufacture and adding a large sum to the national wealth.

XIX. *On the Solution of Linear Differential Equations.* By the Rev. B. Bronwin. *Communicated by* Samuel Hunter Christie, *Esq., Sec. R.S. &c.*

Received February 6,—Read April 18, 1850.

If we consider the very different forms which the solutions of Differential Equations differing very little from each other frequently take, and the very different processes often required in each particular case to obtain the solution, we shall be led to conclude that the discovery of any universal or general method of solving them must be a hopeless case. We cannot therefore regard particular methods, especially when applicable to a large number of cases, as useless speculations. The present paper contains the solution of several classes of these equations effected by means of general theorems in the Calculus of Operations adapted to each particular class. For explanation of the symbols employed, let it be observed that D is put for $\frac{d}{dx}$, and that φ, λ, Ψ, and X denote any functions of x, the independent variable, and are the same as $\varphi(x)$, $\lambda(x)$, &c.; and in like manner $\varphi(\mathrm{D})$, $\lambda(\mathrm{D})$, &c. will be used to denote the same functions of D.

I. FIRST GENERAL THEOREM IN THE CALCULUS OF OPERATIONS.

Let $\tau=\epsilon^{\int\frac{dx}{\varphi}}$, $\varpi=\varphi\mathrm{D}+\lambda$. We easily verify

$$(\varpi+k)u=\tau^{-k}\varpi\tau^{k}u$$

by substituting for τ and ϖ, and then performing the operations indicated in the result. Change u into $(\varpi+k)u$ in the first member, and into its equal $\tau^{-k}\varpi\tau^{k}u$ in the second, and there results

$$(\varpi+k)^{2}u=\tau^{-k}\varpi^{2}\tau^{k}u.$$

A repetition of this process will produce

$$(\varpi+k)^{3}u=\tau^{-k}\varpi^{3}\tau^{k}u.$$

In like manner similar equations will be found for higher powers. But the first gives

$$u=(\varpi+k)^{-1}\tau^{-k}\varpi\tau^{k}u.$$

Change u into $\tau^{-k}\varpi^{-1}\tau^{k}u$, and we have

$$\tau^{-k}\varpi^{-1}\tau^{k}u=(\varpi+k)^{-1}u,$$

or transposing,

$$(\varpi+k)^{-1}u=\tau^{-k}\varpi^{-1}\tau^{k}u.$$

Now change u into $(\varpi+k)^{-1}u$ in the first member, and into its equal $\tau^{-k}\varpi^{-1}\tau^{k}u$ in the

second, and the result is

$$(\varpi+k)^{-2}u=\tau^{-k}\varpi^{-2}\tau^{k}u;$$

and so on. Therefore if $f(\varpi)$ be any function developable in integer powers of ϖ, positive or negative, we shall have

$$f(\varpi+k)u=\tau^{-k}f(\varpi)\tau^{k}u,$$

or

$$\tau^{k}f(\varpi+k)u=f(\varpi)\tau^{k}u. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (a.)$$

The assumption $\tau=\varepsilon^{\int\frac{du}{\varphi}}$ gives $\frac{1}{\varphi}=\frac{\tau'}{\tau}$, and we may regard either τ or φ as the quantity given from which the other is to be found.

Application of the preceding Theorem to the Solution or Reduction of an extensive class of Linear Differential Equations.

To abridge we shall put

$$(\varpi+a)(\varpi+a+k)\ldots(\varpi+a+nk)=\mathrm{P}\begin{Bmatrix}a+nk\\a\end{Bmatrix},$$

$$(\varpi+a)^{-1}(\varpi+a+k)^{-1}\ldots\ldots(\varpi+a+nk)^{-1}=\mathrm{P}\begin{Bmatrix}a+nk\\a\end{Bmatrix}^{-1};$$

and similarly in other cases. But it is to be observed that we may resolve the last into

$$\frac{\mathrm{A}}{\varpi+a}+\frac{\mathrm{A}_1}{\varpi+a+k}+\ldots\ldots+\frac{\mathrm{A}_n}{\varpi+a+nk},$$

which in practice may be more convenient. The operations implied by the reciprocal factors may be readily performed by a well-known theorem due to Mr. Boole.

Thus
$$(\varpi+m)^{-1}\mathrm{X}=(\varphi\mathrm{D}+\lambda+m)^{-1}\mathrm{X}=\left(\mathrm{D}+\frac{\lambda+m}{\varphi}\right)^{-1}\varphi^{-1}\mathrm{X}$$

$$=\varepsilon^{-\int\frac{\lambda+m}{\varphi}du}\mathrm{D}^{-1}\varepsilon^{\int\frac{\lambda+m}{\varphi}u}\varphi^{-1}\mathrm{X}.$$

Now let

$$f(\varpi)\varpi u+pf_1(\varpi)(\varpi+nk)\tau^{k}u=\mathrm{X}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1.)$$

where $f(\varpi)$ and $f_1(\varpi)$ are any rational functions of ϖ, and k may be either positive or negative. To reduce this, assume

$$u=(\varpi+k)(\varpi+2k)\ldots\ldots(\varpi+nk)v.$$

Then by $(a.)$

$$\tau^{k}u=\tau^{k}(\varpi+k)\ldots.(\varpi+nk)v=\varpi(\varpi+k)\ldots.(\varpi+(n-1)k)\tau^{k}v.$$

Substituting these values in (1.), and operating on both members of the result with

$$\varpi^{-1}(\varpi+k)^{-1}\ldots.(\varpi+nk)^{-1}=\mathrm{P}\begin{Bmatrix}nk\\0\end{Bmatrix}^{-1},$$

we find

$$f(\varpi)v+pf_1(\varpi)\tau^{k}v=\mathrm{P}\begin{Bmatrix}nk\\0\end{Bmatrix}^{-1}\mathrm{X}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.)$$

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which is one order lower than the proposed. All equations therefore of the second order included in (1.) may be considered as integrated by this process.

Another form, distinct from the above, is

$$f(\varpi)(\varpi+nk)u+pf_1(\varpi)\varpi\tau^k u=X. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.)$$

To reduce this we must assume

$$u=(\varpi+k)^{-1}(\varpi+2k)^{-1}\ldots.(\varpi+nk)^{-1}v.$$

Then by (*a*.) we have

$$\tau^k u=\varpi^{-1}(\varpi+k)^{-1}\ldots.(\varpi+(n-1)k)^{-1}\tau^k v.$$

Substituting these values and operating with $(\varpi+k)\ldots(\varpi+(n-1)k)$ on both members of the result, we have

$$f(\varpi)v+pf_1(\varpi)\tau^k v=P\begin{Bmatrix}(n-1)k\\k\end{Bmatrix}X. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4.)$$

The observations made with reference to the former example might be repeated here. I shall add, that as k may be both positive and negative, these two examples include every variety of case of this form of equation.

Before I proceed to notice particular examples, it may be as well to give a few more general ones, and thus to point out the whole series of them which are susceptible of reduction by this method.

Let $$f(\varpi)\varpi(\varpi+k)u+pf_1(\varpi)(\varpi+nk)\tau^{2k}u=X. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5.)$$

Make $$u=(\varpi+2k)(\varpi+3k)\ldots..(\varpi+(n+1)k)v,$$

the common difference of the factors being k, as before, which here also may be both positive and negative. This will give

$$\tau^{2k}u=\varpi(\varpi+k)\ldots..(\varpi+(n-1)k)\tau^{2k}v,$$

and proceeding exactly as before, we shall arrive at the reduced equation,

$$f(\varpi)(\varpi+(n+1)k)v+pf_1(\varpi)\tau^{2k}v=P\begin{Bmatrix}nk\\0\end{Bmatrix}^{-1}X. \quad . \quad . \quad . \quad . \quad . \quad . \quad (6.)$$

Again, suppose

$$f(\varpi)(\varpi+nk)(\varpi+nk+k)u+pf_1(\varpi)\varpi\tau^{2k}u=X. \quad . \quad . \quad . \quad . \quad . \quad (7.)$$

The assumption

$$u=(\varpi+2k)^{-1}(\varpi+3k)^{-1}\ldots.(\varpi+(n+1)k)^{-1}v$$

leads to

$$f(\varpi)(\varpi+k)v+pf_1(\varpi)\tau^{2k}v=P\begin{Bmatrix}(n-1)k\\k\end{Bmatrix}X. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8.)$$

The two last examples, like the former, are reduced an order lower; and when they are of the second order, they may be considered as integrated. In order to enable us to effect their reduction, it is necessary that we should have two operating factors, as $\varpi(\varpi+k)$, in one of the terms, these factors having the difference k. I shall only give two more examples, which will suffice to indicate the series of them before mentioned.

Let $$f(\varpi)\varpi(\varpi+k)(\varpi+2k)u+pf_1(\varpi)(\varpi+nk)\tau^{3k}u=X,$$

and

$$f(\varpi)(\varpi+nk)(\varpi+nk+k)(\varpi+nk+2k)u+pf_1(\varpi)\varpi\tau^{3k}u=X.$$

The first of these, by making

$$u=(\varpi+3k)(\varpi+4k)\ldots.(\varpi+(n+2)k)v,$$

will reduce to

$$f(\varpi)(\varpi+nk+k)(\varpi+nk+2k)v+p\tau^{3k}v=P\begin{Bmatrix}nk\\0\end{Bmatrix}^{-1}X,$$

and the second, by putting

$$u=(\varpi+3k)^{-1}\ldots.(\varpi+(n+2)k)^{-1}v,$$

reduces to

$$f(\varpi)(\varpi+k)(\varpi+2k)v+p\tau^{3k}v=P\begin{Bmatrix}(n-1)k\\k\end{Bmatrix}X.$$

It will be observed that the equations in the two last examples are of the third order at the lowest, and those to which they are reduced of the second; and if we were to continue the series, they would rise an order at every step. But we will here leave them and proceed to give a few particular examples.

In (1.) and (2.) make

$$f(\varpi)=\varpi+a,\ f_1(\varpi)=1;$$

and they become

$$\varpi(\varpi+a)u+p(\varpi+nk)\tau^k u=X \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9.)$$

$$(\varpi+a)v+p\tau^k v=P\begin{Bmatrix}nk\\0\end{Bmatrix}^{-1}X.$$

The last gives

$$v=(\varpi+a+p\tau^k)^{-1}P\begin{Bmatrix}nk\\0\end{Bmatrix}^{-1}X,$$

and therefore

$$u=P\begin{Bmatrix}nk\\k\end{Bmatrix}(\varpi+a+p\tau^k)^{-1}P\begin{Bmatrix}nk\\0\end{Bmatrix}^{-1}X.$$

It is not necessary to reduce the value of u any further, as the mode of doing it has been made sufficiently plain, and moreover it is quite as convenient as it stands. We shall only observe that if, to abridge, we make

$$\frac{\lambda+a+p\tau^k}{\varphi}=\Phi,$$

we have

$$(\varpi+a+p\tau^k)^{-1}=(D+\Phi)^{-1}\varphi^{-1}=\varepsilon^{-\int\Phi dx}D^{-1}\varepsilon^{\int\Phi dx}\varphi^{-1}.$$

If we wish to see (9.) under the ordinary form, it is easily reduced, first to

$$\varpi^2u+(a+p\tau^k)\varpi u+(n+1)kp\tau^k u=X;$$

and then by substituting for ϖ to

$$\varphi^2D^2u+\varphi(\varphi'+2\lambda+a+p\tau^k)Du+(\varphi\lambda'+\lambda^2+(a+p\tau^k)\lambda+(n+1)kp\tau^k)u=X.$$

If we would deduce particular integrable equations from this, we may assume τ and λ any functions of x at pleasure, and the relation $\frac{1}{\varphi}=\frac{\tau'}{\tau}$ will give φ. It will be

observed that the accent upon τ, λ, and φ is employed to denote differential coefficients. But as this equation, and many more which may be deduced, contain two arbitrary functions of the independent variable, the number of particular practicable forms is immense. To select examples therefore would be very difficult.

Now make in (1.) and (2.)

$$f(\varpi)=\varpi+a,\ f_1(\varpi)=\varpi+b,$$

and we have

$$\varpi(\varpi+a)u+p(\varpi+b)(\varpi+nk)\tau^k u=X \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10.)$$

$$(\varpi+a)v+p(\varpi+b)\tau^k v=P\begin{Bmatrix} nk \\ 0 \end{Bmatrix}^{-1}X,$$

or

$$(\varpi+a)v+p\tau^k(\varpi+b+k)v=P\begin{Bmatrix} nk \\ 0 \end{Bmatrix}^{-1}X,$$

and therefore

$$(1+p\tau^k)\varpi v+(a+(b+k)p\tau^k)v=P\begin{Bmatrix} nk \\ 0 \end{Bmatrix}^{-1}X.$$

To abridge make

$$\frac{a+(b+k)p\tau^k}{1+p\tau^k}=\Phi,$$

and the last equation becomes

$$(1+p\tau^k)(\varpi+\Phi)v=P\begin{Bmatrix} nk \\ 0 \end{Bmatrix}^{-1}X;$$

whence

$$v=(\varpi+\Phi)^{-1}(1+p\tau^k)^{-1}P\begin{Bmatrix} nk \\ 0 \end{Bmatrix}^{-1}X,\ u=P\begin{Bmatrix} nk \\ k \end{Bmatrix}(\varpi+\Phi)^{-1}(1+p\tau^k)^{-1}P\begin{Bmatrix} nk \\ 0 \end{Bmatrix}^{-1}X.$$

Similarly, from (3.) and (4.) we derive

$$(\varpi+a)(\varpi+nk)u+p\varpi\tau^k u=X, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11.)$$

where

$$u=P\begin{Bmatrix} nk \\ k \end{Bmatrix}^{-1}(\varpi+a+p\tau^k)^{-1}P\begin{Bmatrix} (n-1)k \\ k \end{Bmatrix}X.$$

The last equation, reduced to the ordinary form, making $c=a+nk$, becomes

$$\varphi^2 D^2 u+\varphi(\varphi'+2\lambda+c+p\tau^k)Du+(\varphi\lambda'+\lambda^2+(c+p\tau^k)\lambda+nak+kp\tau^k)u=X.$$

From (3.) and (4.) we also deduce

$$(\varpi+a)(\varpi+nk)u+p\varpi(\varpi+b)\tau^k u=X \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12.)$$

$$(\varpi+a)v+p(\varpi+b)\tau^k v=P\begin{Bmatrix} (n-1)k \\ k \end{Bmatrix}X.$$

From these, exactly as in (10.), we find

$$u=P\begin{Bmatrix} nk \\ k \end{Bmatrix}^{-1}(\varpi+\Phi)^{-1}(1+p\tau^k)^{-1}P\begin{Bmatrix} (n-1)k \\ k \end{Bmatrix}X,$$

where Φ has the same value as in (10.).

The examples (10.) and (12.), if reduced to the ordinary form, would be very different from those which have been so reduced, and they would be considerably more

complex. And we may observe that if the functions $f(\varpi), f_1(\varpi)$ in (1.), (3.), &c. have suitable factors, the reduced equations (2.), (4.), &c. may be still further reduced by the same means, the proposed equations in this case being of an order above the second.

From (5.) and (6.), making $f(\varpi)=f_1(\varpi)=1$, we have

$$\varpi(\varpi+k)u+p(\varpi+nk)r^{2k}u=X \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13.)$$

and

$$(\varpi+(n+1)k)v+pr^{2k}v \doteq P\begin{Bmatrix} nk \\ 0 \end{Bmatrix}^{-1}X;$$

whence

$$v=\{\varpi+(n+1)k+pr^{2k}\}^{-1}P\begin{Bmatrix} nk \\ 0 \end{Bmatrix}^{-1}X,$$

$$u=P\begin{Bmatrix} (n+1)k \\ 2k \end{Bmatrix}\{\varpi+(n+1)k+pr^{2k}\}^{-1}P\begin{Bmatrix} nk \\ 0 \end{Bmatrix}^{-1}X.$$

Similarly, from (7.) and (8.) we find

$$(\varpi+nk)(\varpi+nk+k)u+p\varpi r^{2k}u=X \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14.)$$

$$v=\{\varpi+k+pr^{2k}\}^{-1}P\begin{Bmatrix} (n-1)k \\ k \end{Bmatrix}X,$$

$$u=P\begin{Bmatrix} (n+1)k \\ 2k \end{Bmatrix}^{-1}\{\varpi+k+pr^{2k}\}^{-1}P\begin{Bmatrix} (n-1)k \\ k \end{Bmatrix}X.$$

The two last examples, reduced to the ordinary form, are

$$\varphi^3D^3u+\varphi(\varphi'+2\lambda+k+pr^{2k})Du+(\varphi\lambda'+\lambda^2+\lambda(k+pr^{2k})+(n+2)kpr^{2k})u=X,$$

and

$$\varphi^3D^3u+\varphi(\varphi'+2\lambda+(2n+1)k+pr^{2k})Du+(\varphi\lambda'+\lambda^2+(2n+1)k\lambda+(2k+\lambda)pr^{2k}+n(n+1)k^2)u=X$$

respectively.

I shall only give two other examples, derived from the same source with the two preceding.

$$\varpi(\varpi+k)u+p(\varpi+a)(\varpi+nk)r^{2k}u=X,$$

$$u=P\begin{Bmatrix} (n+1)k \\ 2k \end{Bmatrix}(\varpi+\Psi)^{-1}(1+pr^{2k})^{-1}P\begin{Bmatrix} nk \\ 0 \end{Bmatrix}^{-1}X$$

$$(\varpi+nk)(\varpi+nk+k)u+p\varpi(\varpi+a)r^{2k}u=X,$$

$$u=P\begin{Bmatrix} (n+1)k \\ 2k \end{Bmatrix}^{-1}(\varpi+\Phi)^{-1}(1+pr^{2k})^{-1}P\begin{Bmatrix} (n-1)k \\ k \end{Bmatrix}X;$$

where

$$\Psi=\frac{(n+1)k+(a+2k)pr^{2k}}{1+pr^{2k}}, \quad \Phi=\frac{k+(a+2k)pr^{2k}}{1+pr^{2k}}.$$

II. SECOND GENERAL THEOREM IN THE CALCULUS OF OPERATIONS.

Make $D=D_1+D_2$, D_1 operating upon u only and D_2 upon x only. Then by TAYLOR'S theorem

$$f(D)=f(D_1+D_2)=f(D_1)+D_2f'(D_1)+\tfrac{1}{2}D_2^2f''(D_1)+\ldots.$$

This formula will be of frequent use in the subsequent part of this paper. By it

$$\varepsilon^{kf(D)}=\varepsilon^{kf(D_1)+kD_1f'(D_1)+\dots}=\varepsilon^{kf(D_1)}\varepsilon^{kD_1f'(D_1)+\dots}=\{1+kD_1f'(D_1)+\}\varepsilon^{kf(D_1)}.$$

Operate with both members of this upon $\{\varphi(D)x+\lambda(D)\}u$, and we have

$$\varepsilon^{kf(D)}\{\varphi(D)x+\lambda(D)\}u=\{\varphi(D)x+\lambda(D)+k\varphi(D)f'(D)\}\varepsilon^{kf(D)}u,$$

dropping the mark under D, as being no further needed, D everywhere now operating upon all that follows it. Make

$$f(D)=\int\frac{dD}{\varphi(D)},$$

then

$$f'(D)=\frac{1}{\varphi(D)},$$

and the last equation becomes

$$\varepsilon^{k\int\frac{dD}{\varphi(D)}}\Big\{\varphi(D)x+\lambda(D)\Big\}u=\Big\{\{\varphi(D)x+\lambda(D)+k\Big\}\varepsilon^{k\int\frac{dD}{\varphi(D)}}u.$$

Now let

$$\varphi(D)x+\lambda(D)=w',\quad \varepsilon^{\int\frac{dD}{\varphi(D)}}=\tau',$$

which gives

$$\frac{1}{\varphi(D)}=\frac{1}{\tau}\frac{d\tau'}{dD};$$

and the preceding becomes

$$\tau'^{k}w'u=(w'+k)\tau'^{k}u.$$

Change u into $\tau'^{-k}u$, and there results

$$\tau'^{k}w'\tau'^{-k}u=(w'+k)u,$$

or

$$(w'+k)u=\tau'^{k}w'\tau'^{-k}u$$

by transposing the members.

From the last equation, by the same course of reasoning by which (a.) was established, we find

$$\tau'^{-k}f(w'+k)u=f(w')\tau'^{-k}u. \quad\dots\dots\dots\quad (b.)$$

If in (a.) we change x into D and D into x, and also s into s^{-1}, we convert (a.) into (b.); and the same conversion of symbols will change (b.) into (a.).

Application of the preceding Theorem.

The equations

$$f(w')w'u+pf_1(w')(w'+nk)\tau'^{-k}u=X,$$
$$f(w')(w'+nk)u+pf_1(w')w'\tau'^{-k}u=X,$$
$$f(w')w'(w'+k)u+pf_1(w')(w'+nk)\tau'^{-2k}u=X, \&c.$$

by the assumptions

$$u=(w'+k)\dots.(w'+nk)v,$$
$$u=(w'+k)^{-1}\dots.(w'+nk)^{-1}v,$$
$$u=(w'+2k)\dots.(w'+(n+1)k)v, \&c.$$

will reduce to

$$f(\varpi')v+pf_1(\varpi')\tau^{j-k}v=\mathrm{P}\begin{Bmatrix}nk\\0\end{Bmatrix}^{-1}\mathrm{X},$$

$$f(\varpi')v+pf_1(\varpi')\tau^{j-k}v=\mathrm{P}\begin{Bmatrix}(n-1)k\\k\end{Bmatrix}\mathrm{X},$$

$$f(\varpi')(\varpi'+(n+1)k)v+pf_1(\varpi')\tau^{j-2k}v=\mathrm{P}\begin{Bmatrix}nk\\0\end{Bmatrix}^{-1}\mathrm{X},\ \&c.$$

respectively, the reductions being made by the theorem (*b*.).

But these equations and every step of their reduction may be derived immediately from (1.), (3.), &c., and their reductions by the conversion of symbols before mentioned, which is not a little remarkable. Hence we may derive the solutions of the one series from those of the other merely by the interchange of symbols.

In the equations of which we are now treating, $\varphi(\mathrm{D})$ and $\lambda(\mathrm{D})$ must be rational functions of D; and we must also have $\tau^{j-k}=\chi(\mathrm{D})$, a rational function. Therefore we must have

$$\frac{1}{\varphi(\mathrm{D})}=-\frac{\chi'(\mathrm{D})}{k\chi(\mathrm{D})},$$

and

$$\varphi(\mathrm{D})=-\frac{k\chi(\mathrm{D})}{\chi'(\mathrm{D})}.$$

In the values of u we have operating factors of the form

$$(\varpi'+rk)^{-1}=(\varphi(\mathrm{D})x+\lambda(\mathrm{D})+rk)^{-1}=\left(x+\frac{\lambda(\mathrm{D})+rk}{\varphi(\mathrm{D})}\right)^{-1}\varphi(\mathrm{D})^{-1}=\varepsilon^{\int\frac{\lambda(\mathrm{D})+rk}{\varphi(\mathrm{D})}d\mathrm{D}}x^{-1}\varepsilon^{-\int\frac{\lambda(\mathrm{D})+rk}{\varphi(\mathrm{D})}d\mathrm{D}}\varphi(\mathrm{D})^{-1}$$

by a well-known theorem due to Mr. Boole. Now in order that these may be practicable in finite terms, or as it is usual to say, capable of interpretation, we must have

$$\varepsilon^{\int\frac{\lambda(\mathrm{D})}{\varphi(\mathrm{D})}d\mathrm{D}}=\Phi(\mathrm{D})\varepsilon^{m\mathrm{D}},$$

$\Phi(\mathrm{D})$ being a rational function of D, and m being any constant, positive or negative, or nothing. This is the most general form possible, and it gives

$$\frac{\lambda(\mathrm{D})}{\varphi(\mathrm{D})}=\frac{m\Phi(\mathrm{D})+\Phi'(\mathrm{D})}{\Phi(\mathrm{D})},$$

and

$$\lambda(\mathrm{D})=\frac{\varphi(\mathrm{D})}{\Phi(\mathrm{D})}(m\Phi(\mathrm{D})+\Phi'(\mathrm{D})).$$

The expression $\varepsilon^{\int\frac{rk}{\varphi(\mathrm{D})}d\mathrm{D}}$ has been rendered practicable in making τ^{j-k} so.

The only other operating factor which we have to consider is

$$(\varpi'+h+p\tau^{j-k})^{-1},$$

h being some constant. Now putting for τ^{j-k} its value before found, this will reduce to

$$\left\{x+\frac{\lambda(\mathrm{D})+h+p\chi(\mathrm{D})}{\varphi(\mathrm{D})}\right\}^{-1}\varphi(\mathrm{D})^{-1}.$$

The exponentials depending on $\frac{\lambda(\mathrm{D})}{\varphi(\mathrm{D})}$ and $\frac{h}{\varphi(\mathrm{D})}$ have been considered, and it only re-

mains to consider

$$\varepsilon^{p\int\frac{\chi(D)}{\phi(D)}dD}=\varepsilon^{-\frac{p}{k}\int\chi'(D)dD}=\varepsilon^{-\frac{p}{k}\chi(D)}$$

by substituting for $\frac{1}{\phi(D)}$ its value. But $\varepsilon^{-\frac{p}{k}\chi(D)}$ cannot be interpreted except when $\chi(D)=qD$. We have therefore, finally,

$$\tau^{j-k}=qD,\ \phi(D)=-kD,\ \lambda(D)=-\frac{kD}{\Phi(D)}(m\Phi(D)+\Phi'(D)).$$

This result narrows very much the limits of the practicable cases.

We may however obtain a small increase of generality by making $\chi(D)=qD+r$. There is nothing to prevent this, since

$$\varepsilon^{-\frac{p}{k}(qD+r)}=\varepsilon^{-\frac{pr}{k}}\varepsilon^{-\frac{pq}{k}D},$$

which makes no change in the interpretation of the results, the only difference which it occasions being the introduction of the constant quantity $\varepsilon^{-\frac{pr}{k}}$ into the value of u where we should otherwise have unity in its place. Thus we shall have

$$\tau^{j-k}=qD+r,\ \phi(D)=-k\left(D+\frac{r}{q}\right),\ \lambda(D)=-k\left(D+\frac{r}{q}\right)\left(m+\frac{\Phi'(D)}{\Phi(D)}\right);$$

which values give

$$\varpi'=-k\left(D+\frac{r}{q}\right)\left(x+m+\frac{\Phi'(D)}{\Phi(D)}\right).$$

By the conversion of symbols we derive from (9.),

$$\varpi'(\varpi'+a)u+p(\varpi'+nk)\tau^{j-k}u=X,$$

$$u=P'\begin{Bmatrix}nk\\k\end{Bmatrix}(\varpi'+a+p\tau^{j-k})^{-1}P'\begin{Bmatrix}nk\\0\end{Bmatrix}^{-1}X,$$

which by putting for τ^{j-k} its value becomes

$$\varpi'(\varpi'+a)u+p(\varpi'+nk)(qD+r)u=X \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15.)$$

$$u=P'\begin{Bmatrix}nk\\k\end{Bmatrix}(\varpi'+pqD+pr+a)^{-1}P'\begin{Bmatrix}nk\\0\end{Bmatrix}^{-1}X.$$

From (11.) we derive in like manner

$$(\varpi'+a)(\varpi'+nk)u+p\varpi'(qD+r)u=X \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16.)$$

$$u=P'\begin{Bmatrix}nk\\k\end{Bmatrix}^{-1}(\varpi'+pqD+pr+a)^{-1}P'\begin{Bmatrix}(n-1)k\\k\end{Bmatrix}X.$$

But (10.) and (12.) treated thus would lead to equations of the third order; and as we cannot notice those of all orders, we shall pass these by.

It would render the values of u too long and complex to substitute for ϖ' its value and reduce them further. But this is unnecessary, since the method of doing it has been made sufficiently plain, and indeed is well understood. If we would reduce these equations to the ordinary form, it can easily be effected by the formula

$$f(D)=f(D_1+D_2)=f(D_1)+D_2f'(D_1)+\frac{1}{2}D_2^2f''(D)+\&c.,$$

where D_1 operates upon u, D_2 upon x. Thus we find

$$\varphi(D)xu=x\varphi(D)u+\varphi'(D)u$$

$$\varphi(D)x\varphi(D)xu=x^2\varphi(D)^2u+3x\varphi(D)\varphi'(D)u+\varphi(D)\varphi''(D)u+\varphi'(D)^2u.$$

But this reduction would lead to resulting equations of considerable length, unless we give to $\Phi(D)$ a particular form. This form should be such that negative powers of D may disappear. Or we may make $u=\Phi(D)z$, and so take away such powers, and at the same time may introduce more arbitrary constants into the equation. Thus we should find integrable equations of the second order, having coefficients of the form $a+bx+cx^2$, of considerable generality, owing to the large number of constants which they would contain. The method will be found on trial well adapted to the integration of such equations.

By changing k into $2k$, we shall have

$$\varpi'^{-2k}=qD+r,\ \varphi(D)=-2k\Big(D+\frac{r}{q}\Big),\ \lambda(D)=-2k\Big(D+\frac{r}{q}\Big)\Big(m+\frac{\Phi'(D)}{\Phi(D)}\Big),$$

$$\varpi'=-2k\Big(D+\frac{r}{q}\Big)\Big(x+m+\frac{\Phi'(D)}{\Phi(D)}\Big).$$

With these values we derive from (14.),

$$(\varpi'+nk)(\varpi'+nk+k)u+p\varpi'(qD+r)u=X \quad \ldots\ldots\ldots\ldots\ldots (17.)$$

$$u=P\begin{Bmatrix}(n+1)k\\2k\end{Bmatrix}^{-1}\{\varpi'+pqD+pr+k\}^{-1}P\begin{Bmatrix}(n-1)k\\k\end{Bmatrix}X.$$

Other examples might be given under this head, but I shall now proceed to the solution of two equations somewhat similar to some of those which have been given, but which cannot be solved in the same manner.

III. THIRD AND FOURTH GENERAL THEOREMS.

Make

$$\varrho=\varepsilon^{\int\frac{\lambda}{\varphi}dx},\ \varpi_m=\varphi D+\Psi+m\lambda,$$

and consequently

$$\varpi_n=\varphi D+\Psi+n\lambda.$$

We easily verify the equation

$$\varrho^n\varpi_{m+n}u=\varpi_m\varrho^n u$$

by substituting for ϱ and ϖ_{m+n}, ϖ_m their values, and performing the operations indicated in the result. Therefore by the process followed in the investigation of (*a.*), we find

$$\varrho^n f(\varpi_{m+n})u=f(\varpi_m)\varrho^n u. \quad \ldots\ldots\ldots\ldots\ldots (c.)$$

To establish the other general theorem, we make

$$\varrho'=\varepsilon^{\int\frac{\lambda(D)}{\varphi(D)}dD},\quad \varpi'_m=\varphi(D)x+\Psi(D)+m\lambda(D),$$

and therefore also

$$\varpi'_n=\varphi(D)x+\Psi(D)+n\lambda(D).$$

But we have found

$$\varepsilon^{\Psi(D)}=\{1+nD_\alpha f'(D_1)+\}\varepsilon^{\Psi(D_1)}.$$

Operating with both members of this on

$$\{\varphi(D)x+\Psi(D)+m\lambda(D)\}u,$$

we find

$$\varepsilon^{\Psi(D)}\{\varphi(D)x+\Psi(D)+m\lambda(D)\}u=\{\varphi(D)x+\Psi(D)+m\lambda(D)+n\varphi(D)f'(D)\}\varepsilon^{\Psi(D)}u,$$

dropping the mark under D as no longer needed.

Make

$$f(D)=\int\frac{\lambda(D)}{\varphi(D)}dD,$$

then

$$f'(D)=\frac{\lambda(D)}{\varphi(D)},\ \varphi(D)f'(D)=\lambda(D).$$

Substituting this value of $f'(D)$ in the last equation, the result will be equivalent to

$$\varrho^{\prime n}\pi'_m u=\pi'_{m+n}\varrho^{\prime n}u.$$

Change u into $\varrho^{\prime -n}u$, and transpose the members, and we have

$$\pi'_{m+n}u=\varrho^{\prime n}\pi'_m\varrho^{\prime -n}u.$$

Therefore, as before, we shall have

$$\varrho^{\prime -n}f(\pi'_{m+n})u=f(\pi'_m)\varrho^{\prime -n}u. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (d.)$$

By the interchange of the symbols x and D, the two general theorems (*c.*) and (*d.*) may be converted the one into the other, ε at the same time being changed into ε^{-1}.

Before we can employ these theorems in the way intended, we must find the relations between the arbitrary functions required in order that two others may subsist. The first of which is

$$\pi_m\pi_n u=\pi_n\pi_m u+(n-m)a\varrho u. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (e.)$$

With the values given of π_m and π_n, we easily find that

$$\pi_m\pi_n u=\pi_n\pi_m u+(n-m)\varphi\lambda' u.$$

Therefore we must have

$$a\varrho=\varphi\lambda', \text{ or } a\varepsilon^{\int\frac{\lambda}{\varphi}dx}=\varphi\lambda'.$$

Passing to the logarithms of both members,

$$\log a+\int\frac{\lambda}{\varphi}dx=\log(\varphi\lambda'),$$

and by differentiation

$$\frac{\lambda}{\varphi}dx=\frac{d(\varphi\lambda')}{\varphi\lambda'}.$$

Hence we find successively

$$\lambda\lambda' dx=d(\varphi\lambda'),\ \frac{1}{2}d(\lambda^2)=d(\varphi\lambda'),\ b+\frac{1}{2}\lambda^2=\varphi\lambda', \text{ and } \frac{1}{\varphi}=\frac{\lambda'}{b+\frac{1}{2}\lambda^2},$$

which is the required relation or condition.

From the last we find

$$\int\frac{\lambda}{\varphi}dx=\int\frac{\lambda d\lambda}{b+\frac{1}{2}\lambda^2}=\log\left(b+\tfrac{1}{2}\lambda^2\right)+\log c.$$

We easily find $c=\frac{1}{a}$, therefore $\varphi=\dfrac{b+\frac{1}{2}\lambda^2}{a}$.

The other condition alluded to is

$$\pi'_m\pi'_n u=\pi'_n\pi'_m u+(n-m)a\varphi^{l-1}u. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (f.)$$

From the values of π'_m, π'_n, we find

$$\pi'_m\pi'_n u=\pi'_n\pi'_m u+(m-n)\varphi(D)\lambda'(D).$$

Therefore we must have

$$-a\varphi^{l-1}=\varphi(D)\lambda'(D), \text{ or } -a\varepsilon^{-\int\frac{\lambda(D)}{\varphi(D)}dD}=\varphi(D)\lambda'(D);$$

and as in the former case,

$$-\frac{\lambda(D)}{\varphi(D)}dD=\frac{d\{\varphi(D)\lambda'(D)\}}{\varphi(D)\lambda'(D)},\ -\lambda(D)d\lambda(D)=d\{\varphi(D)\lambda'(D)\}.$$

Therefore, also,

$$b-\tfrac{1}{2}\lambda(D)^2=\varphi(D)\lambda'(D), \text{ and } \frac{1}{\varphi(D)}=\frac{\lambda'(D)}{b-\frac{1}{2}\lambda(D)^2},$$

which is the required relation between $\varphi(D)$ and $\lambda(D)$.

Also

$$-\int\frac{\lambda(D)}{\varphi(D)}dD=\log\left(\frac{b-\frac{1}{2}\lambda(D)^2}{c}\right),$$

where we easily perceive that $c=-a$. Whence $\varphi=\dfrac{a}{\frac{1}{2}\lambda(D)^2-b}$.

Application of the four last Theorems and Formulæ.

Let $$\pi_m\pi_n u+p\varphi u=X. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18.)$$

A single case of this equation was solved by Mr. Boole in No. 7, New Series of the Cambridge Mathematical Journal, the quantities φ and λ being given functions of the independent variable, and having a given relation, Ψ being nothing. In the Philosophical Magazine, vol. xxxii. p. 257, I gave two similar solutions, but contrived to introduce an arbitrary function of x, by which the solution was very greatly extended. Here that method is superseded and the solution rendered much more general. To solve the above, make $u=\pi_{m+1}u_1$; then

$$\pi_m\pi_n\pi_{m+1}u_1+p\varphi\pi_{m+1}u_1=X \text{ by substitution,}$$

$$\pi_m\pi_n\pi_{m+1}u_1+p\pi_m\varphi u_1=X \text{ by } (c.),$$

$$\pi_n\pi_{m+1}u_1+p\varphi u_1=\pi_m^{-1}X,$$

$$\pi_{m+1}\pi_n u_1+p_1\varphi u_1=\pi_m^{-1}X \text{ by } (e.),\ p_1=p-a(n-m-1).$$

After i transformations, we have

$$\pi_{m+i}\pi_n u_i + p\varrho u_i = \pi_{m+i-1}^{-1} \ldots\ldots \pi_m^{-1} X.$$

If $p_i = 0$, or $p = ia\left(n-m-\frac{i+1}{2}\right)$, the equation is solved, and we have

$$u_i = \pi_n^{-1}\pi_{m+i}^{-1} \ldots\ldots \pi_m^{-1} X.$$

From this we easily find u. The success of the method, it will be seen, depends upon the value of p.

Other cases and solutions may be seen in the Philosophical Magazine in the article before mentioned, but they cannot be given here.

Reduced to the ordinary form, (18.) becomes

$$\varphi^2 D^2 u + \varphi(\varphi' + 2\Psi + (m+n)\lambda) Du + (\varphi(\varphi' + n\lambda') + (\Psi + m\lambda)(\Psi + n\lambda) + p\varrho)u = X.$$

If we were to substitute for φ and ϱ their values, this would become very complicated. We see that it differs considerably in form from the preceding examples which have been thus reduced.

There is one equation bearing some analogy to that which has just been solved which deserves to be noticed here, although its solution must be effected by a very different process. It is

$$\pi_m \pi_n u + p\varrho^{\frac{1}{2}(n-m)} u = X. \quad \ldots\ldots\ldots\ldots \quad (19.)$$

By (c.) this may be put under the form

$$\pi_m \varrho^{m-n} \pi_m \varrho^{n-m} u + p\varrho^{\frac{1}{2}(n-m)} u = X.$$

Make $u = \varrho^{\frac{1}{2}(m-n)} v$, and the last, by substituting this value, will become

$$\pi_m \varrho^{m-n} \pi_m \varrho^{m-n} v + pv = X.$$

Now let $\pi_m \varrho^{m-n} = \tau$, and the preceding will be reduced to

$$(\tau^2 + p)v = X.$$

Whence

$$v = (\tau^2+p)^{-1} X = \frac{A}{\tau - p^{\frac{1}{2}}\sqrt{-1}} X - \frac{A}{\tau + p^{\frac{1}{2}}\sqrt{-1}} X,$$

where

$$A = \frac{1}{2p^{\frac{1}{2}}\sqrt{-1}}.$$

If $v = v_1 + v_2$, we may evidently make

$$(\tau - p^{\frac{1}{2}}\sqrt{-1})v_1 = AX, \quad (\tau + p^{\frac{1}{2}}\sqrt{-1})v_2 = -AX;$$

for these lead to the same result. And thus v is found from two equations of a lower order than the given equation.

If we now put for τ its value in the two last equations, we shall have

$$\pi_m \varrho^{m-n} v_1 - p^{\frac{1}{2}}\sqrt{-1}\, v_1 = AX, \quad \pi_m \varrho^{m-n} v_2 + p^{\frac{1}{2}}\sqrt{-1}\, v_2 = -AX.$$

If we put $\varrho^{m-n}v_1=\omega_1$, $\varrho^{m-n}v_2=\omega_2$, these will take the more convenient form

$$\pi_m\omega_1-p^{\frac{1}{2}}\sqrt{-1}\varrho^{n-m}\omega_1=\mathrm{AX},\ \pi_m\omega_2+p^{\frac{1}{2}}\sqrt{-1}\varrho^{n-m}\omega_2=-\mathrm{AX};$$

which are only of the first order, since π_m is of that order. The equation (19.) may therefore be considered as integrated.

By the conversion of symbols (18.) and (19.) will be changed into

$$\left.\begin{aligned}\pi'_m\pi'_n u+p\varrho^{l-1}u&=\mathrm{X},\\ \pi'_m\pi'_n u+p\varrho^{l(m-n)}u&=\mathrm{X}\end{aligned}\right\}\quad\ldots\ldots\ldots\ldots\quad(20.)$$

These may be solved by means of the formulæ (*d.*) and (*f.*) in exactly the same way as (18.) and (19.), and every step of the solution in the one case may be derived from the corresponding step of the solution in the other merely by the conversion of symbols. But every solution of these equations will not be a practicable one, or be susceptible of interpretation in finite terms. The operations π'^{-1}_{m+r} however can be performed if

$$\varrho^{\int\left(\frac{\Psi(\mathrm{D})+(m+r)\lambda(\mathrm{D})}{\varphi(\mathrm{D})}\right)d\mathrm{D}=\chi(\mathrm{D})+q\mathrm{D}},$$

$\chi(\mathrm{D})$ being a rational function of D, and the constant q being positive, negative, or nothing. By differentiating the logarithms of each member relative to D, this will give

$$\frac{\Psi(\mathrm{D})+(m+r)\lambda(\mathrm{D})}{\varphi(\mathrm{D})}=\frac{q\chi(\mathrm{D})+\chi'(\mathrm{D})}{\chi(\mathrm{D})},$$

or

$$\frac{\Psi(\mathrm{D})+(m+r)\lambda(\mathrm{D})}{b-\frac{1}{2}\lambda(\mathrm{D})^2}=\frac{q\chi(\mathrm{D})+\chi'(\mathrm{D})}{\chi(\mathrm{D})\lambda'(\mathrm{D})},$$

by putting for $\frac{1}{\varphi(\mathrm{D})}$ its value, and dividing the equation by $\lambda'(\mathrm{D})$. Therefore

$$\Psi(\mathrm{D})=\frac{q\chi(\mathrm{D})+\chi'(\mathrm{D})}{\chi(\mathrm{D})\lambda'(\mathrm{D})}\left(b-\tfrac{1}{2}\lambda(\mathrm{D})^2\right)-(m+r)\lambda(\mathrm{D}).$$

Such is the value which $\Psi(\mathrm{D})$ must have in order that the solution may be practicable, $\lambda(\mathrm{D})$ being at the same time a rational function of D.

The value of $\Psi(\mathrm{D})$ is too complex for practical utility. But if we make

$$b=\tfrac{1}{2}c^2,\ \lambda(\mathrm{D})=a\mathrm{D}+c,$$

we shall have

$$\frac{b-\frac{1}{2}\lambda(\mathrm{D})^2}{\lambda'(\mathrm{D})}=-\tfrac{1}{2}\mathrm{D}(a\mathrm{D}+2c).$$

If therefore $\chi(\mathrm{D})=a\mathrm{D}+2c$, then we find

$$\Psi(\mathrm{D})=-\tfrac{1}{2}aq\mathrm{D}^2-\left(am+ar+cq+\tfrac{1}{2}a\right)\mathrm{D}-(cm+cr).$$

But if $\chi(D)=\frac{1}{2}D$, we shall have

$$\Psi(D)=-\tfrac{1}{2}aqD^3-\left(am+ar+cq+\tfrac{1}{2}a\right)D-(cm+cr+c).$$

With either of these values, and by suitably assuming m and n, we may find convenient practicable forms for both the equations (20.). And perhaps we may succeed with other assumptions. If we admit complex forms, we may have them in abundance.

IV. SOLUTION OR REDUCTION OF ANOTHER SERIES OF EQUATIONS.

The equations here treated of are generalizations and extensions of one solved by Mr. Boole in the Cambridge Mathematical Journal, No. 1, New Series. In that equation the coefficients are integer functions of x, here they may be any functions of that quantity consistent with the conditions of integrability. Also the symbol D is replaced by ϖ, and arbitrary functions of this last are introduced.

In order to abridge I shall write ϖ_a for $\varpi+a$, and $f(\varpi)$ will denote any rational function of ϖ. This being premised, let

$$f(\varpi)f(\varpi-k)\varpi(\varpi+k)u+p\varpi_a(\varpi_a+(2n+1)k)\tau^{3k}u=X. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21.)$$

Assume

$$u=(\varpi_a+3k)(\varpi_a+5k)\ldots.(\varpi_a+(2n+1)k)v,$$

the common difference of the factors being here $2k$. Then by $(a.)$ we shall have

$$(\varpi_a+(2n+1)k)\tau^{3k}u=(\varpi_a+k)\ldots.(\varpi_a+(2n+1)k)\tau^{3k}v.$$

Substituting these values and reducing the result, we find

$$f(\varpi)f(\varpi-k)\varpi(\varpi+k)v+p\varpi_a(\varpi_a+k)\tau^{3k}v=P\left\{\begin{matrix}(2n+1)k\\3k\end{matrix}\right\}X_1,$$

where the a is suppressed in the factorial of the second member for brevity. If we put

$$f(\varpi)^{-1}f(\varpi-k)^{-1}\varpi^{-1}(\varpi+k)^{-1}P\left\{\begin{matrix}(2n+1)k\\3k\end{matrix}\right\}^{-1}X=X_1,$$

we have

$$v+p\tfrac{(\varpi_a+k)\varpi_a}{f(\varpi)(\varpi+k)f(\varpi-k)\varpi}\tau^{3k}v=X_1,$$

or

$$v+p\left(\tfrac{\varpi_a+k}{f(\varpi)(\varpi+k)}\right)\tau^k\left(\tfrac{\varpi_a+k}{f(\varpi)(\varpi+k)}\right)\tau^k v=X_1, \text{ by } (a.).$$

Make

$$\left(\tfrac{\varpi_a+k}{f(\varpi)(\varpi+k)}\right)\tau^k=\varrho,$$

and the last becomes

$$v+p\varrho^2 v=X_1.$$

The mode of treating this has been already explained. By it the proposed is made to depend upon two others, each of which is of an order only half as high. In certain cases therefore this reduction amounts to a solution.

Let us next take the equation

$$\varpi_a(\varpi_a+(2n+1)k)u+pf(\varpi)f(\varpi-k)\varpi(\varpi+k)\tau^{3k}u=X, \quad . \quad . \quad . \quad . \quad (22.)$$

the arbitrary functions being put in the second term, for this only amounts to changing them and X.

Assume

$$u=(\varpi_a+3k)^{-1}\ldots.:(\varpi_a+(2n+1)k)^{-1}v.$$

Proceeding exactly as heretofore, we find

$$v+p\frac{f(\varpi)(\varpi+k)f(\varpi-k)\varpi}{(\varpi_a+k)\varpi_a}\tau^{3k}v=X_1,$$

if

$$X_1=\varpi_a^{-1}(\varpi_a+k)^{-1}P\left\{\begin{matrix}(2n-1)k\\k\end{matrix}\right\}X.$$

Making

$$\left(\frac{f(\varpi)(\varpi+k)}{\varpi_a+k}\right)\tau^k=\varrho,$$

by (*a*.) the last becomes

$$v+p\varrho^3v=X_1.$$

In the next two examples we shall put the equations under a somewhat different form, on account of their complexity. Suppose

$$u+\frac{\varpi_a(\varpi_a+k)(\varpi_a+(3n+2)k)}{f(\varpi)f(\varpi-k)f(\varpi-2k)\varpi(\varpi+k)(\varpi+2k)}p\tau^{3k}u=X.$$

Here we assume

$$u=(\varpi_a+5k)\ldots.(\varpi_a+(3n+2)k)v,$$

the common difference of the factors being $3k$.

If

$$X_1=P\left\{\begin{matrix}(3n+2)k\\5k\end{matrix}\right\}^{-1}X, \quad \varrho=\left(\frac{\varpi_a+2k}{f(\varpi)(\varpi+2k)}\right)\tau^k,$$

we shall have in this case

$$v+p\varrho^3v=X_1,$$

which may be replaced by three equations, each containing the first power only of ϱ.

As a last example let

$$u+\frac{f(\varpi)f(\varpi-k)f(\varpi-2k)\varpi(\varpi+k)(\varpi+2k)}{\varpi_a(\varpi_a+k)(\varpi_a+(3n+2)k)}p\tau^{3k}u=X.$$

Here

$$u=(\varpi_a+5k)^{-1}\ldots.(\varpi_a+(3n+2)k)^{-1}v, \quad \varrho=\left(\frac{f(\varpi)(\varpi+2k)}{\varpi_a+2k}\right)\tau^k,$$

and also

$$v+p\varrho^3v=X_1.$$

This series of equations may be continued at pleasure; and it is obvious that in the whole series, if we change ϖ into ϖ', and τ into τ'^{-1}, the resulting equations may be reduced by (*b*.) exactly as these have been by (*a*.), and that the solutions or reductions of the one series may be obtained from those of the other by the conversion of symbols.

Mr. Boole's general equation

$$X=u+af(D)\varepsilon^{\theta}u+bf(D)f(D-k)\varepsilon^{2\theta}u+\ldots..$$

may be generalized and extended in the same way. Thus if

$$X=u+af(\varpi)\tau^{k}u+bf(\varpi)f(\varpi-k)\tau^{2k}u+\ldots..,$$

or

$$X=u+af(\varpi')\tau^{l-k}u+bf(\varpi')f(\varpi'-k)\tau^{l-2k}u+\ldots.,$$

by making $\varrho=f(\varpi)\tau^{k}$ in the first, and $\varrho'=f(\varpi')\tau^{l-k}$ in the second, we shall have

$$X=u+a\varrho u+b\varrho^{2}u+\ldots=(1-p_{1}\varrho)(1-p_{2}\varrho)\ldots.u,$$

$$X=u+a\varrho'u+b\varrho'^{2}u+\ldots.=(1-p_{1}\varrho')(1-p_{2}\varrho')\ldots.u.$$

And thus each equation, by the method explained further back, will be reduced to a number of others, each of which is much more simple than the proposed.

It must be observed, that the method which has been applied to the solution of each particular class of equations will not apply to either of the other classes. We see the same thing when employing other methods, and we see no reason to suppose that our means of integrating equations will ever be greatly extended otherwise than by the multiplication or aggregation of particular methods. Such methods therefore ought not to be considered as possessing little interest. The same thing may be inferred from the various conditions of integrability at which we arrive in those cases where we can treat the same sort of equations by different methods.

Some of the examples which have been given in this paper, when reduced to the ordinary form, are very complex; but when particular forms are assigned to the arbitrary functions, results sufficiently simple may be obtained. If we will accept none but such as at first sight present themselves under a very simple form, we must not expect any great extension of our present scanty means of integration; for these equations are usually very easily integrated. The following example may serve as an illustration.

The theorem

$$\varphi D^{n}u=D^{n}\varphi u-nD^{n-1}\varphi' u+\frac{n(n-1)}{2}D^{n-2}\varphi'' u-\&c.$$

is easily verified by performing the operations D^{n}, D^{n-1}, &c. in the second member by means of the formula

$$D^{n}=D_{1}^{n}+nD_{2}D_{1}^{n-1}+\frac{n(n-1)}{2}D_{2}^{2}D_{1}^{n-2}+\&c.$$

Now let there be given the equation

$$X=\Psi u+\Psi_{1}Du+\Psi_{2}D^{2}u+\Psi_{3}D^{3}u+\ldots..$$

and let this, by the preceding theorem, be transformed into

$$X=\Phi u+D\Phi_{1}u+D^{2}\Phi_{2}u+D^{3}\Phi_{3}u+\ldots..$$

We shall have

$$\Phi=\Psi-\Psi_{1}'+\Psi_{2}''-\Psi_{3}'''+\ldots$$

$$\Phi_{1}=\Psi_{1}-2\Psi_{2}'+3\Psi_{3}''-\ldots.$$

$$\Phi_{2}=\Psi_{2}-3\Psi_{3}'+\ldots..$$

$$\Phi_{3}=\Psi_{3}-\ldots..\ \&c.$$

If $\Phi=0$, the equation is integrable once, and we shall have

$$D^{-1}X=\Phi_1 u+D\Phi_2 u+D^2\Phi_3 u+\ldots.$$

If both $\Phi=0$, and $\Phi_1=0$, it is integrable twice, and we have

$$D^{-2}X=\Phi_2 u+D\Phi_3 u+\ldots.$$

Let us take as an example the following equation of the second order,

$$\Psi_2 D^2 u+\Psi_1 Du+(\Psi'_1-\Psi''_2)u=X.$$

The transformed gives

$$D\Phi_2 u+\Phi_1 u=D^{-1}X,$$

or

$$\Psi_2 Du+(\Psi_1-\Psi'_2)u=D^{-1}X,$$

and

$$u=\left(D+\frac{\Psi_1-\Psi'_2}{\Psi_2}\right)^{-1}\Psi_2^{-1}D^{-1}X=\Psi_2^{-1}e^{-\int\frac{\Psi_1}{\Psi_2}dx}D^{-1}\Psi_2^{-1}e^{\int\frac{\Psi_1}{\Psi_2}dx}D^{-1}X.$$

This example was given by Mr. HARGREAVE in the Philosophical Transactions, 1848.

But the proposed equation may at once be put under the form

$$D\{\Psi_2 Du+(\Psi_1-\Psi'_2)u\}=X,$$

which is immediately integrable, giving

$$\Psi_2 Du+(\Psi_1-\Psi'_2)u=D^{-1}X,$$

as before found, but this is far from being a solitary example.

V. FIFTH AND SIXTH GENERAL THEOREMS.

The theorems now to be given bear no resemblance to those which have been heretofore investigated, nor is their mode of application at all similar. Making

$$\varphi\chi'=\chi_1,\ \varphi\chi'_1=\chi_2,\ \varphi\chi'_2=\chi_3,\ \&c.,$$

$$\chi\varpi^n u=\varpi^n\chi u-n\varpi^{n-1}\chi_1 u+\frac{n(n-1)}{2}\varpi^{n-2}\chi_2 u-\&c. \quad . \quad . \quad . \quad . \quad . \quad (g.)$$

will be verified by putting for ϖ its value $\varphi D+\lambda$, and actually performing the operations denoted by D, which may be readily done by the method which has now been repeatedly explained.

Again, making

$$\varphi(D)\chi'(D)=\chi_1(D),\ \varphi(D)\chi'_1(D)=\chi_2(D),\ \varphi(D)\chi'_2(D)=\chi_3(D),\ \&c.,$$

we shall have

$$\chi(D)\varpi'^n u=\varpi'^n\chi(D)u+n\varpi'^{n-1}\chi_1(D)u+\frac{n(n-1)}{2}\varpi'^{n-2}\chi_2(D)u+\&c. \quad . \quad . \quad . \quad (h.)$$

This also may be verified by putting for ϖ' its value $\varphi(D)x+\lambda(D)$, and performing the operations D, where required, by the formula

$$f(D)=f(D_1)+D_x f'(D_1)+\frac{1}{2}D_x^2 f''(D_1)+\&c.,$$

and afterwards dropping the marks of distinction.

Application of the Theorem (g.) *to Integration.*

To apply the theorem (*g.*), let us suppose that we have given the equation

$$X=\Psi u+\Psi^{1}\varpi u+\Psi^{2}\varpi^{2}u+\&c. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23.)$$

Let this by (*g.*) be transformed into

$$X=\Phi u+\varpi\Phi_{1}u+\varpi^{2}\Phi_{2}u+\&c.$$

We shall have

$$\Phi=\Psi-\Psi_{1}^{1}+\Psi_{2}^{2}-\Psi_{3}^{3}+\ldots$$

$$\Phi_{1}=\Psi^{1}-2\Psi_{1}^{2}+3\Psi_{2}^{3}-\ldots.$$

$$\Phi_{2}=\Psi^{2}-3\Psi_{1}^{3}+\ldots.$$

$$\Phi_{3}=\Psi^{3}-\ldots.\&c.$$

The figure at the top of Ψ denotes the place of the term in the given equation, that at the bottom marks the term in (*g.*) corresponding to it*. By going a little into the operations, the meaning of these symbols will be easily understood.

If $\Phi=0$, or $\Psi=\Psi_{1}^{1}-\Psi_{2}^{2}+\ldots$ the equation thus transformed will be integrable once, and we shall have

$$\varpi^{-1}X=\Phi_{1}u+\varpi\Phi_{2}u+\varpi^{2}\Phi_{3}u+\&c.$$

If, moreover, $\Phi_{1}=0$, it will be twice integrable.

Let us take as a more particular example the equation

$$\Psi^{2}\varpi^{2}u+\Psi^{1}\varpi u+(\Psi_{1}^{1}-\Psi_{2}^{2})u=X,$$

which is integrable because $\Phi=0$. Putting for Ψ_{1}^{1}, Ψ_{2}^{2} their values, this becomes

$$\Psi^{2}\varpi^{2}u+\Psi^{1}\varpi u+\rho(\Psi^{1\prime}-\rho'\Psi^{2}-\rho\Psi^{2\prime})u=X. \quad . \quad . \quad . \quad . \quad . \quad (24.)$$

The transformed equation will be

$$\varpi^{2}\Phi_{2}u+\varpi\Phi_{1}u=X,$$

or

$$\varpi\Phi_{2}u+\Phi_{1}u=\varpi^{-1}X;$$

which by substituting for ϖ its value, is easily reduced to

$$Du+\left(\frac{\Phi_{2}'}{\Phi_{2}}+\frac{\lambda}{\rho}+\frac{\Phi_{1}}{\rho\Phi_{2}}\right)u=(\rho\Phi_{2})^{-1}\varpi^{-1}X.$$

In order to abridge, make

$$\frac{\Phi_{2}'}{\Phi_{2}}+\frac{\lambda}{\rho}+\frac{\Phi_{1}}{\rho\Phi_{2}}=\frac{\lambda}{\rho}+\frac{\Psi^{1}}{\rho\Psi^{2}}-\frac{\Psi^{2\prime}}{\Psi^{2}}=\theta;$$

then the last becomes

$$(D+\theta)u=(\rho\Psi^{2})^{-1}\varpi^{-1}X,$$

which gives

$$u=(D+\theta)^{-1}(\rho\Psi^{2})^{-1}\varpi^{-1}X,$$

or

$$u=\varepsilon^{-\int\theta dx}D^{-1}\varepsilon^{\int\theta dx}(\rho\Psi^{2})^{-1}\varepsilon^{-\int\frac{\lambda}{\rho}dx}D^{-1}\varepsilon^{\int\frac{\lambda}{\rho}dx}\phi^{-1}X.$$

* The figures at the top of Ψ are not exponents of powers, but distinct functional marks.

If (24.) be reduced to the ordinary form, it becomes

$$D^2u+\left(\frac{\varphi'+2\lambda}{\varphi}+\frac{\Psi^1}{\varphi\Psi^2}\right)Du+\left(\frac{\varphi\lambda'+\lambda^2}{\varphi^2}+\frac{\Psi'(\varphi+\lambda)}{\varphi^2\Psi^2}+\frac{\Psi''-\varphi'\Psi''-\varphi\Psi'''}{\varphi\Psi^2}\right)u=\frac{X}{\varphi^2\Psi^2}.$$

This would give a very large number of integrable equations by assigning particular forms to the arbitrary functions φ, λ, Ψ^1, and Ψ^2. And indeed every one of the equations treated of in this paper, by giving particular forms to the arbitrary functions, would furnish a large number of particular ones. This circumstance makes the chance of our being able to put a given equation under some one of these forms the greater, and consequently in this respect enhances the value of the methods employed.

Application of the Theorem (h.).

The equations to be reduced by (*h.*) are of the form

$$X=\Psi(D)u+\Psi^1(D)\varpi'u+\Psi^2(D)\varpi'^2u+\&c. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25.)$$

This is deduced from (23.) by the interchange of the symbols D and x, but the theorems (*g.*) and (*h.*) by which the transformations are effected cannot be thus deduced the one from the other.

The above will be transformed by (*h.*) into

$$X=\Phi(D)u+\varpi'\Phi_1(D)u+\varpi'^2\Phi_2(D)u+\ldots\ldots,$$

where

$$\Phi(D)=\Psi(D)+\Psi_1^1(D)+\Psi_2^2(D)+\Psi_3^3(D)+\ldots$$

$$\Phi_1(D)=\Psi^1(D)+2\Psi_1^2(D)+3\Psi_2^3(D)+\ldots.$$

$$\Phi_2(D)=\Psi^2(D)+3\Psi_1^3(D)+\ldots\ldots$$

$$\Phi_3(D)=\Psi^3(D)+\ldots\&c.$$

If

$$\Phi(D)=\Psi(D)+\Psi_1^1(D)+\Psi_2^2(D)+\ldots=0,$$

or

$$\Psi(D)=-\Psi_1^1(D)-\Psi_2^2(D)-\ldots.,$$

we shall have

$$\varpi'^{-1}X=\Phi_1(D)u+\varpi'\Phi_2(D)u+\ldots.$$

We might take as examples equations of a higher order than the second, but as these last are of the greater importance I have hitherto selected them, and shall take here the equation

$$\Psi^2(D)\varpi'^2u+\Psi^1(D)\varpi'u-\left(\Psi_1^1(D)+\Psi_2^2(D)\right)u=X,$$

which by putting for $\Psi_1^1(D)$ and $\Psi_2^2(D)$ their values, the figures here at the top and bottom of Ψ signifying the same as in the former case, becomes

$$\Psi^2(D)\varpi'^2u+\Psi^1(D)\varpi'u-\varphi(D)\{\Psi''(D)+\varphi'(D)\Psi'''(D)+\varphi(D)\Psi''''(D)\}u=X. \quad . \quad (26.)$$

The transformed equation gives

$$\varpi'\Phi_2(D)u+\Phi_1(D)u=\varpi'^{-1}X;$$

which, reduced to the most simple form, is equivalent to

$$\varphi(D)\Phi_2(D)xu+\{\Phi_1(D)+\lambda(D)\Phi_2(D)-\varphi(D)\Phi_2'(D)\}u=\varpi^{j-1}X.$$

Make

$$\theta(D)=\frac{\Phi_1(D)}{\varphi(D)\Phi_2(D)}+\frac{\lambda(D)}{\varphi(D)}-\frac{\Phi_2'(D)}{\Phi_2(D)}$$
$$=\frac{\Psi^1(D)}{\varphi(D)\Psi^2(D)}+\frac{\lambda(D)}{\varphi(D)}+\frac{\Psi^2(D)}{\Psi^2(D)}.$$

Substituting this value in the last equation, after dividing both members by $\varphi(D)\Phi_2(D)$, or in other words, operating with $\varphi(D)^{-1}\Phi_2(D)^{-1}$ on both members, we have

$$\{x+\theta(D)\}u=\{\varphi(D)\Phi_2(D)\}^{-1}\varpi^{j-1}X$$

and

$$u=\{x+\theta(D)\}^{-1}\{\varphi(D)\Psi^2(D)\}^{-1}\varpi^{j-1}X,$$

where the value of $\Phi_2(D)$ has been substituted. By further reduction,

$$u=\varepsilon^{\int\theta(D)dD}x^{-1}\varepsilon^{-\int\theta(D)dD}\Big(\varphi(D)\Psi^2(D)\Big)^{-1}\varepsilon^{\int\frac{\lambda(D)}{\varphi(D)}dD}x^{-1}\varepsilon^{-\int\frac{\lambda(D)}{\varphi(D)}dD}\varphi(D)^{-1}X.$$

The equation itself requires that $\Psi^1(D)$, $\Psi^2(D)$, $\varphi(D)$, and $\lambda(D)$ should be rational functions of (D); and that the solution may be practicable, we must have

$$\varepsilon^{\int\frac{\lambda(D)}{\varphi(D)}dD}=\chi(D)\varepsilon^{mD},$$

$\chi(D)$ being a rational function. Also the term $\frac{\Psi^1(D)}{\varphi(D)\Psi^2(D)}$ in the value of $\theta(D)$ requires that we should have

$$\varepsilon^{\int\frac{\Psi^1(D)dD}{\varphi(D)\Psi^2(D)}}=\Phi(D)\varepsilon^{pD},$$

$\Phi(D)$ being a rational function. These assumptions give, by taking the differentials of their logarithms,

$$\lambda(D)=\varphi(D)\Big(m+\frac{\chi'(D)}{\chi(D)}\Big),\ \Psi^1(D)=\varphi(D)\Psi^2(D)\Big(p+\frac{\Phi'(D)}{\Phi(D)}\Big).$$

If we were to substitute these values in (26.), we should have a very complex resulting equation; but by giving suitable particular values to the arbitrary functions, and perhaps by changing u into $f(D)z$, giving to $f(D)$ a convenient form, we might obtain resulting equations sufficiently simple, and we might obtain some very general of their kind, remembering to introduce as many arbitrary constants as we can.

Make

$$\chi(D)=\varphi(D),\ \Phi(D)=\Psi^2(D);$$

then

$$\lambda(D)=m\varphi(D)+\varphi'(D),\ \Psi^1(D)=\varphi(D)\Big(p\Psi^2(D)+\Psi^{2\prime}(D)\Big).$$

With these values (26.) becomes, dropping the figure 2 on the top of Ψ, as being no longer needed,

$$\Psi(D)\varpi^{j}u+\varphi(D)\Big(p\Psi(D)+\Psi'(D)\Big)\varpi^{j}u-\varphi(D)\frac{d}{dD}\Big\{\varphi(D)\Big(p\Psi(D)+2\Psi'(D)\Big)\Big\}u=X, \quad (27.)$$

which is of a remarkable form. Also

$$u=\varphi(\mathrm{D})\Psi(\mathrm{D})^{2}e^{(m+p)\mathrm{D}}x^{-1}\{\varphi(\mathrm{D})\Psi(\mathrm{D})^{2}\}^{-1}e^{-p\mathrm{D}}x^{-1}\varphi(\mathrm{D})^{-2}e^{-m\mathrm{D}}\mathrm{X}.$$

The equation (27.) might very well be reduced to the ordinary form without particularizing the arbitrary functions.

If we make

$$\chi(\mathrm{D})=\varphi(\mathrm{D}),\ \Phi(\mathrm{D}=\varphi(\mathrm{D}),$$

and change Ψ^{2} into Ψ, we shall have

$$\lambda(\mathrm{D})=m\varphi(\mathrm{D})+\varphi'(\mathrm{D}),\ \Psi^{1}(\mathrm{D})=\Psi(\mathrm{D})\big(p\varphi(\mathrm{D})+\varphi'(\mathrm{D})\big).$$

With these values (26.) will become, operating with $\Psi(\mathrm{D})^{-1}$ on both members,

$$w''u+\big(p\varphi(\mathrm{D})+\varphi'(\mathrm{D})\big)w'u-\frac{\varphi(\mathrm{D})}{\Psi(\mathrm{D})}\frac{d}{d\mathrm{D}}\Big\{p\varphi(\mathrm{D})\Psi(\mathrm{D})+\frac{d}{d\mathrm{D}}\big(\varphi(\mathrm{D})\Psi(\mathrm{D})\big)\Big\}u=\Psi(\mathrm{D})^{-1}\mathrm{X},\quad(28.)$$

which is of a form not a little singular, and which might be put under the common form, retaining the arbitrary functions.

The value of u in this example is

$$u=\varphi(\mathrm{D})^{2}\Psi(\mathrm{D})e^{(m+p)\mathrm{D}}x^{-1}\{\varphi(\mathrm{D})\Psi(\mathrm{D})\}^{-2}e^{-p\mathrm{D}}x^{-1}\varphi(\mathrm{D})^{-2}e^{-m\mathrm{D}}\mathrm{X}.$$

In the two last examples we see from the expression of the value of u that they are practicable, or that their solutions can be interpreted.

It may be well to observe, that if we make

$$\varphi(\mathrm{D})=\alpha(\mathrm{D})\chi(\mathrm{D}),\ \Psi^{2}(\mathrm{D})=\beta(\mathrm{D})\Phi(\mathrm{D}),$$

which give

$$\lambda(\mathrm{D})=\alpha(\mathrm{D})\big(m\chi(\mathrm{D})+\chi'(\mathrm{D})\big),\ \Psi^{1}(\mathrm{D})=\alpha(\mathrm{D})\beta(\mathrm{D})\chi(\mathrm{D})\big(p\Phi(\mathrm{D})+\Phi'(\mathrm{D})\big);$$

and if we eliminate both $\varphi(\mathrm{D})$ and $\lambda(\mathrm{D})$ from the value of w', and $\Psi^{1}(\mathrm{D})$, $\Psi^{2}(\mathrm{D})$ from (26.), our equation by the substitution of these values will then contain the four arbitrary functions $\alpha(\mathrm{D})$, $\beta(\mathrm{D})$, $\chi(\mathrm{D})$ and $\Phi(\mathrm{D})$. These should then be made integer functions of D; but after actual substitution the resulting equation would be very complicated, unless we first give particular forms to the arbitrary functions. If this be done, and as many constants as possible be introduced, particular integrable equations of interest and value, and of sufficient simplicity, may be obtained.

I shall terminate this paper by observing, that in our attempts at integration, we are apt to seek for equations which immediately present themselves under simple forms, and by this means fall chiefly, or altogether, upon such as could previously be integrated.

Gunthwaite Hall, near Barnsley, Yorkshire,
February 4th, 1850.

XX. *Additional Observations on the Diffusion of Liquids.* (*Third Memoir.*)

By Thomas Graham, *Esq.*, *F.R.S.*, *F.C.S.* &c.

Received March 27,—Read May 22, 1851.

THE experiments on diffusion to be presently described are in continuation of those detailed in my last publication on the subject, the same method of observing being followed without change. The diffusion is generally made from solutions in four different proportions, so as to exhibit pretty fully the characters of the property in question for each salt, so far as is possible at a constant temperature. The salts operated upon are of two bases only, potash and soda; but the acids are considerably varied, so as to include hydrates, carbonates, sulphates, sulphites, hyposulphites, sulphovinates, oxalates, acetates and tartrates. Salts of almost every class of acids come thus to be represented, either in the present or former series of experiments on diffusion, while at the same time much information is elicited respecting the diffusive relation of the two important bases named.

Comparison of Salts of Potash and Soda.

Hydrate of Potash.—Time of diffusion 4·041 days. The usual number of eight cells of the 1 and 2 per cent. solutions of this substance were diffused, and four cells of the 4 and 8 per cent. solutions. The whole diffusates of each proportion were then mixed together, and the quantity of hydrate of potash diffused for two cells determined by an alkalimetric experiment, which was always repeated twice.

1. One per cent. of hydrate of potash, density 1·00978, diffused at 63°·4, gave 6·56 grs. of hydrate of potash for two cells.

2. 2·005 per cent. of hydrate of potash, density 1·01878, diffused at 63°·4, gave 12·88 grs. for two cells; calculated for 2 per cent., 12·84 grs. of hydrate of potash in two cells.

3. 4·01 per cent. of hydrate of potash, density 1·0366, diffused at 63°·4, gave 12·56 grs. for one cell; calculated for 4 per cent., 12·52 grs. of hydrate of potash in one cell.

4. 8·02 per cent. of hydrate of potash, density 1·07069, diffused at 63°·4, gave 26·20 grs. for one cell; calculated for 8 per cent., 26·12 grs. of hydrate of potash in one cell.

Diffusion of Hydrate of Potash in 4·04 days at 63°·4; two cells.

	Grs.	Ratio.
From 1 per cent. solution . . .	6·56	1·022
From 2 per cent. solution . . .	12·84	2
From 4 per cent. solution . . .	25·04	3·900
From 8 per cent. solution . . .	52·24	8·137

Hydrate of Soda.—Time of diffusion 4·9497 days; the times chosen as hitherto for the corresponding potash and soda salts being always, when squared, in the proportion of 2 to 3. The usual number of eight cells of the 1 and 2 per cent. solutions were diffused, and four cells of the 4 and 8 per cent. solutions. The whole diffusates of each proportion were then mixed together, and the quantity of hydrate of soda diffused for two cells determined by alkalimetric experiments.

1. 1·043 per cent. of hydrate of soda, density 1·01256, diffused at 63°·3, gave 6·06 grs. for two cells; calculated for 1 per cent., 5·81 grs. of hydrate of soda in two cells.

2. 2·087 per cent. of hydrate of soda, density 1·0242, diffused at 63°·3, gave 11·58 grs. for two cells; calculated for 2 per cent., 11·09 grs. of hydrate of soda in two cells.

3. 4·177 per cent. of hydrate of soda, density 1·04666, diffused at 63°·3, gave 10·90 grs. for one cell; calculated for 4 per cent., 10·43 grs. of hydrate of soda in one cell.

4. 8·35 per cent. of hydrate of soda, density 1·08846, diffused at 63°·3, gave 21·10 grs. for one cell; calculated for 8 per cent., 20·22 grs. of hydrate of soda in one cell.

Diffusion of Hydrate of Soda in 4·95 days at 63°·3; two cells.

	Grs.	Ratio.
From 1 per cent. solution . . .	5·81	1·048
From 2 per cent. solution . . .	11·09	2
From 4 per cent. solution . . .	20·86	3·765
From 8 per cent. solution . . .	40·44	7·30

The nearest approach to equality of diffusion in the hydrates of potash and soda, is exhibited by the 1 per cent. solutions, which are as 6·56 to 5·81, or as 100 to 88·57, being a difference of so much as 11·43 per cent.

Comparative Diffusion of Hydrate of Potash in 4·041 days at 63°·4, and of Hydrate of Soda in 4·95 days at 63°·3.

	From solutions of			
	1 per cent.	2 per cent.	4 per cent.	8 per cent.
Hydrate of potash	100	100	100	100
Hydrate of soda.....	88·57	86·37	83·30	77·41

The marked departure from the usual ratio of diffusion, here exhibited, must not be overlooked in considering the relation between potash and soda salts.

Carbonate of Potash.—Time of diffusion 8·083 days, or double the time of hydrate of potash. The usual number of eight cells of the 1 and 2 per cent. solutions were diffused, with four cells of the 4 and 8 per cent. solutions, and the diffusates of each proportion determined by alkalimetric experiments as with the preceding alkaline hydrates.

1. 1·002 per cent. of carbonate of potash, density 1·00957, diffused at 63°·7, gave 6·15 grs. for two cells; calculated for 1 per cent., 6·13 grs. of carbonate of potash in two cells.

2. 2·005 per cent. of carbonate of potash, density 1·01843, diffused at 63°·7, gave 11·95 grs. for two cells; calculated for 2 per cent., 11·92 grs. of carbonate of potash in two cells.

3. 4·01 per cent. of carbonate of potash, density 1·03577, diffused at 63°·7, gave 11·48 grs. for one cell; calculated for 4 per cent., 11·44 grs. of carbonate of potash in one cell.

4. 8·02 per cent. of carbonate of potash, density 1·06935, diffused at 63°·7, gave 22·79 grs. for one cell; calculated for 8 per cent., 22·72 grs. of carbonate of potash in one cell.

5. 2·005 per cent. of carbonate of potash, density 1·01843, diffused at 68°·2, gave 12·64 grs. for two cells; calculated for 2 per cent., 12·62 grs. of carbonate of potash in two cells.

Diffusion of Carbonate of Potash in 8·08 days at 63°·7; two cells.

	Grs.	Ratio.
From 1 per cent. solution . . .	6·13	1·028
From 2 per cent. solution . . .	11·92	2
From 4 per cent. solution . . .	22·88	3·839
From 8 per cent. solution . . .	45·44	7·624

Comparative Diffusion of Hydrate of Potash in 4·04 days at 63°·4, and Carbonate of Potash in 8·08 days at 63°·7.

	From solutions of			
	1 per cent.	2 per cent.	4 per cent.	8 per cent.
Hydrate of potash	100	100	100	100
Carbonate of potash	93·48	92·81	91·36	86·97

The diffusion of hydrate of potash appears to be excessive when compared with the carbonate of potash, as well as when compared with the hydrate of soda.

Carbonate of Soda.—Time of diffusion 9·8994 days. The usual number of eight cells of the 1 and 2 per cent. solutions of this substance were diffused, and four cells of the 4 and 8 per cent. solutions, and the diffusates determined by alkalimetric experiments.

1. 1·003 per cent. of carbonate of soda, density 1·01120, diffused at 63°·4, gave 6·04 grs. for two cells; calculated for 1 per cent., 6·02 grs. of carbonate of soda in two cells.

2. 2·007 per cent. of carbonate of soda, density 1·0216, diffused at 63°·4, gave 11·74 grs. for two cells; calculated for 2 per cent., 11·70 grs. of carbonate of soda in two cells.

3. 4·015 per cent. of carbonate of soda, density 1·04156, diffused at 63°·4, gave 10·74 grs. for one cell; calculated for 4 per cent., 10·71 grs. of carbonate of soda in one cell.

4. 8·03 per cent. of carbonate of soda, density 1·0800, diffused at 63°·4, gave 19·93 grs. for one cell; calculated for 8 per cent., 19·87 grs. of carbonate of soda in one cell.

In the two following additional experiments the diffusate was treated with hydrochloric acid, and estimated from the weight of chloride of sodium produced.

5. 2·007 per cent. of carbonate of soda, density 1·02166, diffused at 68°·1, gave 12·06 grs. for two cells; calculated for 2 per cent., 12·02 grs. of carbonate of soda in two cells at 68°·1.

6. 2·13 per cent. of carbonate of soda, density 1·02246, diffused at 59°·6 in eight cells, 11·12, 11·34, 11·17, 11·58; mean 11·38 grs. for two cells; calculated for 2 per cent., 10·65 grs. of carbonate of soda in two cells at 59°·6.

Diffusion of Carbonate of Soda in 9·9 days at 63°·4; two cells.

	Grs.	Ratio.
From 1 per cent. solution . . .	6·02	1·028
From 2 per cent. solution . . .	11·70	2
From 4 per cent. solution . . .	21·42	3·661
From 8 per cent. solution . . .	39·74	6·792

The similarity of diffusion between the carbonates of potash and soda is remarkable in the 1 per cent. solution, which are as 6·13 to 6·02, or as 100 to 98·2; also in the 2 per cent. solutions, which were observed at two different temperatures.

Diffusion of 2 per cent. solutions of Carbonate of Potash in 8·083 days, and Carbonate of Soda in 9·9 days.

	Grs.	Ratio.
Carbonate of potash at 63°·7 . . .	11·92	100
Carbonate of soda at 63°·4	11·70	98·15
Carbonate of potash at 68°·2 . . .	12·62	100
Carbonate of soda at 68°·1	12·02	95·24

Comparative Diffusion of Carbonate of Potash in 8·083 days at 63°·7, and Carbonate of Soda in 9·9 days at 63°·4.

	From solutions of			
	1 per cent.	2 per cent.	4 per cent.	8 per cent.
Carbonate of potash	100	100	100	100
Carbonate of soda	98·20	98·15	93·63	87·44

The carbonate of soda appears to preserve more analogy to the hydrate of soda in its diffusion than the carbonate of potash to the hydrate of potash.

Comparative Diffusion of Hydrate of Soda in 4·937 days at 63°·3, and Carbonate of Soda in 9·9 days at 63°·4.

	From solutions of			
	1 per cent.	2 per cent.	4 per cent.	8 per cent.
Hydrate of soda	100	100	100	100
Carbonate of soda	103·62	105·50	102·68	98·27

Here we have a nearly equal diffusion with the times as 1 to 2 for the hydrate and carbonate of soda respectively.

Sulphate of Potash.—Time of diffusion 8·083 days. The solution of diffused salt was evaporated to dryness and weighed.

1. 1·005 per cent. of sulphate of potash, density 1·00844, diffused at 60°·3, in eight cells, 6·11, 6·21, 6·12, 6·32; mean 6·19 grs. in two cells; calculated for 1 per cent., 6·16 grs. of sulphate of potash in two cells.

2. 2·01 per cent. of sulphate of potash, density 1·01653, diffused at 60°·3, in eight cells, 11·61, 11·58, 11·50, 11·97; mean 11·66 grs. in two cells; calculated for 2 per cent., 11·60 grs. of sulphate of potash in two cells.

3. 4·02 per cent. of sulphate of potash, density 1·03240, diffused at 60°·3, in four cells, 10·80, 11·33, 11·20, 12·07; mean 11·35 grs. for one cell; calculated for 4 per cent. 11·35 grs. of sulphate of potash in one cell.

4. 8·04 per cent. of sulphate of potash, density 1·06306, diffused at 60°·3, in four cells, 21·91, 21·93, 22·00, 22·47; mean 22·08 grs. for one cell; calculated for 8 per cent., 21·96 grs. of sulphate of potash in one cell.

Diffusion of Sulphate of Potash in 8·083 days at 60°·3; two cells.

	Grs.	Ratio.
From 1 per cent. solution . . .	6·16	1·062
From 2 per cent. solution . . .	11·60	2
From 4 per cent. solution . . .	22·70	3·914
From 8 per cent. solution . . .	43·92	75·72

The diffusion of sulphate of potash above, at 60°·3, appears to be similar to that of carbonate of potash at 63°·7, a higher temperature by 3°·4 (p. 413). The numbers for the sulphate of potash and hydrate of potash (p. 412) would probably correspond

closely at the same temperature, the times of diffusion of the two substances being taken as 2 to 1.

Sulphate of Soda.—Time of diffusion 9·9 days. The diffusate was evaporated to dryness and weighed.

1. One per cent. of sulphate of soda, density 1·00940, diffused at 59°·9, in eight cells, 6·31, 6·47, 6·21, 6·32; mean 6·33 grs. for two cells.

2. 1·995 per cent. of sulphate of soda, density 1·01832, diffused at 59°·9, in eight cells, 12·02, 12·10, 11·66, 12·16; mean 11·98 grs. for two cells; calculated for 2 per cent., 12·00 grs. of sulphate of soda in two cells at 59°·9.

3. 3·99 per cent. of sulphate of soda, density 1·03594, diffused at 59°·9, in four cells, 10·00, 10·80, 11·53, 11·53; mean 10·96 grs. for one cell; calculated for 4 per cent., 10·98 grs. of sulphate of soda in one cell at 59°·9.

4. 7·98 per cent. of sulphate of soda, density 1·06960, diffused at 59°·9, in four cells, 20·50, 21·36, 20·01, 20·68; mean 20·64 grs. for one cell; calculated for 8 per cent., 20·69 grs. of sulphate of soda in one cell at 59°·9.

Diffusion of Sulphate of Soda in 9·9 days at 59°·9; two cells.

	Grs.	Ratio.
From 1 per cent. solution . . .	6·33	1·055
From 2 per cent. solution . . .	12·00	2
From 4 per cent. solution . . .	21·96	3·66
From 8 per cent. solution . . .	41·38	6·896

The 2 per cent. solution of sulphate of soda above, at 59°·9, appears to be more diffusive than the corresponding solution of carbonate of soda at 59°·6, as 12·00 to 10·65 grs., or as 100 to 88·75.

The assumed relation in the times of equal diffusion for salts of potash and soda appears to derive support from the sulphates, particularly in the lower proportions of salt.

Comparative Diffusion of Sulphate of Potash in 8·083 days at 60°·3, and Sulphate of Soda in 9·9 days at 59°·9.

	From solutions of			
	1 per cent.	2 per cent.	4 per cent.	8 per cent.
Sulphate of potash	100	100	100	100
Sulphate of soda	102·74	103·45	96·74	94·21

Another set of experiments made upon these two salts, simultaneously with certain oxalates and acetates which follow, may be placed here to illustrate the same relation. The preceding times of diffusion were observed.

5. 2·013 per cent. of sulphate of potash, diffused at 59°·8, in eight cells, gave 11·90, 11·94, 11·41, 11·71; mean 11·74 grs. for two cells; calculated for 2 per cent., 11·67 grs. of sulphate of potash in two cells.

6. Two per cent. of sulphate of soda, density 1·01846, diffused at 59°·6, in eight cells, gave 11·58, 11·56, 11·94, 11·97; mean 11·76 grs. of sulphate of soda in two cells.

The present results, from 2 per cent. solutions of these salts, are as follows:—

Sulphate of potash in 8·083 days at 59°·8 . . .	11·67	100
Sulphate of soda in 9·9 days at 59°·6	11·76	100·94

The diffusion in the selected times appears therefore to be, as nearly as possible, equal at the present temperatures.

Another experiment may be recorded at present, of which the object was to ascertain the influence of free sulphuric acid upon the diffusion of sulphate of soda.

7. Sulphate of soda, 2·02 per cent., to which an equivalent quantity of sulphuric acid was added, had the density 1·02703. This solution, diffused into pure water, in eight cells, at 59°·6, for the same time as the preceding sulphate of soda, gave a diffusate of 9·70, 10·06, 10·17 and 10·29 grs. of sulphate of soda; mean 10·05 grs. in two cells; calculated for 2 per cent., this diffusate becomes 9·95 grs. The presence of one equivalent of sulphuric acid appears therefore to diminish the diffusion of sulphate of soda considerably, namely, from 11·76 to 9·95 grs., or from 100 to 84·61. The free sulphuric acid diffused in the experiment was not determined.

Sulphite of Potash.—It was curious to observe how far this and other salts of the oxygen acids of sulphur correspond with the sulphates. Time of diffusion 8·083 days. The diffusate was treated with sulphuric acid and estimated in the form of neutral sulphate of potash.

1·90 per cent. of sulphite of potash, density 1·01753, diffused at 59°·5, in eight cells, gave 11·36, 10·72, 11·22 and 10·90 grs. in two cells; mean 11·05 grs. in two cells; calculated for 2 per cent., 11·63 grs.

Making the comparison for 2 per cent. solutions we have—

Sulphate of potash at 60°·3 . . .	11·60	100
Sulphite of potash at 59°·5 . . .	11·63	100·26

The two salts appear to have the same diffusibility.

Sulphite of Soda.—Time of diffusion 9·9 days. The diffusate was estimated in the same manner as the preceding salt.

2·26 per cent. of sulphite of soda, density 1·02330, diffused at 59°·6, in eight cells, gave 12·93, 13·35, 13·56 and 13·62 grs. in two cells; mean 13·37 grs. in two cells; calculated for 2 per cent., 11·83 grs.

The comparative diffusion of the sulphites of the two bases is as follows:—

Sulphite of potash in 8·083 days at 59°·5 . . .	11·63	100
Sulphite of soda in 9·9 days at 59°·6	11·83	100·72

The sulphites of potash and soda appear therefore to exhibit the usual relation of these two bases.

Hyposulphite of Potash.—Time of diffusion 8·083 days. The diffusate was evaporated to dryness with sulphuric acid, ignited and estimated from the sulphate produced.

1·925 per cent. of anhydrous hyposulphite of potash, density 1·0150, diffused at 59°·8, in eight cells, gave 11·66, 11·67, 12·01, 12·26 grs.; mean 11·90 grs. for two cells; calculated for 2 per cent., 12·37 grs. of hyposulphite of potash in two cells.

Comparing this salt with the sulphate of the same base, we have—

Sulphate of potash at 60°·3 . . .	11·60	100
Hyposulphite of potash at 59°·8 . .	12·37	106·44

A slight error in excess may possibly have been introduced from the circumstance that the hyposulphite of potash employed was not absolutely free from sulphate.

Hyposulphite of Soda.—Time of diffusion 9·9 days. The diffusate was estimated in the same manner as the preceding salt.

2·136 per cent. of anhydrous hyposulphite of soda, density 1·01778, diffused at 59°·9, in eight cells, gave 13·11, 12·77, 12·92, 11·99 grs.; mean 12·70 grs. for two cells; calculated for 2 per cent., 11·89 grs. of hyposulphite of soda in two cells.

The comparative diffusion of the salts of potash and soda appears thus:—

Hyposulphite of potash in 8·083 days at 59°·8 . . .	12·37	100
Hyposulphite of soda in 9·9 days at 59°·9	11·89	96·21

The relation of the sulphate and hyposulphite of soda appears still closer.

Sulphate of soda at 59°·6	11·76	100
Hyposulphite of soda at 59°·9 . .	11·89	101·10

Sulphovinate of Potash.—Time of diffusion 8·083 days. The diffusate was evaporated to dryness and ignited with sulphuric acid.

1·966 per cent. of sulphovinate of potash, of density 1·00977, diffused at 59°·8, in eight cells, gave 12·57, 12·48, 12·12 and 12·39 grs.; mean 12·39 grs. for two cells; calculated for 2 per cent., 12·60 grs. of sulphovinate of potash in two cells.

The diffusion of this salt appears in excess when compared with sulphate of potash.

Sulphate of potash at 60°·3 . . .	11·60	100
Sulphovinate of potash at 59°·8 . .	12·60	108·62

Sulphovinate of Soda.—Time of diffusion 9·9 days. The diffusate was estimated in the same manner as the preceding salt.

2·063 per cent. of sulphovinate of soda, of density 1·00944, diffused at 59°·6, in eight cells, gave 13·32, 13·50, 13·78 and 13·15 grs.; mean 13·44 grs. for two cells; calculated for 2 per cent., 13·03 grs. of sulphovinate of soda in two cells.

Comparing the two sulphovinates together, we have—

Sulphovinate of potash in 8·083 days at 59°·8 . . .	12·60	100
Sulphovinate of soda in 9·9 days at 59°·6	13·03	103·41

Neither of these sulphovinates appeared to suffer decomposition in the act of diffusion, at least to any considerable extent. But the experiments on both of these salts are less precise than usual, from the difficulty of obtaining them entirely free from sulphates.

Oxalate of Potash.—Time of diffusion 8·083 days. The diffusate was evaporated to dryness, ignited, and then converted into chloride of potassium.

1. 0·981 per cent. of anhydrous oxalate of potash, density 1·00775, diffused at 59°·8, in eight cells, 6·24, 6·22, 5·91, 5·97; mean 6·08 grs. for two cells; calculated for 1 per cent., 6·20 grs. of oxalate of potash in two cells.

2. 1·92 per cent. of oxalate of potash, density 1·01463, diffused at 59°·9, in eight cells, 11·54, 11·63, 11·94, 11·63; mean 11·68 grs. for two cells; calculated for 2 per cent., 12·17 grs. of oxalate of potash, in two cells.

3. 3·84 per cent. of oxalate of potash, density 1·02864, diffused at 59°·9, in four cells, 11·10, 11·16, 11·09, 10·92; mean 11·07 grs. for one cell; calculated for 4 per cent., 11·52 grs. of oxalate of potash in one cell.

4. 7·68 per cent. of oxalate of potash, density 1·05604, diffused at 59°·9, in four cells, 20·45, 20·63, 20·99, 20·15; mean 20·55 grs. for one cell; calculated for 8 per cent., 21·41 grs. of oxalate of potash in one cell.

Diffusion of Oxalate of Potash in 8·083 days at 59°·9; two cells.

	Grs.	Ratio.
From 1 per cent. solution . . .	6·20	1·019
From 2 per cent. solution . . .	12·17	2
From 4 per cent. solution . . .	23·04	3·789
From 8 per cent. solution . . .	42·82	7·042

The oxalate of potash corresponds well with sulphate of potash.

Comparative diffusion in 8·083 days of Sulphate of Potash at 60°·3, and Oxalate of Potash at 59°·9.

	From solutions of			
	1 per cent.	2 per cent.	4 per cent.	8 per cent.
Sulphate of potash	100	100	100	100
Oxalate of potash	100·65	104·91	101·50	97·47

The sensible excess in the diffusion of the 2 per cent. solution of oxalate of potash is, I believe, to be accounted for by this proportion having been estimated in the form of carbonate of potash instead of chloride of potassium.

Oxalate of Soda.—Time of diffusion 9·9 days. The diffusate was evaporated to dryness, ignited and then converted into chloride, as with the preceding oxalate of potash.

1. 0·925 per cent. of oxalate of soda, density 1·00882, diffused at 59°·9, in eight cells, 5·99, 5·81, 5·69, 5·59; mean 5·77 grs. for 2 cells; calculated for 1 per cent., 6·24 grs. of oxalate of soda in two cells.

It was found impossible to form a 2 per cent. solution of this salt from its sparing solubility, so that the observations respecting it are limited to the lowest proportion of salt.

The comparative diffusion from 1 per cent. solutions of these alkaline oxalates is as follows:—

Oxalate of potash in 8·083 days at 59°·9	6·20	100
Oxalate of soda in 9·9 days at 59°·9	6·24	100·65

Here again a nearly equal diffusion is observed in times of which the squares are as 2 to 3.

Acetate of Potash.—Time of diffusion 8·083 days. The diffusate was converted into chloride of potassium and weighed in that form.

1. 0·951 per cent. of anhydrous acetate of potash, density 1·00540, diffused at 60°·3, in eight cells, 6·01, 6·18, 6·27, 6·06; mean 6·13 grs. for two cells; calculated for 1 per cent., 6·44 grs. of acetate of potash in two cells.

2. 1·903 per cent. of acetate of potash, density 1·00976, diffused at 60°·3, in eight cells, 11·90, 11·91, 11·65, 12·24; mean 11·92 grs. for two cells; calculated for 2 per cent., 12·52 grs. of acetate of potash in two cells.

3. 3·807 per cent. of acetate of potash, density 1·01928, diffused at 60°·3, in four cells, 10·91, 11·08, 11·48, 11·18; mean 11·16 grs. for one cell; calculated for 4 per cent., 11·72 grs. of acetate of potash in one cell.

4. 7·614 per cent. of acetate of potash, density 1·03743, diffused at 60°·3, in four cells, 22·62, 21·72, 23·23, 22·42; mean 22·50 grs. for one cell; calculated for 8 per cent., 23·63 grs. of acetate of potash in one cell.

Diffusion of Acetate of Potash in 8·08 days at 60°·3; two cells.

	Grs.	Ratio.
From 1 per cent. solution . . .	6·44	1·028
From 2 per cent. solution . . .	12·52	2
From 4 per cent. solution . . .	23·44	3·744
From 8 per cent. solution . . .	47·26	7·549

The acetate will be found to exceed sensibly the sulphate and oxalate of potash in diffusibility at the preceding temperature.

Comparative diffusion of Sulphate and Acetate of Potash in 8·083 days, at 60°·3.

	From solutions of			
	1 per cent.	2 per cent.	4 per cent.	8 per cent.
Sulphate of potash	100	100	100	100
Acetate of potash	104·55	107·93	103·26	107·58

The acetate of potash appears to possess that increased diffusibility which is observed in bicarbonate of potash. But a parallelism still holds between the acetate and sulphate, and the diffusion of the two salts would probably coincide if that of the acetate were observed at a temperature 3° or 4° lower than the sulphate.

Acetate of Soda.—Time of diffusion 9·9 days. The diffusate was converted into chloride of sodium and weighed in that form.

1. 0·958 per cent. of anhydrous acetate of soda, density 1·00530, diffused at 59°·6, in eight cells, 6·33, 6·56, 6·63, 6·05; mean 6·39 grs. for two cells; calculated for 1 per cent., 6·67 grs. of acetate of soda in two cells.

2. 1·917 per cent. of acetate of soda, density 1·01032, diffused at 59°·6, in eight cells, 11·70, 12·04, 11·90, 12·19; mean 11·96 grs. for two cells; calculated for 2 per cent., 12·46 grs. of acetate of soda in two cells.

3. 3·835 per cent. of acetate of soda, density 1·02039, diffused at 59°·6, in four cells, 12·19, 11·74, 12·15, 11·93; mean 12·00 grs. for one cell; calculated for 4 per cent., 12·52 grs. of acetate of soda in one cell.

4. 7·67 per cent. of acetate of soda, density 1·03968, diffused at 59°·6, in four cells, 23·49, 22·26, 23·87, 22·49; mean 23·03 grs. for one cell; calculated for 8 per cent., 24·02 grs. of acetate of soda in one cell at 59°·9

Diffusion of Acetate of Soda in 9·9 days at 59°·6; two cells.

	Grs.	Ratio.
From 1 per cent. solution . . .	6·67	1·070
From 2 per cent. solution . . .	12·46	2
From 4 per cent. solution . . .	25·04	4·019
From 8 per cent. solution . . .	48·04	7·711

The diffusion of acetate of soda presents a general parallelism to that of acetate of potash for the times chosen, the temperatures of the two series of experiments differing only 0°·7.

Comparative diffusion of Acetate of Potash in 8·083 days at 60°·3, and Acetate of Soda in 9·9 days at 59°·6.

	From solutions of			
	1 per cent.	2 per cent.	4 per cent.	8 per cent.
Acetate of potash	100	100	100	100
Acetate of soda	103·57	99·52	106·82	101·65

Tartrate of Potash.—Time of diffusion 8·08 days. The diffusate was evaporated to dryness, ignited, and then converted into chloride of potassium.

1·82 per cent. of anhydrous tartrate of potash, density 1·01227, diffused at 59°·9, in eight cells, 9·96, 10·02, 10·06, 9·87; mean 9·98 grs. for two cells; calculated for 2 per cent., 10·96 grs. of tartrate of potash in two cells.

Tartrate of Soda.—Time of diffusion 9·9 days. The diffusate was evaporated to dryness, ignited, and then converted into chloride of sodium.

2·03 per cent. of anhydrous tartrate of soda, density 1·01460, diffused at 59°·6, in

eight cells, 10·84, 10·77, 10·81, 10·82; mean 10·81 grs. for two cells; calculated for 2 per cent., 10·65 grs. of tartrate of soda in two cells.

Diffusion of 2 per cent. solutions of Tartrate of Potash in 8·08 days at 59°·9, and Tartrate of Soda in 9·9 days at 59°·6.

Tartrate of potash	10·96	100
Tartrate of soda	10·65	97·18

The tartrate of soda happens to correspond absolutely with the carbonate of soda, the diffusate of the latter salt obtained at the same temperature being also 10·65 grs. (p. 486).

The double *Tartrate of Potash and Soda* afforded an interesting instance of decomposition produced by diffusion. Instead of the usual diffusion cells, a single plain cylindrical jar was employed about 5 inches in diameter and 10 inches in height. A portion of a 4 per cent. solution of Rochelle salt, amounting to 5000 grs. of liquid, was covered over by nine times as much pure water, in this jar, and the salt allowed to diffuse upwards for three weeks at 56°. The upper third of the fluid column, amounting to 17,000 grs. of liquid, was then drawn off, evaporated to dryness, and the salt ignited and converted, by the addition of hydrochloric acid, into chloride, which weighed 3·14 grs. The salt proved to be a mixture of the two chlorides in the proportion of 2·43 grs. of chloride of potassium and 0·71 gr. of chloride of sodium. This gives the following as the composition of the diffusate:

Tartrate of potash	3·68
Tartrate of soda	1·17
	4·85

It hence follows that the proportion of potash, or of tartrate of potash, in the diffusate is nearly three times greater than existed in the original Rochelle salt. It will be recollected that the salt decomposed is, strictly speaking, a bibasic tartrate of potash and soda, and not a double tartrate of potash and soda. The diffusate was observed to be exactly neutral to test-paper. This mode of diffusing upwards in water from the lower part of a deep jar obviously gives the greatest degree of separation attainable in consequence of unequal diffusibility.

I may confine myself at present to the conclusion that of the nine pairs of potash and soda salts, of which the diffusion is compared in this paper, the potash salt uniformly exceeds in diffusibility the corresponding soda salt; that the ratio between the two classes is always sensibly the same, or exhibits only a small range of variation at the temperature of the experiments, which was near 60°; with the peculiar exception of the hydrates of potash and soda.

XXI. *The Figure and Primitive Formation of the Earth, or Researches in Terrestrial Physics.*—Part I. *By* HENRY HENNESSY, *M.R.I.A. &c. Communicated by Major* LUDLOW BEAMISH, *F.R.S.*

Received October 29,—Read December 17, 1846.

1. THE hypothesis by which the figures of the heavenly bodies are theoretically explained has long engaged the attention of geologists and geometers, without having acquired any important improvements. Great as have been the discoveries which have originated from this hypothesis, it remains in nearly the same philosophical position as that to which it had arrived when CLAIRAUT published his Theory of the Figure of the Earth. During the period of more than a century which has elapsed since the appearance of that immortal work, the improvements which have been made in the theory of the figures of the planets, appear to consist chiefly in discoveries connected with the attractions of bodies, and in some important generalizations of the equilibrium and motions of fluids.

Comparatively few positive discoveries have however been as yet made in geology as deductions from the hypothesis of the primitive fluidity of the earth. In explanation of this circumstance several causes may be assigned, one of which appears to be the limited nature of the hypothesis. It has not been considered sufficient to suppose that the earth was originally a mass of heterogeneous matter in a fluid state; an additional supposition has been made, in support of which I am not aware that any evidence has been ever adduced. The supposition alluded to is, that the volume of the entire mass and the law of density of the fluid have not been changed by the solidification of a part of that fluid, no matter how far the solidification may have proceeded; or in other words, that the distribution of the molecules composing the earth is the same in the present state of our planet as that which they had when the mass was fluid. Although no precise evidence can be brought forward for the examination of this portion of the hypothesis, it appears not to be entirely consistent with what is known respecting the solidification of fluids. It is thus defective from appearing to involve an unproved and improbable physical law. Its exclusion from the hypothesis will make the latter more general and more applicable to the explanation of certain cosmical phenomena. Making the simplified hypothesis the basis of my investigations, it shall be their object further to generalize the theory of the figure of the earth, and to examine the nature and the energy of the physical and mechanical actions which may have been exerted upon its surface during its different geological transformations.

2. Let the earth be considered to have been originally a heterogeneous fluid mass, possessing only such general properties as those which have been established for fluids. Let x, y, z be the rectangular coordinates of μ, an elementary molecule of the fluid; and let the components of the forces, parallel respectively to the axes of x, y, z, be X, Y, Z, if the mass were not in rotation but at rest and in equilibrium. Let the mass rotate about the axis of z with an angular velocity at unity of distance represented by α, let the density at μ be represented by ϱ, a function of x, y, z; and let p represent the entire pressure upon μ, then*

$$dp=\varrho(\mathrm{X}dx+\mathrm{Y}dy+\mathrm{Z}dz)+\alpha^2(xdx+ydy).$$

The equation of the exterior surface of the fluid, and of the surfaces of equilibrium in its interior, will be

$$\mathrm{X}dx+\mathrm{Y}dy+\mathrm{Z}dz+\alpha^2(xdx+ydy)=0.$$

Let M represent the sum of the moments by which the rotation of the fluid mass was at first produced, and let the axis of z pass through the centre of gravity, then†

$$\alpha=\frac{\mathrm{M}}{\Sigma\mu(x^2+y^2)}.$$

The numerator at the right side of the above equation is evidently constant, and the denominator is a quantity depending upon the distribution of the molecules of the fluid about the centre of gravity of the entire mass. Let I represent the value of $\Sigma\mu(x^2+y^2)$ when α was the angular velocity of rotation of the body, and let I′ be its value corresponding to an angular velocity of rotation denoted by α'. If α should not be known independently, but if I, I′ and α' be known, then the equation to the free surface of the fluid can be transformed into

$$\mathrm{X}dx+\mathrm{Y}dy+\mathrm{Z}dz+\frac{\alpha'^2\mathrm{I}'^2}{\mathrm{I}^2}(xdx+ydy)=0. \quad \ldots\ldots\ldots \quad (1.)$$

In any solutions heretofore given of the problem of the determination of the figure of the earth, the coefficient $\frac{\mathrm{I}'^2}{\mathrm{I}^2}$ seems not to have been noticed from the nature of the assumption which has been mentioned in (Art. 1.).

When X, Y, Z are the components produced by the mutual attractions of the molecules of the fluid according to a certain function of x, y, z, then the values of X, Y, Z will in general depend upon the form of the entire mass and of its surfaces of equilibrium, and again that form will depend upon the values of the components and of α'. As the value of I depends upon the form of the mass, it must also depend upon X, Y, Z; but it may be considered independently if the conditions upon which any particular solution of equation (1.) is to be made are such as to assign what function it may be of the coordinates. Considered under this point of view, the three first terms, and the fourth or last term at the left side of the equation (1.), may be considered as independent quantities whose values may be found separately, and $\frac{\mathrm{I}'^2}{\mathrm{I}^2}$ may

* Poisson, Mécanique, tom. ii. p. 536, 2ième edit. † Ibid. p. 79.

be regarded as a coefficient affecting α'^n of such a nature as to satisfy the equation

$$\alpha = \alpha' \frac{\Gamma'}{\Gamma} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.)$$

3. In order to arrive at a solution of the problem of the figure of the earth, it has been found necessary to suppose that the primitive fluid mass deviated but little from the spherical form. If the spheroid be in rotation, it must follow that its axis of rotation should be that one of all the axes passing through the centre of gravity, with respect to which the sum of the forces acting upon the system would be a maximum. This axis may therefore be supposed to coincide very nearly with the position of the present axis. If no other forces be supposed to act upon the mass but the mutual attractions of its molecules and centrifugal force, then equation (1.) will rigorously represent the general equation of its surface, and the consideration of its physical or mechanical changes could be treated with less difficulty.

Whatever may be the cause of the primitive fluidity of the earth, and whatever may be the nature of the process of solidification by which at least a portion of it has arrived at a solid state, if that process should be such that a shell would be first formed surrounding a nucleus of matter in a state of fluidity, and if solidification be accompanied by a change in density of the matter solidified, it appears that some change would be produced in the motions of the entire mass about its axis, and in the arrangement and composition of that portion of the fluid constituting the nucleus. The manner in which, from physical or chemical causes, the solidifying process may proceed, will to a great extent determine what may be the actions mutually exercised by the nucleus and shell. The combination of such actions, with the changes which may have occurred in the velocity of rotation of the mass, will be thus known, and their influence upon the geological phenomena occurring during the different epochs of the physical history of the earth can be thus appreciated. In this way it may be possible to carry our ideas back to the remoter periods of the earth's existence, and to assist in explaining the phenomena which geological observers are constantly unfolding.

The application of physical and mechanical science to questions connected with geology, has been made only within a comparatively recent period, and the subject seems yet to require a more complete systematizing in order to bring it into the class of sciences which belong properly to what may be called the mechanics of the universe. An important step towards connecting geology with physical astronomy has already been made by Mr. Hopkins in his researches concerning the constitution and thickness of the present solid shell of the globe. One of the objects of the present memoir will be fulfilled if it should in any way tend to increase that connexion.

4. Adopting in its greatest generality the hypothesis that the earth was originally a fluid mass in rotation, the particles of which attracted each other with forces varying inversely as the squares of their distances, and that the figure of the mass was nearly spherical, it will be impossible to proceed with the general investigation of the

figure which the forces acting on its different parts would ultimately produce without any knowledge of the peculiar physical nature of the fluid. It appears to be sufficiently demonstrated, that the figure which such a fluid mass would assume in virtue of the mutual attractions of its particles and of centrifugal force would be an ellipsoid of revolution. It is superfluous therefore to enter upon that portion of the subject. Assuming then the figure of the mass to be an ellipsoid of revolution, I shall endeavour to obtain general expressions for its ellipticity, and for the variation of gravity at its surface.

From what has been stated in the last paragraph of art. 2, it appears to be unnecessary to transform equation (1.) into the equation of a similar form which the conditions of the present problem would require, and from this cause the whole investigation may be considerably abridged.

Let the origin of the coordinates be at the centre of gravity of the fluid mass, let r represent the radius vector drawn from this origin to any point in any surface of equilibrium of the fluid, let θ represent the angle formed between this radius and the axis of z or axis of rotation, let ω represent the angle formed by the plane which passes through the radius r and the axis of rotation with the plane of zx, then

$$z=r\cos\theta, \quad x=r\sin\theta\cos\omega, \quad y=r\sin\theta\sin\omega.$$

Let J represent in this case the fourth term at the left side of the integral of (1.), and let f represent the centrifugal force of a point situated at distance unity from the axis, hence

$$\mathrm{J}=\tfrac{1}{2}f(x^2+y^2)=\tfrac{1}{3}fr^2-\tfrac{1}{2}fr^2\left(\cos^2\theta-\tfrac{1}{3}\right).$$

If it be assumed that the polar and equatorial axes of the spheroid always differ but very little, we shall have for the determination at any period of f_1 the centrifugal force at the equator,

$$f_1=f'\frac{a_1\mathrm{l}'^2}{a'\mathrm{l}_1^2}, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3.)$$

where a_1 represents the semipolar axis at that period, and a' the same semiaxis at present, f' being the centrifugal force at the equator at present. Let ϱ represent the density at any point of a surface of equilibrium, the polar radius of which is a, and the ellipticity e, then as the figures of the mass and of its surfaces of equilibrium differ so little from the spherical form, r may be supposed nearly equal to a, and hence for the determination of e we shall have the equation*

$$\mathrm{U}^{(0)}\left\{\tfrac{4}{5}\pi a^5 f\varrho de-\tfrac{4}{3}\pi a^3 e f\varrho d.a^2+\tfrac{4}{3}\pi f\varrho d(a^5 e)\right\}-\frac{fa^5}{2\beta}\left(\mu^3-\tfrac{1}{3}\right)=0;$$

where $\mathrm{U}^{(0)}$ is a function of the coordinates, βa constant, and $\mu=\cos\theta$. If E represent the ellipticity, and ϱ' the density at the exterior surface, it follows that

$$\mathrm{E}=\frac{f_1\mathrm{E}}{\frac{8}{5}\pi(5\mathrm{E}f\varrho a^3 da+fa^5\mathrm{E}d\varrho-\mathrm{E}\varrho')}. \quad \ldots\ldots\ldots\ldots \quad (4.)$$

* Mécanique Céleste, tom. ii. p. 88.

If m' represent the ratio of centrifugal force f' to gravity g' at the equator, then

$$f_1=\frac{m'g'a_1\mathrm{I}^2}{a'\mathrm{I}^2}.$$

If this value of f_1 be substituted in (4.), and if all the integrations be performed according to the given value of the function ϱ, an equation may be obtained after due reduction of the form

$$\mathrm{E}=\frac{a_1m'g'\mathrm{I}^2}{a'}\varphi(a), \quad \ldots\ldots\ldots\ldots \quad (5.)$$

in which equation I is considered as a function of a, and where φ is a functional symbol. That I may be considered as a function of a, is evident from the transformation of x, y, z into polar coordinates, and from the assumed identity of r and a. The above equation appears therefore to be general, or applicable to the consideration of the figure of any fluid mass upon the conditions which have formed the groundwork for its investigation.

In applying equation (5.) to the case of the earth's ellipticity, the values of m', g', a' and I', known at the present day, may be introduced. If it be supposed that the present ellipticity of the earth is the same as that which it had at the time when the first external coat of the shell had solidified from the fluid, I' will be a function of E, which can be then found if I' should be introduced under such a form by some further transformations.

The quantities I and I' are evidently the moments of inertia of the mass with respect to the axis of z during certain states of the arrangement of its molecules about the centre of gravity. Hence from the properties of ellipsoids of revolution

$$\mathrm{I}'=\frac{2\mathrm{M}_1a'^2}{5\sigma},$$

σ being a number depending upon the arrangement of the particles in the interior of the mass, M_1 representing the earth's mass, and a' the present polar radius of the earth, which, without sensible error, may be used for the equatorial radius. From the consideration of the phenomena of precession and nutation, the following expression for σ' has been obtained*—

$$\sigma=\frac{\mathrm{P}}{\mathrm{E}-\frac{1}{2}m'},$$

where P represents a number deduced from the phenomena alluded to. If the above values of I' and σ be substituted in (5.), it will be transformed into

$$\mathrm{E}=m\mathrm{Q}\frac{\left(\mathrm{E}-\frac{1}{2}m'\right)^2}{\mathrm{P}^2},$$

* Pontécoulant, Théorie Analytique du Système du Monde, tome ii. p. 475.

where Q is used for brevity instead of

$$\frac{4}{25}a_1 g' M_1^2 a'^3 \varphi(a).$$

The above expression gives

$$E=\frac{1}{2mQ}\left\{m^2Q+P^2+p\sqrt{2m^2Q+P^2}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6.)$$

If from any cause, E_1, the present ellipticity of the earth, should not be identical with E, and that we should wish to obtain the latter, the numerical value of σ, calculated from the formula

$$\sigma=P\left(E_1-\frac{1}{2}m\right)^{-1},$$

must be introduced in the value of I′ and thence in equation (5.).

5. The variation of gravity at the surface of the spheroid, during its fluid state and during the process of its solidification, may now be considered, the mass being supposed in both cases to consist of spheroidal shells having a density increasing from the external surface to the centre.

From the supposition which has been just made, if γ represent the entire force of gravity at the surface of the spheroid, we shall have

$$\gamma=\frac{4\pi}{3r^2}\int\rho d.a^3+\frac{4\beta\pi}{r^2}\int\rho d.\left(a^3Y_0+\frac{3a^5}{5r^2}Y_2\right)-J,$$

Y_0 and Y_2 being rational functions of the coordinates; and let it be remembered that at the surface

$$r=a_1[1+\beta(Y_0+Y_2)],$$

and that

$$\frac{4\beta\pi}{5}\int\rho.d.a^3Y_2=\frac{4\beta\pi}{3}Y_2\int\rho.da^3+\frac{1}{2}f\left(\cos^2\theta-\frac{1}{3}\right).$$

Let for brevity

$$G=\frac{4\pi}{a_1^2}\int\rho a^2da-\frac{8\beta\pi Y_0}{a_1^2}\int\rho a^3da+\frac{4\beta\pi}{a_1^2}\int\rho da^3Y_0-\frac{2}{3}f.$$

Let G′ represent the attraction at the surface of the earth at present, then if all quantities of the order β^2 and βf_1 be neglected, we shall have after a few reductions,

$$\gamma=G+\left(\frac{5}{2}mG'\frac{I'^2a_1}{I'^2a'}-EG\right)\left(\cos^2\theta-\frac{1}{3}\right);$$

$m'G'\frac{I'^2a_1}{I'^2a'}$, the value of f, being here substituted for it.

If after the solidification of the external portion of the fluid into a spheroidal shell the axes of the shell undergo no great changes in magnitude, then whatever may be the physical changes occurring within the exterior surface of the shell, it will follow, from the little difference between the figure of the mass and that of a sphere, that

the value of G will be invariable. Hence we may make $G'=G$, and $a_1=a'$, therefore

$$\gamma=G'\left[1+\left(\frac{5}{2}m'\frac{I'^2}{I_1^2}-E\right)\left(\cos^2\theta-\frac{1}{3}\right)\right];$$

I_1 representing the value of $\Sigma\mu\ (x^2+y^2)$ at any time after the formation of the shell. Let

$$\Gamma=G'-\frac{1}{3}G'\left(\frac{5}{2}m'\frac{I'^2}{I_1^2}-E\right);$$

then neglecting all powers of the ellipticity higher than the first,

$$\gamma=\Gamma\left[1+\left(\frac{5}{2}m'\frac{I'^2}{I_1^2}-E\right)\cos^2\theta\right]. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7.)$$

If $\theta=90°$, then $\gamma=\Gamma$, so that Γ will be the expression for gravity at the equator. The expression for Γ_1, gravity at the poles, will be

$$\Gamma_1=\Gamma\left[1+\left(\frac{5}{2}m'\frac{I'^2}{I_1^2}-E\right)\right].$$

Hence if we designate by S the excess of gravity at the pole above gravity at the equator divided by the latter quantity, we shall have the general theorem

$$S+E=\frac{5}{2}m'\frac{I'^2}{I_1^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8.)$$

If $I'=I_1$, this expression will be identical with what is generally called CLAIRAUT'S theorem.

If the earth had been originally a homogeneous fluid mass, and if in solidifying it continued so, then $E=\frac{5}{4}m'$, $S=E$.

If the spheroid were heterogeneous, the excess of gravity at the pole above gravity at the equator at any period of time divided by the latter quantity, and the excess of the equatorial axis above the polar axis divided by the latter axis, would form two fractions, the sum of which multiplied by the square of the ratio of the moment of inertia of the mass at the same period to its present moment of inertia, would form a constant product which would be equal to twice the ellipticity which the spheroid would have in the case of homogeneity.

This and the foregoing article appear to contain all that it may be at present possible to deduce from the improved hypothesis of the primitive state of the earth, with relation to its figure, and to the variation of gravity at its surface. It may be hereafter possible to assign to the quantities entering $\varphi\ (a)$ in equation (5.) such values deduced from considering the physical properties of the primitive fluid, as would enable us to find a numerical value for the ellipticity.

6. In art. 3, a short general sketch has been given of the consequences which may result from the improved hypothesis of the primitive fluidity of the earth to physical geology, or to the changes occurring upon the external crust of the earth during

the process of its solidification. It is now proposed to commence the investigation of these geological consequences.

Before proceeding to the purely mathematical portions of this investigation, it seems necessary that some preliminary remarks should be made on the nature of the solidification of the globe. It seems the more necessary that these remarks should be now given, as they will not probably be found to agree entirely with those which have been made on the same subject by an eminent geologist and geometer*. It is, however, satisfactory to reflect that even if my views respecting the solidification of the earth should be more correct than those of Mr. HOPKINS, the real value of that gentleman's researches will still remain unaltered.

Although until now no allusion has been made to the probable cause of the primitive fluidity of the earth, it becomes necessary in order to proceed with the geological portion of these investigations that this cause should be assigned. Intense heat is that which is generally supposed to be the cause, and which the advocates of the earth's central heat seem to support by numerous facts. The solidification of this heated mass of fluid is so connected with its refrigeration, that some remarks on the latter subject must accompany or precede any investigations about the former.

If the whole earth from the action of a high temperature were reduced to that state of fluidity which forms the groundwork of the hypothesis upon which the theory of its figure is founded, it appears probable that the fluid mass would be extremely heterogeneous chemically, as well as mechanically. New chemical combinations would be formed, of whose physical properties it would be difficult to form an accurate idea. After the most energetic combining tendencies of the different substances composing the fluid would have been satisfied, the mass may be supposed to have arrived at a state of comparative chemical stability. Such properties of the different compounds as density and compressibility, would then exert a direct influence upon the nature of their arrangement around the centre of gravity of the spheroid. Every portion of the fluid, denser than the stratum, in which from any cause it may happen to be, would sink until it would meet a stratum of equal density. The light portions of the fluid which may happen to be in heavier strata than themselves, would on the contrary ascend until equilibrium would be obtained. The fluid mass would thus arrive at length at such a state, that it would consist of a series of spheroidal strata, each of uniform density throughout its own mass, and having that density expressible as a function of its axes.

At some time after the arrival of the fluid at this state, and perhaps before it, portions of its heat would radiate in space. The exterior portions of the fluid would cool first, until they would acquire, according to the particular circumstances which may influence their cooling, certain densities. If the effect of refrigeration be in general an increase in density of the matter cooled, then the cooled portions of the fluid will sink. The higher temperature of the matter yet unexposed to cooling influences,

* HOPKINS, Philosophical Transactions, 1839.

would then tend to change the acquired densities of the cooled matter. The as yet uncooled fluid would have its temperature reduced from contact with the cold particles from above, and it would tend to change its position in a similar manner to the portions at first cooled. As each cooled portion of the fluid descends, the three following causes would therefore impede its descent.

1. From the arrangement already indicated of the denser strata about the centre of the mass, and from the nature of the law of density of the strata, each stratum into which the cooled fluid would descend, would be denser than the preceding.

2. If the general effect of the refrigeration be to increase the density of the cooled matter, each stratum would have its density augmented by the passage through it of the cooler matter from above.

3. The descending portions will have their densities diminished by the increase in their temperature.

During this process of circulation the interior matter in a state of fusion will be losing some of its heat by conduction. It must be evident however that if the temperature of the fluid be very high, the cooling would be carried on at first chiefly by the process of circulation. If, as analogy seems to point out, the conducting power of the fluid were small, the process of circulation would long continue to perform a principal part in the refrigeration of the fluid.

Although from the imperfect state of our knowledge with respect to the cooling of fluids it would be impossible to arrive at the precise laws and nature of the process of circulation, it yet seems probable from the three impeding causes which have been adduced, that the energy of this process would diminish from the surface of the spheroid to its centre. *The principal oscillations of the fluid produced by this cause would therefore be at first confined to the vicinity of the surface.* A solid crust may therefore be formed at the surface, long before the process of circulation could have extended to any great depths. We should thus have an external solid crust with strata of the imperfect fluid which may have been subjected to the process of circulation below it, and in the centre a mass of matter still retaining a high state of fluidity. The existence at the centre of the spheroid of a solid nucleus of small mass compared to the entire mass of the earth, cannot, as will hereafter appear, exercise any important influence on geological phenomena. If from compression, or any other cause, this solid nucleus should exist, it is evident that solidification would proceed much more slowly upon it upwards, than upon the interior surface of the shell downwards; or in other words, the increase of its mass after its first formation would be small compared to the increase of the mass of the shell subsequent to its first solidification. Whether such an internal solid nucleus exists or not, certain phenomena will attend the formation of the shell which it may be important to remark. From what is known respecting the solidification of fused substances, it may be inferred that at the moment when a portion of the fluid assumes the solid

state a considerable quantity of latent heat will be eliminated. A portion of this heat will pass upwards through the shell, and another portion will pass into the imperfect fluid immediately below, tending to bring it back to a state of more perfect fluidity. If the quantity of latent heat thus eliminated should be great, it may exert an influence in changing the arrangement of the parts of the shell which it may be hereafter useful to consider. The effect of the elimination of latent heat upon the process of circulation would be evidently such as to render that process still more complex than what it was before, and its examination would thus be attended with still more difficulty and uncertainty. A very accurate examination of this portion of the phenomena attending the earth's refrigeration, appears not to be necessary for the present, and it will be found most convenient to defer its consideration, if it should be required in any future portion of these researches.

7. The motions of the shell and internal fluid matter, and their mutual actions depend so much upon the arrangement of the molecules of the primitive fluid mass, that it would be desirable before attempting their investigation to form if possible a few precise ideas respecting the last mentioned subject. The results which may be thus obtained with respect to the dimensions and law of density of the primitive fluid, will also enable us to find the probable values for $\varphi(a)$, or $\mathbf{Q}$, which should be substituted in this case respectively in either equations (5.) or (6.), in order to obtain the ellipticity of the spheroid.

Let the mass be supposed to have arrived at the state alluded to in the foregoing article, when "it would consist of a series of spheroidal strata, each of uniform density throughout its own mass, and having that density expressible as a function of its axes." Each stratum would have to sustain the normal pressures of all the strata above it, whether these pressures be produced by the action of gravity or other causes. The compressive force exercised by the upper strata upon those farther from the surface, would tend to increase the density of the latter. How far this increase of density from compression may proceed without solidification, we have no precise experimental evidence by which to judge. The celebrated experiments of Sir James Hall appear, however, to show that from the action of a great compressive force, matter in a fused state may very nearly retain the volume, and therefore the specific gravity which it had when solid. If from the combined action of increased pressure and increased temperature the specific gravity of the matter should be augmented, while its fluidity should be retained, it would fulfil the conditions of the hypothesis made in explanation of the earth's figure. If it can be assumed that the substances composing those portions of the globe inaccessible to observers are similar in constitution to those which are found at the surface, this portion of the subject appears to be susceptible of experimental examination. Until such an examination shall have been made, the mass may be supposed to have an arrangement such as that indicated in art. 6. If a solid nucleus of comparatively small mass

should be formed, the densities of its strata may be supposed such, as to be expressible according to the same function of their axes as the densities of the fluid strata.

Let the fluid mass be conceived to consist of an infinite number of pyramids or columns meeting at its centre. Let, in conformity with the supposition mentioned by LAPLACE, the relation between the pressure Π at any point in one of these fluid columns, and the density ϱ be expressed by the equation*

$$\frac{d\Pi}{d\rho}=2k\varrho,$$

$2k$ being a constant depending on the peculiar physical properties of the fluid. The value of ϱ deduced from this equation, may after a few reductions be expressed under the form

$$\varrho=\frac{A}{a}\sin an;$$

where a is the semipolar axis of the spheroidal stratum in which the point may be, A a constant, and n a constant depending on the constitution of the fluid, for n is here used to express $\sqrt{\frac{2\pi}{k}}$.

Let M_1, as before, represent the mass of the spheroid which is supposed to remain constant, let ϱ' represent its density at the surface when fluid, and let D represent its mean density, so that we shall have very nearly

$$D=\frac{3M_1}{4\pi a_1^3},$$

a_1 representing the semipolar axis of the fluid mass, and π, as in every other part of this memoir, representing the ratio of the diameter of a circle to its circumference. Let c represent the amount of compression of the fluid produced by the pressure of a column l units in height, and the ratio of which to the earth's radius is represented by m_1. Then the values of n and a_1 can be determined from the equations

$$\frac{na_1}{\tan na_1}=1-\frac{c}{m};\quad \frac{D}{\varrho'}=\frac{3c}{mn^2a_1^3}.$$

Remembering the value of D, and that $m_1=\frac{l}{a_1}$, we can obtain from the second of the above equations,

$$a_1=\frac{n}{2}\sqrt{\frac{M_1 l}{c\pi\varrho'}}. \qquad (9.)$$

Let for brevity the quantity under the radical be called h^3. Then for the determination of n, we shall have the equation

$$1-\frac{cnh}{2l}-\frac{n^2h}{2\tan\frac{1}{2}n^2h}=0. \qquad (10.)$$

* Méc. Cél., tome v. p. 49.

The value of n found from this expression by successive approximations, may be substituted in (9.), whence the value of a_1 can be obtained.

By means of the operations above indicated, if sufficient data existed, some knowledge could be obtained of the arrangement of the molecules of the primitive fluid about their centre of gravity. At present no experimental data appear to exist by which numerical values could be assigned to c or l. The experiments of MM. Colladon and Sturm show that the compressibilities of different liquids vary considerably, according to their physical properties. It would be impossible therefore, from analogy with their experiments, or those of M. Œrsted, to form any accurate idea of the compressibility of the fluid constituting the earth when in its supposed state of fusion. The calculation of a precise value for n or a_1 seems then to be at present impossible, but geological considerations appear to impose certain limits on the value which a_1 may have had. The value of a_1 may have been equal to, or greater or less than the present semipolar axis of the earth. If it were greater, the difference should be small compared to its whole length; for if this difference were great, the external shell first formed should (no matter from what cause) arrive at its present dimensions, and in doing so it would have to undergo immense contortions of whose existence no evidence can be adduced. If a surface be conceived to pass through the centres of gravity of the infinite number of pyramidal segments into which we may conceive the shell to be divided, it is evident that all of these segments could not descend below that surface, or could not all approach in the same degree towards the centre of the mass. The shell should be broken in many places to permit the displacement of the segments, as the amount of common displacement from the contraction of the shell by refrigeration would not effect the reduction of its dimensions to those which it has at present. The irregularities of the surface of the fractured shell would evidently be in proportion to its primitive axes.

If a_1 were less than the present semipolar axis of the earth, then in order that the external coat of the shell first formed should arrive at its present dimensions, it should receive additions in some parts. If the difference between a_1 and the present semipolar axis should have been great, the effect of the contraction of the shell from refrigeration may, as in the foregoing paragraph, be neglected. From some cause, no matter what, the earth's mass should expand, and in the expansion rents would be produced in the shell. These empty spaces may be afterwards filled up with injected matter pushed forward by further expansion during the subsequent formation of the under strata of the shell. The entire shell formed of the cemented portions of the first solidified mass, and the new coats of matter added from within, may be again ruptured, and a process similar to the first may be again repeated. As the whole shell would increase in thickness, the extent and frequency of these ruptures would probably lessen, as whatever may be the mechanical action producing them, it would then have an increased resistance to overcome. The ejected matter which would arrive at the surface would also probably not amalgamate so completely with the per-

fectly cooled solidified matter there, as the matter formerly ejected did when the solid surface was itself at a very high temperature. From the foregoing considerations, it appears that a value of a_1 less than the present semipolar axis of the earth would be at least as much in accordance with observed geological phenomena as a value greater than that semiaxis. The protrusion of the primary and igneous rocks through the strata composing the external crust of the earth, would be in some measure a necessary consequence of the smaller value of a_1; and it would not follow that the surface of the earth should necessarily undergo any great contortions.

In either of the cases which have been just examined, the arrangement of the molecules constituting the fluid globe should be different from their present arrangement. Their arrangement may or may not be different, if the value of a_1 coincided with the present semipolar axis of the earth, in so far as the value of a_1 would have any influence upon that arrangement.

If the dimensions of the globe had been increasing in the manner which should happen if a_1 had at first a small value, the rate of that increase could be determined by finding the areas of the fissures through which the fused matter may have been ejected during the progress of the different geological formations. The geometrical relations between the surface of the spheroid and its axes would evidently be of some assistance in the course of such an investigation. If the sum of the areas of these fissures was small, compared with that of the surface of the earth, then it may be generally concluded that a_1 could not be much less than the present semipolar axis of the terrestrial spheroid.

The three following general conclusions may be now stated as deductions from the foregoing remarks:—

1. If a_1 were greater than the present semipolar axis of the earth, the difference should be small.

2. If a_1 were less than that semiaxis, it becomes more difficult to assign a limit to their difference, but it appears probable that this difference would be greater than what it would be in the foregoing case.

3. The value of a_1 may be identical with that of the present semipolar axis without contradicting any properties of the primitive fluid mass of which we may be aware, or without discordance with geological phenomena.

In order that the value of a_1 should be greater than the present semipolar axis, it is evident that the fluid should have either a low density or a small compressibility. If a_1 were, on the contrary, much less than that semiaxis, the specific gravity of the fluid should be high or its compressibility should be great. If a_1 were equal to the semipolar axis, the fluid's compressibility should still be small compared with those of all the fluids, except mercury, upon which experiments have been made, unless its density were much less than the mean density of the present external crust of the earth.

The above considerations, in addition to what have been previously presented, appear to prove that a_1 could not much exceed the present semipolar axis of the earth, and

that if a_1 differed in any way from that semiaxis the difference would most probably be owing to the comparative smallness of a_1.

8. The application of equations (5.) or (6.) to the determinations of the earth's ellipticity, may now be attempted before proceeding to any further investigations. In making use of (6.) or any other similar equation for the determination of the ellipticity of the spheroid, the numerical results should be always considered as hypothetical. Such an equation could not give the present ellipticity, unless that which the earth had immediately after the solidification of the outer coat of the shell had remained unaltered. It is not improbable that many causes may have existed to produce such an alteration.

If from any cause the fluid nucleus exerted unequal pressures upon different parts of the shell, it is evident that their tendency would be to alter its form. It is possible to conceive that any alteration so produced in the figure of the earth may consist in a reduction as well as in an increase of its ellipticity. In a more advanced stage of these researches I may be able to fully consider the influence of such causes. It is enough for the present to point out the possibility of their existence.

If it be admitted that the causes which have been referred to in the foregoing paragraphs are of sufficient importance to produce appreciable changes in the general figure of the earth, it must appear at least a doubtful assumption to assign to n such a value in the expression for the density of the strata of the primitive fluid, as that which would give precisely the same ellipticity as the present.

In order to illustrate the application of equation (6.) and to show that from its form a change in the value of n can make but a comparatively small change in the value of E, I shall now proceed to find an expression for Q.

If the dimensions of the fluid mass be supposed to be the same as those of the earth at present, then g' will be the same as g, gravity at the period of its fluid state.

It is evident that $\varphi(a)$ can be expressed under the form

$$\varphi(a)=\frac{\psi(a)}{1^2},$$

where

$$\mathrm{I}=\frac{8}{3}\pi\int_0^{a_1}\varrho a^4 da.$$

When a_1 is taken as unity, we shall have

$$\psi(a)=\frac{\frac{5}{2}\left(1-\frac{3q}{n^2}\right)}{\left(3-q-\frac{n^2}{q}\right)},$$

where for brevity q is used instead of $1-\frac{n}{\tan n}$; but when $\varrho=\frac{\mathrm{A}}{a}\sin an$,

$$\int_0^1 \varrho a^4 da=\frac{\mathrm{A}}{n^4}[(n^2-6)q+2n^2a^2],$$

but

$$A=\frac{q'}{\sin n},\quad D=\frac{3qq'}{n^3},\quad M_1=\frac{4}{3}\pi D,$$

hence

$$I=\frac{2}{3}\frac{M_1}{qn^3}[(n^3-6)q+2n^3];$$

whence, finally,

$$\varphi(a)=\frac{45}{8M_1^2}\frac{q^3\left(1-\frac{3q}{n^3}\right)}{\left\{2\left(1-\frac{3q}{n^3}\right)+q\right\}^2\left(3-q-\frac{n^2}{q}\right)}.$$

If this expression be written in the form

$$\frac{45}{8}\frac{K}{M^2}=\varphi(a),$$

then $Q=\frac{9}{10}K$, and (5.) will become

$$E=\frac{9}{10}\frac{m'K}{a^3}. \qquad (11.)$$

In making use of either (6.) or (11.), a quantity P is introduced which depends upon certain numbers obtained from observation. One of these numbers is the ratio of the earth's mass to that of the moon, and some uncertainty seems yet to exist as to its value*. The results obtained from using in (6.) or (11.) the different values of this number, which may be obtained by different methods, are much affected by their differences.

In order to compare equation (6.) with that used in the Mécanique Céleste for finding the earth's ellipticity, the following calculations have been made.

If the equation in the Mécanique Céleste be used, we shall have

$$\begin{cases} n=\frac{5}{6}\pi, & E=\frac{1}{305} \\ n=\frac{11}{12}\pi, & E=\frac{1}{336}. \end{cases}$$

If equation (6.) be used,

$$\begin{cases} n=\frac{5}{6}\pi, & E=\frac{1}{308} \\ n=\frac{11}{12}\pi, & E=\frac{1}{317}. \end{cases}$$

The value of P which has been used in the application of (6.) is that which is given in equation (*k*.), Livre V. of the work of M. De Pontécoulant.

9. After the formation of the external shell of the earth, the entire mass composed of that shell and of the internal fluid nucleus, may be supposed to rotate about an axis not differing much in position from the present terrestrial axis of rotation. If the angular velocities of the shell and nucleus about this axis were from any cause

* Pontécoulant, Théorie, &c., tom. iv., Note 3, p. 651.

different at first, it seems probable that the influence of friction, and of the cohesion of their surfaces of contact, would at length establish an uniform motion for the whole mass. The direct influence which the rotatory motion of the shell and nucleus may exercise upon geological phenomena, will not require therefore the motions of both to be separately considered.

The examination of the motions of the shell and nucleus, and of their mutual actions, whether directly, or from the influence of exterior disturbing causes, may therefore be comprehended under the following divisions:—

1. The phenomena attending the rotation of the entire mass during the gradual solidification of the shell.

2. The pressures exerted by the fluid upon the interior surface of the shell, whether from molecular action, the agency of the heavenly bodies, or from centrifugal force.

3. The influence of the changes of temperature which may occur during the solidification of the shell in contracting or expanding its parts, and in producing physical changes in its interior structure.

From the intimate connexion between the first and second of the above divisions, it would be impossible to examine one of them in a perfectly general manner without introducing the consideration of the other. The complete consideration of both could with great advantage be treated in another memoir than in the present, and I hope soon to have it in my power to complete all the necessary investigations. It will be enough for the present to indicate the importance of the first division in a geological point of view, by referring to one of the phenomena comprehended under it. If from the formation of the solid shell, and from the gradual diminution in the mass of the fluid nucleus, any change should be produced in the moment of inertia of the whole mass with respect to its axis of rotation, the angular velocity of the globe about its axis would be changed. The manner in which the solidification of the mass may take place, and the probable constitution of the fluid, will determine how far such a change may extend. If such a change should be great, it would exert an important influence upon the motions of the liquids or gases surrounding the earth, and upon a multitude of organic phenomena which its surface may have presented at different times.

XXII. *Researches in Terrestrial Physics.*—Part II. *By* Henry Hennessy, *M.R.I.A. &c. Communicated by Major* Ludlow Beamish, *F.R.S.*

Received February 8,—Read March 15, 1849.

Preliminary Remarks.

In the first part of these researches, I have endeavoured, by generalizing the hypothesis on which is usually founded the theory of the earth's figure, not only to improve that theory, but also to establish a secure basis for researches into the changes which may have taken place within and at the surface of the earth during the epochs of its geological history. Although I stated that no precise physical evidence could be adduced for the examination of the assumption that the molecules of the primitive fluid, supposed to have constituted the earth, retained their positions after solidification, it yet appears that such evidence exists, if we may be permitted to draw any conclusions relative to the physical properties of substances in the earth's interior, from the observed physical properties of substances at its surface. Professor Bischof of Bonn, has shown* that Granite contracts in volume in passing from the fluid to the solid crystalline state, from 1 to ·7481, Trachyte from 1 to ·8187, and Basalt from 1 to ·8960. The first of these rocks appears, as far as can be observed, to constitute the greater part of the earth; hence the assumption alluded to must be considered not only as superfluous, but as erroneous.

In this Part it is my object to discover relations between the interior structure of the earth and phenomena observed at its surface, and also the effects of the reaction of the fluid nucleus, described in Article 6, Part I., upon the solid crust. I divide this Part into sections, each containing a distinct investigation, the order of arrangement of these sections being determined according to their fundamental importance. The statement of the geological results capable of being deduced from these investigations is, for greater clearness, reserved for the end. Such of these results as chiefly depend on the validity of the reasonings used in Section III. are presented with some diffidence, owing to the imperfect experimental knowledge we possess respecting the subjects discussed in that section. The diminution of the earth's mean radius by refrigeration is neglected all through, except where the contrary is specially mentioned.

I. THE PRESSURES OF THE SHELL AND NUCLEUS AT THEIR SURFACE OF CONTACT.

1. In this investigation the earth is supposed to consist of a nucleus of fluid matter inclosed in a solid shell. The inner and outer surfaces of the shell are sup-

* Leonhard und Bronn's Neues Jahrbuch, 1841, p. 565. See also a paper by the same writer in the N. Jahrbuch for 1843, p. 1. These results seem also to be confirmed by others subsequently found by M. Deville.

posed to be spheroidal, oblate, and nearly spherical. Both shell and nucleus are supposed to be formed of strata increasing in density, according to unknown laws, as the radii of these strata decrease.

If the earth were solid from its surface to its centre, all the phenomena of its rotation could be determined by the integration of three differential equations relative to its motion about its centre of gravity, and of three more relative to the motions of its principal axes referred to three rectangular axes fixed in space. The constants contained in these equations would depend on the attractions of exterior bodies, on the impressed forces, and on the arrangement of the particles composing the earth. It is evident that similar equations would suffice for the case of an empty shell constituted in the manner above mentioned, after the substitution of the constants depending on the magnitude and internal constitution of the shell for those depending on the magnitude and internal constitution of the entirely solid spheroid. By adopting this method, no new analytical transformations are required in discovering the phenomena of the shell's independent rotation, for we can thus avail ourselves of the researches already made by geometers relative to the rotation of the entirely solid spheroid. I have therefore thought it sufficient for our present purposes to merely present the following conclusions to which I have arrived, by using the method above indicated.

1st. If the original impressed forces were the same in direction and intensity for the shell as for the entirely solid spheroid, the angular velocity of the former about its instantaneous axis would be greater than the angular velocity of the latter about its instantaneous axis.

2nd. The influence of exterior disturbing forces would be insensible relative to the rotation of the shell about its centre, provided that its strata possess nearly similar laws of arrangement with those of the solid spheroid.

3rd. The motions in space of the axes of the shell due to the action of exterior disturbing bodies, would be affected to a greater degree than similar motions of the axes of the solid spheroid.

If no other forces acted on the fluid nucleus besides the attractions of its own particles, the attraction of the shell, and centrifugal force, its surface would be an ellipsoid of revolution, and it would rotate steadily about the axis of its greatest moment of inertia. If the action of exterior disturbing bodies be added to these forces, tidal oscillations in the surface of the nucleus would tend to be produced; but it is evident, that unless the disturbing forces were very great, the position in space of the axis of rotation of the mass would be much less affected than if it existed in the solid state. Let, in addition to the forces already enumerated as acting on the fluid nucleus, certain molecular forces be conceived to act on its particles, by which the whole mass might in general acquire a tendency to expand or contract, and also to change its form. This class of forces may be conceived to be resolved into two pressures acting at the inner surface of the shell, one of them being supposed to be constant for every point, and the other variable. If, moreover, the constitution of the shell

and nucleus should be favourable to a great amount of friction at their surface of contact, it is at least possible to conceive that if the sum of the pressures existing at the same surface be sufficiently great, the whole mass might rotate nearly as if solid from its surface to its centre. As it will hereafter appear that such conditions must almost necessarily exist, it is superfluous for our present object to further examine the general problem of the rotation of the shell and nucleus. I shall therefore proceed to consider the pressures which might take place at the surface of contact of the shell and nucleus.

2. Abstracting the action of exterior disturbing bodies, the pressure of the nucleus at the inner surface of the shell will result from the attractions of all the particles of the shell and nucleus, centrifugal force, and molecular actions. Unless the last mentioned class of forces should have a tendency to disturb the position of the axes of rotation of the shell and nucleus, it is plain that these axes may be considered as coincident.

Of the two resultant pressures mentioned in the preceding article, it is evident that the variable pressure may be conceived to result from the difference in form of the free surface of the nucleus and the shell's inner surface. If these surfaces be very nearly spheroids of revolution, described about the shell's axis of rotation, it follows that the greatest pressure will be at the equator or poles of the shell, according as the tendency of the nucleus may be to become more or less oblate. In either case it is evident that every point on certain lines situated between the equator and poles will not be subjected to any pressure from this cause, or in other words, the sum of all the pressures at the points in question will be equal to the constant pressure. The general expression for the pressure at any point of the shell may therefore be made to contain the coordinates of the lines in question as constant quantities. As these lines in the case considered are evidently circles parallel to the equator of the shell, we may call them the parallels of mean pressure.

Let a straight and indefinitely narrow canal of fluid be conceived to reach from the centre to the surface of the nucleus. Let the origin of the coordinates of any point in it be fixed at the centre of gravity of the mass, and let the plane of x, y be perpendicular to the axis of rotation. Let r represent the radius drawn from the origin to a molecule in the canal, θ the angle made by this radius with the axis of rotation, and ω the angle formed by the projection of the radius on the plane of x, y with the axis of y; then

$$z = r\cos\theta,\ y = r\sin\theta\cos\omega,\ x = r\sin\theta\sin\omega.$$

The pressure p at the point in the canal having these coordinates will be expressed by the equation

$$dp = \varrho[\mathrm{X}dx + \mathrm{Y}dy + \mathrm{Z}dz + \alpha^2(xdx + ydy)],$$

in which, as in art. 2, Part I., X, Y, Z represent the components of the attractions parallel to the three rectangular axes of x, y and z, α the angular velocity of rota-

tion of the nucleus, and ϱ the density at the point in the canal. This equation may be written as follows, after the substitution of the above values, and it being remembered that θ and ω are constant for the canal,

$$dp=\varrho(dV+\alpha^2\sin^2\theta.rdr),$$

V being a function of r, θ and ω. On integration this gives

$$P-P'=\int_0^r \varrho dV+\alpha^2\sin^2\theta\int_0^r \varrho r dr,$$

P standing for the pressure on an unit of surface of the stratum of the nucleus having the radius r, and P'_0 the pressure at the centre. With the angular velocity a_1, all other things remaining as before,

$$P''-P'''=\int_0^r \varrho dV+a_1^2\sin^2\theta\int_0^r \varrho r dr.$$

But when θ becomes θ_1, its value at the parallel of mean pressure,

$$P'=\Pi-\int_0^{r'} \varrho dV-a^2\sin^2\theta_1\int_0^{r'} \varrho r dr,$$

$$P'''=\Pi-\int_0^{r'} \varrho dV-a_1^2\sin^2\theta_1\int_0^{r'} \varrho r dr,$$

Π representing the constant pressure on the stratum in question, and r' the value of r at the parallel of mean pressure. If α_1 be that angular velocity which would cause the surface of the nucleus to coincide with that of the shell $P''=\Pi$, we shall obtain by combining the foregoing expressions and neglecting the difference between r and r',

$$P=\Pi+(\alpha^2-\alpha_1^2)(\sin^2\theta-\sin^2\theta_1)\int_0^r \varrho r dr. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1.)$$

For the pressure P_1 at the shell's inner surface, we shall have

$$P_1=\Pi_1+(\alpha^2-\alpha_1^2)(\sin^2\theta-\sin^2\theta_1)\int_0^{r_1} \varrho r dr, \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.)$$

r_1 and Π_1 representing respectively the radius and pressure at the surface stratum of the nucleus. If Π_1 be not negative, this expression will be always positive with respect to some portion of the shell's inner surface; for when $\alpha > \alpha_1$ the greatest pressure is at the equator, and $\theta > \theta_1$; when $\alpha < \alpha_1$, the greatest pressure is at the poles, and consequently $\theta < \theta_1$.

3. The determination of θ_1 may be thus easily effected. Let the equations of the generating ellipses of the surfaces of the nucleus and shell be respectively

$$A_1^2x'^2+B_1^2z'^2=A_1^2B_1^2,\ A_2^2x'^2+B_2^2z'^2=A_2^2B_2^2,$$

A_1, B_1 and A_2, B representing respectively the less and greater axes of these surfaces. The volumes contained within these surfaces being equal, we have $A_1B_1^2=A_2B_2^2$, and hence at the parallels of mean pressure, where $x=x'$ and $z=z'$,

$$z=\pm A_1A_2\sqrt{\frac{B_1^2-B_2^2}{B_1^2A_2^2-B_2^2A_1^2}}.$$

This expression shows that the parallels of mean pressure are symmetrically situated at each side of the equator.

If for z its value be substituted, and the condition of equality of volumes be remembered, we shall obtain

$$\cos\theta_1 = \pm \frac{A_2}{\sqrt{A_1^2 + A_1 A_2 + A_2^2}}, \quad \ldots\ldots\ldots \quad (3.)$$

r_1 being supposed to differ very little from A_1.

If the changes in the oblateness of the nucleus be small, $\cos\theta_1$ will be nearly constant, and the parallels of mean pressure will oscillate but slightly about their mean position. Hence if $A_2 = A_1(1+\varepsilon)$, ε will be a very small quantity depending on the difference of the ellipticities of the two surfaces, but by TAYLOR'S theorem

$$F(A_2) = F(A) + \varepsilon F'(A) + \frac{\varepsilon^2}{1.2} F''(A_1) + \ldots.$$

Hence if the surface of the shell should change with every change of form of the nucleus, ε will be infinitely small, and consequently

$$\cos\theta_1 = \pm \frac{1}{\sqrt{3}}. \quad \ldots\ldots\ldots\ldots \quad (4.)$$

If the surface of the shell should not change its form for every change in form of the nucleus, ε might yet be so small as to be negligible compared with other quantities entering into our analytical expressions, and hence in such a case the above expression for θ_1 will be approximately true.

As θ_1 is the complement of the latitude at the parallel of mean pressure, we may conclude from the preceding investigation, that, *at the parallel the square of the sine of the latitude of which is one-third, the pressure of the fluid is always the same as if the surface of the nucleus were one of equal pressure for the shell.*

Hence if we represent by f the centrifugal force at the equator of the shell's outer surface, where the radius is supposed equal to unity, and by f_1 its value corresponding to a_1, equation (1.) becomes

$$P = \Pi - (f - f_1)\left(\cos^2\theta - \frac{1}{3}\right)\int_0^{a_1} \varrho a da, \quad \ldots\ldots\ldots \quad (5.)$$

where a_1 represents the radius of a sphere equal in volume to the spheroid included within the stratum of the nucleus with the radius r, and which consequently differs but little from r.

The simple expressions for the pressure on any stratum of the nucleus and on the shell's inner surface obtained in this section, will be found useful in the course of these researches, and particularly in the succeeding section.

II. THE VARIATION OF GRAVITY AT THE EARTH'S SURFACE.

4. It is in general evident, that if the laws of arrangement of the molecules composing the shell and nucleus be different, the variation of gravity at the surface of

the former will not be the same as if in solidifying all the particles retained the same positions which they had when constituting the entirely fluid mass. All investigations heretofore made of the variation of gravity at the earth's surface being grounded on this very untenable hypothesis, it seems desirable that a more general solution of the question should be obtained, in which such a supposition would not be involved.

The forces acting on a particle at the shell's outer surface which we have here to examine, are—1st, the shell's attraction; 2nd, the attraction of the nucleus; 3rd, centrifugal force. If the laws of arrangement of the matter composing the shell be continuous, the first of these forces will be equal to the difference between the attraction of the entirely solid spheroid and that of the spheroid produced by the complete solidification of the nucleus, the particles in both spheroids arranging themselves according to the influence of the forces acting on them.

5. *Attraction of the Shell.*—In a spheroid differing but little from a sphere, and composed of homogeneous strata varying in form and density, the expression upon which its attraction on an exterior point depends, is thus written*,—

$$V=\frac{4\pi}{r'}\int_0^{a'}\varrho_s a^2 da+\frac{4\pi\beta}{r'}\int_0^{a'}\varrho_s d\left(a^3W_0+\frac{a^4}{3r}W_1+\&c.\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad (6.)$$

the radius of each stratum being of the form $a(1+\beta w)$, a being the radius of the sphere equal in volume to the mass included within the surface of that stratum, and $w=W_0+W_1+W_2+\&c.$, W_0, W_1, &c. being functions satisfying the equation of LAPLACE's coefficients, ϱ_s the density of any stratum, a' the value of a at the surface, and r' the radius of the attracted point.

If the above expression be supposed to refer to the spheroid included within the shell's outer surface, it is evident that a corresponding expression may be obtained for the spheroid included within the shell's inner surface by merely changing β into β', and a' into a_1; β' being a constant depending on the ellipticity of that surface, and a_1 the value of a corresponding to it, we shall have therefore

$$V_1=\frac{4\pi}{r'}\int_0^{a_1}\varrho_s a^2 da+\frac{4\pi\beta'}{r'}\int_0^{a_1}\varrho_s d\left(aW_0+\frac{a^4}{3r}W_1+\text{etc.}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (7.)$$

If (6.) and (7.) be differentiated with respect to r, and the first then subtracted from the second, we shall obtain for G_1, the attraction of the shell, the expression

$$G_1=\frac{4\pi}{r'^2}\int_{a_1}^{a'}\varrho_s a^2 da+\frac{4\pi}{r'^2}\left\{\beta\int_0^{a'}\varrho_s d\left(a^3W_0+\frac{2a^4}{3r}W_1+\frac{3a^5}{5r^2}W_2+\ldots.\right)\right.$$
$$\left.-\beta'\int_0^{a_1}\varrho_s d\left(a^3W_0+\frac{2a^4}{3r}W_1+\frac{3a^5}{5r^2}W_2+\ldots.\right)\right\} \quad . \quad . \quad . \quad . \quad . \quad (8.)$$

Let us conceive the surfaces of both spheroids to be covered with a homogeneous fluid to a small depth compared with their radii. The bounding surfaces of these fluid strata will depend on the attractions of the spheroids, and also on the attractions of the fluid particles. If the density of each of these strata be nearly the same as that

* See PONTÉCOULANT, Théorie Analytique, &c., Livre V. No. 23.

of the solid stratum in immediate contact with it, its superposition on the spheroid will evidently not much increase the mass of the entire attracting body. The equation of equilibrium at the surface of the fluid on the greater spheroid is

$$\int_0^{a_1'}\frac{dp}{\varrho_2}=\frac{4\pi}{3a_1'}(1-\beta w)\int_0^{a_1'}\varrho_2 da^3+\frac{4\beta\pi}{a_1'}\int_0^{a_1'}\varrho_2 d\left[a^3W_0+\frac{a^4}{3r}W_1+\frac{a^5}{5r^2}W_2+\text{etc.}\right]$$

$$+\frac{1}{3}fa_1'^2-\frac{1}{2}fa'^2\left(\cos^2\theta-\frac{1}{3}\right),$$

a_1' being the value of a' at the fluid's surface, and p the pressure on the stratum. On developing and then comparing similar terms, we obtain

$$\int_0^{a_1'}\frac{dp}{\varrho_2}-\frac{4\pi}{3a}\int_0^{a_1'}\varrho_2 d.a^3+\frac{4\pi\beta W_0}{3a_1'^2}\int_0^{a_1'}\varrho_2 da^3-\frac{4\beta\pi}{a_1'}\int_0^{a_1'}\varrho_2 da^3W_0-\frac{1}{3}fa_1'^2=0.$$

The strata of the shell being bounded by spheroids of revolution and the origin of the coordinates being at the centre of gravity,

$$Wi=0,$$

i being different from 2. From the equation for W_0, it can be immediately inferred that W_0 may receive an arbitrary value. For W_2 we obtain

$$\frac{4\beta\pi}{5a_1'^3}\int_0^{a_1'}\varrho_2 d.a^5W^2-\frac{4\pi\beta W_2}{3a_1'}\int_0^{a_1'}\varrho_2 da^3+\frac{1}{2}fa_1'^2\left(\cos^2\theta-\frac{1}{3}\right)=0,$$

whence

$$\beta W_2=-\beta h\left(\cos^2\theta-\frac{1}{3}\right), \quad \ldots\ldots\ldots\ldots \quad (9.)$$

in which

$$\beta h=mf(a_1'), \quad \ldots\ldots\ldots\ldots \quad (10.)$$

m standing for the ratio of centrifugal force to gravity at the equator, and f being a functional symbol. If we make $W_0=-\frac{1}{3}h$, and substitute in the expression for r these values, we would have

$$r'=a_1'(1-\beta h\cos^2\theta),$$

and therefore βh must represent the ellipticity of the fluid stratum.

Similarly for the surface of the fluid stratum on the smaller spheroid, where $\overline{a}_1$ represents what $\overline{a}_1$ becomes for the fluid

$$r_1=\overline{a}_1(1-\beta' h\cos^2\theta), \quad \beta' h=mf(\overline{a}_1). \quad \ldots\ldots\ldots \quad (11.)$$

Besides these general expressions for βW_0, &c., $\beta' W_0$, &c., it is possible to find others in a particular case, which expressions will subsequently be found useful. The particular case referred to, is that in which the shell's strata are supposed to have all the same ellipticity. On this hypothesis W_0, W_1, &c. are not functions of a, and consequently (8.) becomes

$$G=\frac{4\pi}{r^2}\int_{a_1}^{a'}\varrho a^2 da+\frac{4\pi\beta}{r^2}\left[3W_0\int_0^{a'}\varrho a^2 da+\frac{8}{3}\frac{W_1}{r}\int_0^{a'}\varrho a^3 da+\frac{3W_2}{r^2}\int_0^{a'}\varrho a^4 da+\ldots\right]$$

$$-\frac{4\pi\beta'}{r^2}\left[3W_0\int_0^{a_1}\varrho a^2 da+\frac{8}{3}\frac{W_1}{r}\int^{a_1}\varrho a^3 da+\frac{3W_2}{r^2}\int_0^{a_1}\varrho.a^4 da+\ldots\right].$$

From the values attributed to a' and a_1, W_2 will disappear, and from the origin chosen for the coordinates W_1 will also disappear*.

Let M represent the mass of the greater spheroid, I its moment of inertia with respect to its axis of rotation, M_1 and I_1 being the corresponding quantities relative to the smaller spheroid, then

$$G=\frac{M-M_1}{r^2}+\frac{\beta}{r^4}\left\{\frac{q}{2}IW_2+c_3W_3+\ldots\right\}-\frac{\beta'}{r'^4}\left\{\frac{q}{2}I_1W_2+c'_3W_3+\ldots\right\},$$

c_3 and c'_3 being constant coefficients.

The artifice already employed may be used here in order to find expressions for βW_2, $\beta' W_2$, &c. In this case the equation of equilibrium of the fluid at the surface of the greater spheroid is

$$C=\frac{M'}{r'_1}\left\{1+\frac{\beta}{r'^3_1}\left[\frac{3}{2}\frac{I'}{M'}W_2+\ldots\right]\right\}-ra_1^2f\left(\cos^2\theta-\frac{1}{3}\right),$$

M' and I' standing for the mass and moment of inertia respectively of the whole mass composed of the fluid stratum and solid spheroid. This becomes, on developing r, and making $C=\frac{M'}{a'_1}+\frac{a'^2_1}{3}f$,

$$\frac{\beta w}{a}=\beta\left[\frac{3}{2}\frac{W_2}{a^3}\frac{I}{M}+\ldots.\right]-\frac{a^2f}{2M'}\left(\cos^2\theta-\frac{1}{3}\right).$$

But also

$$w=W_2+W_3+W_4+\ldots.+W_i.$$

The truth of these two simultaneous expressions requires that

$$W_2=\frac{3}{2}\frac{W_2}{a^3}\frac{I'}{M'}-\frac{a^3f}{2M\beta}\left(\cos^2\theta-\frac{1}{3}\right).$$

But $I=\frac{2}{5}\frac{Ma^2}{\sigma}$, σ being a number depending on the internal constitution of the spheroid, and $\frac{fa'^3}{M'}=m$ nearly; hence neglecting very small quantities, or making $a'_1=1$,

$$\beta W_2=-\frac{5\sigma m}{2(5\sigma-3)}\left(\cos^2\theta-\frac{1}{3}\right). \quad \ldots\ldots\ldots \quad (12.)$$

And in a similar manner we may obtain

$$\beta' W_2=-\frac{5a_1^3\sigma_1 m}{2(5\sigma_1-3)\mu_1}\left(\cos^2\theta-\frac{1}{3}\right), \quad \ldots\ldots\ldots \quad (13.)$$

μ_1 being used for brevity to represent $\frac{M_1}{M}$, and σ_1 being a number analogous to σ. Now if r be developed in (8.), and all terms of the order β^2 be neglected, it will become, remembering that $r'=a'(1+\beta y)$, y being a function of the polar coordinates of the point,

$$G_1=M\left\{1-\mu-\beta\left[2(1-\mu)y-\frac{q}{5}\left(\frac{1}{\sigma}-\frac{\mu\beta' a_1^3}{\beta\sigma_1}\right)W_2\right]\right.$$

$$\left.=M\left\{1-\mu+m\left[2(1-\mu)e-\frac{qm}{2(5\sigma-3)}\left(1-a_1^5\frac{5\sigma-3}{5\sigma_1-3}\right)\right]\left(\cos^2\theta-\frac{1}{3}\right), \quad \ldots\ldots \quad (14.)\right.$$

* See the work already cited, No. 21.

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it being remembered that the surface of the earth is a spheroid of revolution,

$$\beta y = -e\left(\cos^2\theta - \frac{1}{3}\right),$$

e representing the ellipticity. The expression for the attraction of a homogeneous shell with surfaces of equal ellipticity, is evidently

$$G_1 = M\left\{1 - a_1^2 + \frac{1}{2}\left[5(1-a_1^2)e - \frac{qm}{2}(1-a_1^4)\right]\right)\cos^2\theta - \frac{1}{3}\right)\Big\}. \quad . \quad . \quad . \quad . \quad . \quad (15.)$$

6. *Attraction of the Nucleus.*—The forces acting on any stratum of the nucleus, by changing the arrangement of its particles, must in general influence their action on exterior bodies; hence to obtain the attraction of the nucleus, it is necessary to take all such forces into consideration. These forces are in the present case, the attractions and pressures of masses of fluid within and without the stratum, the attraction of the shell, and centrifugal force. Hence for any stratum we have the following equation of equilibrium,

$$\begin{aligned}\int\frac{dp}{p} = {} & \frac{4\pi}{3r}\int_0^a \varrho da^3 + \frac{4\beta\pi}{r}\int_0^a \varrho d\left(a^2 Y_0 + \frac{a^4}{3r}Y_1 + \frac{a^5}{5r^2}Y_2 + \text{etc.}\right) \\ & + 2\pi\int_a^{a_1} \varrho_1 d.a^3 + 4\beta_1\pi\int_0^{a_1} \varrho_1 d\left(a^2 Y_0 + \frac{ar}{3}Y_1 + \frac{r^2}{5}Y_2 + \text{etc.}\right) \\ & + 2\pi\int_{a_1}^{a'} \varrho_2 d.a^3 + 4\beta\pi\int_{a_1}^{a'} \varrho_2 d\left(a^2 W_0 + arW_1 + \frac{r^2}{5}W_0 + \text{etc.}\right) \\ & + \frac{1}{3}fr^2 - \frac{1}{2}fr^2\left(\cos^2\theta - \frac{1}{3}\right),\end{aligned}$$

where p represents the pressure at any point in the stratum; Y_0, Y_1, &c. are functions of the coordinates satisfying the equation of LAPLACE's coefficients, and also possessing the property of forming the terms of the series into which the radius of the point may be developed.

But as the nucleus is inclosed in a rigid shell, its surface is constrained to take a form different from that which it would have if the shell were removed. The integral at the left side of the above equation is consequently variable. The conditions of equilibrium require that it should be constant, and these conditions may be fulfilled by separating the variable from the constant terms in that quantity and transposing the former terms to the right side, the quantities Y_0, Y_1, &c. being supposed to undergo any variations which may be required by the new conditions.

By article 2,

$$p = \Pi - (f - f_1)\left(\cos^2\theta - \frac{1}{3}\right)\int_0^a \varrho a da,$$

Π being such a pressure that

$$p_1 = \Pi - (f - f_1)\sin^2\theta_1 \int_0^{a_1} \varrho a da,$$

θ_1 being that value of θ which makes $p = \Pi$, and p_1 a function of a_1, and consequently a constant for the nucleus.

On developing $r=a(1+\beta y)$, and substituting the value of p, we obtain

$$\int_0^a \frac{dp_1}{\varrho}=\frac{4\pi}{3a}(1-\beta y)\int_0^a \varrho.da^3+\frac{4\beta\pi}{a}\Big[\int_0^a \varrho d.a^3Y+\frac{1}{3a}\int_0^a \varrho.da^4Y_1+\frac{1}{5a^2}\int_0^a \varrho.da^5Y_2+\text{etc.}\Big]$$
$$+2\pi\int_a^{a_1}\varrho da^3+4\beta\pi\Big[\int_a^{a_1}\varrho.da^3Y_0+\frac{a}{3}\int_a^{a_1}\varrho daY_1+\frac{a^2}{5}\int_a^{a_1}\varrho dY_2+\text{etc.}\Big]$$
$$+2\pi\int_{a_1}^{a'}\varrho da^3+4\beta\pi\Big[\int_{a_1}^{a'}\varrho da^3W_0+\frac{a}{3}\int_{a_1}^{a'}\varrho daW_1+\frac{a^2}{5}\int_{a_1}^{a'}\varrho dW_2+\text{etc.}\Big]$$
$$-\frac{1}{2}(f-f_1)a^3\sin^3\theta-\frac{1}{2}fa^3\Big(\cos^3\theta-\frac{1}{3}\Big)+\frac{1}{3}fa^3.$$

But

$$y=Y_0+Y_1+Y_2+\ldots.Y_i.$$

Hence if this value be substituted in the above equation, and if after substitution, similar functions of θ and ω, the coordinates of the point in the nucleus be equated to zero, we shall find

$$\int\frac{dp_1}{\varrho}-\frac{1}{3}fa^3-\frac{1}{2}(f_1-f)a^3-\frac{4\pi}{3a}(1-\beta Y_0)\int\varrho d.a^3-\frac{4\pi}{a}\int_0^a\varrho da^3Y_0$$
$$-2\pi\Big(\int_0^{a_1}\varrho da^3+\int_{a_1}^{a_1}\varrho_2 da^3\Big)-4\pi\Big(\beta\int_0^a\varrho.d.a^3Y_0+\beta\int_{a_1}^{a'}\varrho_2 da^3W_0\Big)=0.$$

But (art. 5) as $W_0=0$, and $\int\frac{dp_1}{\varrho}$ is constant, this expression shows that Y_0 may receive an arbitrary value. $Y_1=0$ by the property of the centre of gravity, which is the origin of the coordinates. When $i=2$,

$$\frac{4\beta_1\pi a^2}{5}\int_a^{a_1}\varrho.dY_2+\frac{4\beta_1\pi a^2}{5}\int_{a_1}^{a'}\varrho_2 dW_2-\frac{4\beta_1\pi}{3a}Y_2\int_0^a\varrho.da^3+\frac{4\beta\pi}{5a^5}\int_0^a\varrho.da^5Y_2$$
$$+\frac{1}{2}(f-f_1)a^3\cos^2\theta-\frac{1}{2}fa^3\Big(\cos^3\theta-\frac{1}{3}\Big)=0.$$

But we have already found

$$\beta W_2=-mf(a')\Big(\cos^3\theta-\frac{1}{3}\Big),$$

hence the above becomes

$$\frac{4\beta_1\pi a^2}{5}\int_a^{a_1}\varrho dY_2-\frac{4\beta\pi Y_2}{3a}\int_0^a\varrho d.a^3+\frac{4\beta\pi}{5a^5}\int_0^a\varrho da^5Y_2+\frac{1}{2}\Big(f_1\cos^3\theta-\frac{1}{3}f\Big)a^3$$
$$-\frac{a^2m}{a}\Psi(a_1,a')\Big(\cos^3\theta-\frac{1}{3}\Big),$$

where $\Psi(a_1, a')$ is a function depending on the attraction of the shell, or in other words, depending on the arrangement of the shell's strata.

When i is any number greater than 2, we have

$$\frac{4\pi a^i}{2i+1}\Big[\beta_1\int_a^{a_1}\varrho\frac{dY_i}{a^{i-3}}+\beta\int_{a_1}^{a'}\varrho_2\frac{dW_i}{a^{i-3}}\Big]-\frac{4\pi\beta}{3a}Y_i\int_0^a\varrho.da^3+\frac{4\pi}{(2i+1)a^{i+1}}\int_0^a\varrho da^{i+3}Y_i=0.$$

But as $W_i=0$, and as the surface of the nucleus is a spheroid of revolution, $Y_i=0$. At

the surface of the nucleus

$$\frac{4\beta\pi}{5a_1^3}\int_0^{a_1}\varrho d.a^5 Y_2-\frac{4\beta\pi Y_2}{3a_1}\int_0^{a_1}\varrho d.a^3+\frac{1}{2}a_1^2 f_1\left(\cos^2\theta-\frac{1}{3}\right)-a_1^2 m\Psi(a_1,a')\left(\cos^2\theta-\frac{1}{3}\right), \quad (a.)$$

because

$$f_1\left(\cos^2\theta-\frac{1}{3}\frac{f}{f_1}\right)=f_1\left(\cos^2\theta-\frac{1}{3}\frac{1}{1+f'}\right)=f_1\left[\cos^2\theta-\frac{1}{3}(1-f')\right],$$

where f' is a very small quantity, and where consequently $f_1 f'$ is a negligible product.

Solving (*a.*) with respect to $\beta_1 Y_2$, we obtain

$$\beta_1 Y_2=-\beta h\left(\cos^2\theta-\frac{1}{3}\right), \text{ (16.)}, \qquad \beta_1 h=f_1\Phi(a_1)+a_1^2 m\Psi(a_1,a'). \quad (17.)$$

If we make $Y_0=\frac{1}{3}h$ and substitute the value of y in r, we shall have

$$r=a_1(1-\beta h\cos^2\theta).$$

Hence $\beta h=e_1$, e_1 being the ellipticity of the surface of the nucleus, and consequently of the shell's inner surface.

The general form of the function upon which depends the attraction of the nucleus on an exterior point, may be found by substituting these values in (7.), having first changed ϱ_2 into ϱ and W_0, W_1, &c. into Y_0, Y_1, &c.; hence

$$V_2=\frac{M_1}{r'}\left\{1-\frac{e_1}{r}-\frac{e_1}{r^3}\left(\cos^2\theta-\frac{1}{3}\right)a_1^3\right\}+\frac{a_1^5}{r^3}\left[\frac{1}{2}f_1-m\Psi(a_1,a')\right]\left(\cos^2\theta-\frac{1}{3}\right), \quad (18.)$$

assuming that the mass of the nucleus is equal to the mass of the solid spheroid included within its surface, having the same law of density as the shell. The truth of this assumption will appear more manifest further on.

7. If, in a similar manner, we substitute the values of βW_2, &c., $\beta' W_2$, &c. in (8.), then develope r in both (8.) and (18.), after the differentiation of (18.) we shall have the sum of the attractions of the nucleus and shell on a point at the shell's outer surface, by adding the resulting expressions. If to this sum we add the term produced by the centrifugal force, we shall have for G, gravity at any point of the surface having θ for the complement of its latitude, the expression

$$G=G'(1+\lambda\cos^2\theta), \quad (19.)$$

where

$$\lambda=m\left[\frac{5}{2}-3\left(f(a)-\mu_1 a_1^2 f(a_1)\right)\right]+\frac{3}{2}\left[m_1-m(1+\Psi(a',a_1)\right]a_1^4+2e-3e_1\mu a_1^2, \quad (20.)$$

all terms of the second order being as usual neglected, m_1 standing for the value of m corresponding to f_1, and G' for gravity at the equator. When the mass is supposed to be entirely fluid $a_1=a'=1$, $m_1=m$, $\Psi(a')=0$, $\mu=1$, and consequently $\lambda=\frac{5}{2}m-e$, as should be expected. The foregoing is the most general expression for the variation of gravity at the earth's surface yet obtained; gravity at any point being expressed

as a function of the latitude of that point, of the radii and ellipticities of the shell's inner and outer surfaces, and of functions depending on the constitution of the shell and nucleus. Its value is not merely speculative, for it will be found to assist in explaining certain apparent anomalies detected by observation in the variation of gravity at the earth's surface, as well as in pointing out the limits assigned by observation to the thickness of the solid crust.

III. THE LAWS OF DENSITY OF THE SHELL AND NUCLEUS.

8. The density ϱ of a stratum of the mass when entirely fluid has been shown in Part I.* to depend on the pressure to which it may be subjected, and to the molecular properties of the fluid. The density of a stratum of the nucleus must evidently depend on the same circumstances, and hence we may assume its expression to have the same general form as for the entirely fluid mass, but yet containing variable indeterminate coefficients.

If the solidification of the nucleus proceeded entirely from its centre to its surface no shell could at any time exist, and as it will appear that it could not simultaneously solidify both from its centre towards its surface, and from its surface towards its centre, we can here consider only the latter case. When the solidification of the nucleus proceeds in this manner, its superficial stratum in contact with the inner surface of the shell will be the first to assume the solid state. As solidification must proceed very slowly from the slowness of the refrigeration of the entire mass, it is evident that the thickness of the stratum may be considered indefinitely small compared with the radius of the nucleus. Let its density in the fluid state be ϱ_1, in the solid state ϱ_2, and let $\varrho_1 = k\varrho_2$, k being a function depending on the contraction of the fluid when solidifying.

If, in conformity with the preceding remarks, we assume

$$\varrho = \frac{c_1 \sin a n_1}{a},$$

c_1 and n_1 varying with a_1 the mean radius of the nucleus, we shall have to determine four quantities in order to arrive at a knowledge of the laws of density of the shell and nucleus. It appears, however, that the number of known conditions which these quantities must satisfy will not suffice for their complete determination, although it is possible to conceive how a new condition could be experimentally found by which the required number would be made up. For if the physical properties of the matter composing the earth's interior resemble those of the matter at its surface, the form of k could be found with some degree of approximation, by a series of experiments on the contraction of fused matter at different densities resulting from differences of pressure. The conditions which can be at present determined are easily found thus: If the law of density of the shell be continuous, which must result from its mode of formation, and if the variations of a' and a_1 from refrigeration be neglected, we shall

* Articles 6 and 7.

have

$$4\pi\int_0^{a'}\varrho_2 a^2 da = 4\pi\left(\int_{a_1}^{a'}\varrho_2 a^2 da + \int_0^{a_1}\varrho a^2 da\right),$$

and consequently

$$\int_0^{a_1}\varrho_2 a^2 da = \int_0^{a_1}\varrho a^2 da, \quad \ldots\ldots\ldots\ldots \quad (21.)$$

$$C_1 = \frac{n_1^2\int_0^{a_1}\varrho_2 a^2 da}{\sin a_1 n_1 - a_1 n_1 \cos a_1 n_1};$$

and from the condition $\varrho_1 = k\varrho_2$,

$$k = \frac{\frac{n_1^2}{a_1\varrho_2}\int_0^{a_1}\varrho_2 a^2 da}{1 - a_1 n_1 \cot a_1 n_1}. \quad \ldots\ldots\ldots\ldots \quad (22.)$$

9. Let now an indefinitely thin canal of fluid be conceived to reach from the centre in a straight line to the surface of the nucleus, and consequently to pass through the surface stratum having the thickness da_1. Conceive this stratum to consist of an infinite number of elementary parallelopipeds, having their bases resting against the shell's inner surface. As the oscillations of the shell's surface are small, the simultaneous contraction of all these parallelopipeds can take place only in a direction perpendicular to their bases, and consequently da_1 will become $k_1 da_1$, k_1 being the cubical contraction of a mass of the fluid composing the stratum. The increase in volume of the shell, by the addition of the new stratum, will be less than the decrease in volume of the nucleus by its abstraction from its mass; and hence the nucleus must tend to expand in order to fill the empty space which would otherwise exist. This would evidently result as a necessary consequence of the cause of the variation in density of the strata from its centre to its surface.

The radius of the nucleus after the solidification of its superficial stratum being $a_1 - kda_1$, and it being manifest that the entire mass of the nucleus before the solidification took place is equal to the mass of the solid stratum with the thickness kda_1, and the mass of the new nucleus,

$$\int_0^{a_1}\varrho a^2 da = \int_0^{a_1-kda}\varrho' a^2 da + \int_{a_1-kda_1}^{a_1}\varrho_2 a^2 da,$$

or

$$\int_{a_1-kda_1}^{a}(\varrho_2-\varrho)a^2 da - \int_0^{a_1-kda_1}(\varrho-\varrho')a^2 da = 0,$$

where ϱ' is the density of the stratum with the radius a, after the solidification of the superficial stratum. As ϱ_2 and ϱ are constant in the interval $k_1 da$, we have

$$(\varrho_2-\varrho_1)k_1 a_1^2 da_1 = \int_0^{a_1-k_1 da_1}(\varrho-\varrho')a^2 da. \quad \ldots\ldots\ldots \quad (23.)$$

But ϱ being a function of a, and ϱ' being what this function becomes after the expansion of the stratum to which it refers, we may write

$$\varrho = f(a), \quad \varrho' = f(a + c_1 k_1 da),$$

c_1 being indeterminate, hence

$$\varrho - \varrho' = -c_1 k_1 \frac{d\varrho}{da} da - \frac{c_1^2 k_1^2}{1.2} \frac{d^2\varrho}{da^2} da^2 - \ldots.$$

consequently (23.) becomes after substitution and differentiation with respect to a_1,

$$\frac{d\varrho_2}{da} - c' \frac{d\varrho_1}{da} + \frac{2(1-k_1)}{a} \varrho_2 = 0, \quad \ldots \quad (24.)$$

neglecting terms of the second order, and making $c_1 - 1 = -c'$.

If we assume $\varrho_2 = \frac{C_2 \sin a_1 n_2}{a_1}$, C_2 and n_2 being constants, and make $q_1 = 1 - a_1 n_1 \cot a_1 n_1$, $q_2 = 1 - a_1 n_2 \cot a_1 n_2$ (24.) becomes

$$1 - k_1 - \frac{1}{2} q_2 + \frac{1}{2} c' - \frac{\varrho_1}{\varrho_2} q_1 = 0;$$

but as k_1 and k are evidently identical, we shall obtain after reduction

$$k = \frac{q_2 - 2}{q_1 - 2}, \quad \ldots \quad (25.)$$

whence

$$c' = q_2 - \frac{2[1-(k)]}{(k)q}, \quad \ldots \quad (26.)$$

in which

$$q = 1 - n \cot n, \quad q_2 = 1 - n_2 \cot n_2,$$

n being the value of n_1 at the surface of the entirely fluid mass, and (k) the value of k determined by experiment at the surface under atmospheric pressure.

But from (22.) we have

$$k = \left(\frac{n_1}{n_2}\right)^2 \frac{q_2}{q_1}; \quad \ldots \quad (27.)$$

hence for the determination of n_1 we obtain

$$n_1^2 \left(c' - \frac{2}{q_1}\right) = \left(1 - \frac{2}{q_2}\right) n_2^2. \quad \ldots \quad (28.)$$

In obtaining these results, some assumptions had to be made which in the absence of experimental data can be considered only provisional; but it will be satisfactory to perceive that the results themselves seem to be in accordance with what we would expect from known physical laws. When $a_1 = 0$, $q_1 = 0$, $q_2 = 0$ and $k = 1$, its greatest value, k consequently increases as the thickness of the shell increases, or as the density of the surface stratum of the nucleus increases. But the cubical contraction is equal to the cubical dilatation of the mass if reduced again to the fluid state under the same pressure, and by a recognized physical principle the dilatation of a body is equal to its resistance to compression, or inversely proportional to its compressibility. But the compressibility of a stratum of the nucleus decreases as its density increases, hence the above result appears to be in conformity with an observed physical law.

Equation (28.) may be thus written,

$$m'^{\frac{1}{3}}\left(c'-\frac{2}{q_1}\right)=a_1^2 n_3^2\left(1-\frac{2}{q_3}\right), \quad \ldots\ldots\ldots \quad (29.)$$

making $n_1=\frac{m'}{a_1}$; from which it is evident that m' decreases with a_1, and consequently that the less the volume of the nucleus, the more does it approach the state of homogeneity.

10. If any grounds existed for believing that experiment could never even approximately disclose the laws of density and contraction of the matter composing the earth's interior, we should not be justified in attaching much importance to any hypothesis, however plausible, which may be formed relative to the forms of these laws. Such grounds would partially exist as long as doubts could be formed relative to the great or small specific density of the matter of the earth's interior. The variation in density of the spheroidal strata composing the earth, could be due to an accumulation near its centre of substances possessing a high specific density. From what we know of the matter composing the earth, I believe that almost all substances having a density much greater than that of the outer portion of its crust are metals, bodies which from their chemical nature do not probably exist to any great extent within the globe. Recent experiments tend to show that many metals are but compound radicals which exist only under certain conditions, and it is probable that the constituents of any one of these radicals have densities less than that of the metal itself. Kane, Laurent and Gerhardt are of opinion, for instance, that the known metals do not exist in their oxides, but are eliminated when the oxides are decomposed. The metals alluded to are also simple bodies in the sense generally used, namely, that of being the lowest terms as yet known of compound bodies. Recent chemical researches seem to show that such simple bodies existed in the earth's primitive state, even in less quantity than we can at present discover them*. The chemical laws in virtue of which this would be true, would under certain conditions act as well in the interior as at the earth's surface, if we are entitled at all to admit their generality, and hence we must conclude that the specific density of the substances in the earth's interior must be subordinate to the effects of mechanical and physical causes in producing the observed variation in the density of its strata.

IV. THE FORMS OF THE STRATA OF THE SHELL.

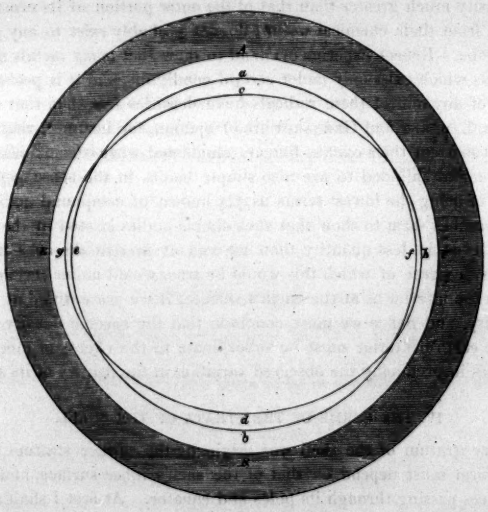
11. As every stratum of the shell was originally the surface stratum of the fluid nucleus, its form must depend on that of the shell's inner surface, and of the isothermal surfaces passing through its poles and equator. At first I shall abstract the influence of the isothermal surfaces in order to simplify the problem of the determination of the form of the stratum.

As in art. (1.), the shell is here supposed to be rigid, and to be perfectly filled

* See Bischof, Lehrbuch der Chem. und Phys. Geologie, Bd. I. s. 584, and Bd. II. s. 6.

with the fluid matter, but here we cannot consider the transition from solidity to fluidity as abrupt. Many physical analogies lead us to believe that, on the contrary, this transition must be gradual, or that a stratum of matter not completely fluid must exist between the highly fluid matter of the nucleus and the solid matter of the shell. The transition from this imperfectly fluid stratum to the perfectly fluid matter within it, will be also gradual, but it is possible to conceive that a portion of it in immediate contact with the shell may merely possess that kind of fluidity which in solids would be considered extreme softness. This matter will cohere to the shell with a considerable force, compared to the cohesion of the fluid particles; and this fact, combined with its viscidity, must serve to remove any doubt as to its not being subjected to the same hydrostatical laws as the perfect fluid. If now a surface be conceived to exist which may be called the effective surface of separation of the perfect from the imperfect fluid, it is evident that its form will depend on the pressures which the former tends to exert.

Let AEBF in the accompanying figure represent the profile of the inner surface of the shell, *aebf* of the similar and concentric surface of the perfect fluid. In



this case the pressure exerted by the nucleus would be the same for all points of the shell, and the thickness of the stratum of imperfect fluid would be everywhere the same. If, however, the pressure of the perfect fluid were not constant, its surface would tend to assume a different form, in order to re-establish equilibrium; it might

for example, tend to assume the form of which *cgdh* is the profile. The thickness of the stratum of imperfect fluid would then be greatest at A, and least at E.

On solidifying, the new stratum thus added to the shell would have the same proportional thickness, and its interior surface would in this particular case be more elliptical than its exterior surface. If, as seems extremely probable, the imperfectly fluid stratum be thin, and if it strongly adhere to the shell, it will exercise no sensible pressure on the perfect fluid, and must consequently take the form impressed on it without any sensible resistance.

12. It can be easily shown that in general the pressure of the perfect fluid will not be constant. Let a spheroidal mass of fluid be conceived, consisting of nearly similar spheroidal strata, these strata increasing in density as their radii decrease. The ellipticities of the bounding surfaces of any stratum will depend on the constitution and thickness of the strata outside it. If possible let the mass outside the stratum be removed, without altering the law of density of the remaining fluid: it is manifest that equilibrium will be obtained only where the surface of the stratum at any point becomes perpendicular to the resultant of all the forces acting on that point. With the same law of density as the entire mass, and urged by the same forces, the ellipticity of its surface must be the same as that of the surface of the primitive mass. If, however, the law of density changed in such a way as to render the remaining fluid more homogeneous, the ellipticity of its surface would be greater than that of the primitive fluid spheroid*. Hence we may conclude—(1.) that if the angular velocity and law of density of the nucleus remained unchanged after the formation of the first stratum of the shell, the outer and inner surfaces of that stratum would be similar, and its attraction on the interior mass would consequently, by a well-known theorem, be evanescent. The next formed stratum would thus also have similar surfaces, and so on with every stratum, until the mass should have completely solidified. In this case, therefore, the surfaces of all the strata would have the same ellipticity as the outer surface. (2.) If the forces acting on the nucleus after the solidification of the first stratum of the primitive fluid were such as to give the surface of the nucleus a tendency to become less oblate, the ellipticity of the interior surface of the imperfectly fluid stratum would be less than that of its exterior surface, and the resulting attraction of the shell after the solidification would manifestly tend to increase in the same direction the effect of the forces previously in action. (3.) If, on the contrary, the resultant of all these forces were such as to increase the oblateness of the perfect fluid, it is evident, from the preceding considerations, that the inner surface of the shell so produced would always be more elliptical than its outer surface.

These conclusions will be necessarily modified when the influence of the isothermal surfaces in the interior of the spheroid is considered. It seems that no complete and general solution of the problem of finding the forms of these surfaces has been yet

* See Poisson, Mécanique, ii. p. 546, 2nd edition.

achieved, but an approximate solution, founded on an hypothesis to which no weighty objections can be urged, has been given by Mr. HOPKINS*. This solution will suffice for my present purpose, particularly when I only refer to the general result of his analysis. The result in question is, that the oblateness of the isothermal surfaces increases with their distance from the earth's surface. This result, combined with the first of the three conclusions in the foregoing paragraph, seems to show that if the earth solidified from its surface to its centre without changing its law of density, the ellipticity of the shell's inner surface would be greater than that of its outer surface. The truth of the second conclusion would be considerably weakened, while that of the third would be strengthened to the same amount.

13. It will immediately appear that this last is the conclusion which must be definitely adopted, if we admit that the matter composing the nucleus becomes denser in assuming the solid state.

Using the notation of art. 6, and neglecting the effect of isothermal surfaces, the ellipticity of the surface of the perfect fluid may be generally expressed thus,

$$e_1 = m_1\Phi(a_1) \pm m\Psi(a_1, a'),$$

the double sign being placed before $m\Psi(a_1 a')$ to show that, according as the ellipticity of the shell's inner surface is greater or less than that of its outer surface, this term should be added or subtracted, as must appear from the theory of the attraction of spheroids. If, when e_1 is supposed to increase, the small term alluded to be neglected, the truth of any conclusion as to the rapid increase of e_1 deduced from an examination of the remaining term will be only rendered still more manifest.

Let at any period of the shell's existence the surface of the nucleus be supposed to coincide with the shell's inner surface, then $m_1 = m$, and consequently, from the expressions given in art. 6, we may deduce

$$e_1 = \frac{e_1 m a_1^3 \int_0^{a'} \varrho_2 a^2 da}{2\left\{e_1 \int_0^{a_1} \varrho_2 a^2 da - \frac{1}{5a^2}\int \varrho_2 da^5 e_1\right\}} = \frac{m a_1^3 \int_0^{a'} \varrho_2 a^2 da}{2\left(1-\frac{3}{5\sigma_2}\right)\int_0^{a_1} \varrho_2 a^2 da - \frac{2e_1}{5a_1^3}\int_0^{a_1} \varrho a_1^5 da} = \frac{m a_1^3}{2\mu\left(1-\frac{3}{5\sigma_2}\right)},$$

σ_2 being a number depending on the law of density of the nucleus, and analogous to σ of article 5. But as the centrifugal force at the surface of the spheroid is proportional to the square of its angular velocity, and as the angular velocity is inversely proportional to the moment of inertia of the mass, we shall have

$$m = m'\left(\frac{\mathrm{I}'}{\mathrm{I}}\right)^2,$$

I' representing the moment of inertia of the mass in its state of entire fluidity, I the moment of inertia corresponding to m, and m' the value of m corresponding to I'. From the general expression for the moment of inertia of a spheroid, we can write

$$\frac{\mathrm{I}'}{\mathrm{I}} = \frac{\frac{a'^2}{\sigma_1'}}{\frac{a'^2}{\sigma} - \mu a_1^2\left(\frac{1}{\sigma_1} - \frac{1}{\sigma_2}\right)}$$

(σ_2' being the value of σ_2 when $a_1 = a'$).

* Philosophical Transactions, 1842, p. 45.

Hence, making $a'=1$,

$$e_1=\frac{m'}{2}\left(\frac{\sigma}{\sigma'_2}\right)^3\left\{\frac{a_1^2}{\mu}+a_1^2\sigma\frac{(\sigma_2-\sigma_1)}{\sigma_1\sigma_2}\right\}^3\left(1-\frac{3}{5\sigma_1}\right)^{-1}=\frac{m'}{2}\left(\frac{\sigma}{\sigma'_2}\right)^3\left[\nu^3+2\nu a_1^2\sigma\frac{(\sigma_2-\sigma_1)}{\sigma_1\sigma_2}\right]\left(1-\frac{3}{5\sigma_1}\right)^{-1},$$

where ν^3 represents a number which always differs but little from 1. The coefficient of a_1^2 being small, it is evident that the numerator of this expression cannot vary rapidly with a_1.

From article 12, it is evident that e_1 must always either increase or decrease as a_1 increases; in order to know how it varies, it will therefore be only necessary to examine its value for any two values of a_1.

Let $a_1=1$, then

$$e_1=\frac{m'}{2}\left(\frac{\sigma}{\sigma'_2}\right)^3\left[1+2\frac{(\sigma'_2-\sigma)}{\sigma'_2}\right]\left(1-\frac{3}{5\sigma}\right)^{-1}=\frac{9}{10}\frac{m}{(\sigma)^3}\mathrm{K},$$

(σ) being determined by observation, and K having the value assigned to it in art. 8, Part I. It will appear that the greatest value of K which can be assigned will make e a little less than $\frac{1}{300}$.

When $a_1=0$,

$$e'=\frac{5}{2}m\left(\frac{\sigma}{\sigma'_2}\right)^3\frac{\nu_1^3\sigma'_1}{5\sigma'_1-3},$$

σ'_1 and ν_1 being nearly equal to unity; and σ'_2 and σ not much differing, e' will not much differ from m, and consequently it will be greater than e. This remark can be verified by actual calculation, when we shall have obtained the numerical values of the quantities contained in the preceding expression.

V. THE PRINCIPAL MOMENTS OF INERTIA OF THE EARTH.

14. The dependence of all the phenomena of the earth's rotation upon its principal moments of inertia, and the resulting connexion which thus subsists between these phenomena and the internal constitution of the earth, renders a complete investigation of their values of some importance for the objects to be fulfilled by the present memoir.

Let the moment of inertia with respect to the axis of rotation be C, and A and B those for the other two axes. Let C_1, &c. represent the moments of inertia of the solidified spheroid included within the shell's outer surface, C_2, &c. the moments of inertia of the spheroid included within the shell's inner surface, and C_3, &c. the moments of inertia of the nucleus, then

$$C=C_1-C_2+C_3,\ B=B_1-B_2+B_3,\ A=A_1-A_2+A_3,$$

and representing an element of the earth's mass by dm,

$$C=\int(x^2+y^2)dm,\ B=\int(x^2+z^2)dm,\ A=\int(y^2+z^2)dm,$$

x, y, z being the rectangular coordinates of the element dm. In finding C_1, &c., C_2, &c., C_3, &c., different values for dm must in general be introduced in the foregoing expres-

sions, and then the integrals may be taken between the limits proper for each particular case.

Let x, y and z be transformed into polar coordinates as in article 2, and the sums of their squares taken two and two be put in such a form as to satisfy the equation of LAPLACE's coefficients. We shall then have, putting μ for $\cos\theta$,

$$C_1=\int_{-1}^{+1}\int_0^{2\pi}\int_0^r \varrho_2 r^4\left\{\frac{2}{3}+\left(\frac{1}{3}-\mu^2\right)\right\}dr\,d\omega\,d\mu,$$

$$B_1=\int_{-1}^{+1}\int_0^{2\pi}\int_0^r \varrho_2 r^4\left\{\frac{2}{3}+\left[\frac{1}{3}-(1-\mu^2)\sin^2\omega\right]\right\}dr\,d\omega\,d\mu,$$

$$A_1=\int_{-1}^{+1}\int_0^{2\pi}\int_0^r \varrho_2 r^4\left\{\frac{2}{3}+\left[\frac{1}{3}-(1-\mu^2)\cos^2\omega\right]\right\}dr\,d\omega\,d\mu.$$

But

$$\int_0^r \varrho_2 r^4 dr=\frac{1}{5}\int_0^{a'}\varrho_2\frac{d(r^5)}{da}da=\frac{1}{5}\int_0^{a'}\varrho_2\frac{d}{da}[a^5(1+\beta\omega)]da,$$

and

$$W=W_0+W_1+W_2+\ldots.+W_i.$$

From article 5 $W_i=0$, except when $i=2$ or 1; and in the latter case it is made to disappear when a' is the radius of a sphere equal in volume to the spheroid; we have, therefore, on making

$$\int_0^{a'}\varrho_2\frac{d(a^5)}{da}da=\chi(a'),$$

and

$$\int_0^r \varrho r^4 dr=\frac{1}{5}\chi(a')+\int_0^{a'}\varrho_2 d\frac{(a^5\beta W_2)da}{da}=\frac{1}{5}\chi(a)-5m\left[f(a')\int_0^a \varrho_2 a^2 da-\frac{1}{2}\int_0^{a'}\varrho_2 a^2 da\right]\left(\frac{1}{3}-\mu^2\right)$$

by article 5. Consequently

$$\left.\begin{aligned}
C_1&=\frac{4}{3}\left[\frac{2\pi}{5}\chi(a')-\frac{Mm}{3}\left(f(a')-\frac{1}{2}\right)\right],\\
C_2&=\frac{4}{3}\left[\frac{2\pi}{5}\chi(a_1)-\frac{Mm}{3}\left(f(a_1)\mu_1-\frac{1}{2}a_1^3\right)\right],\\
C_3&=\frac{4}{3}\left[\frac{2\pi}{5}\chi_1(a_1)-\frac{Mm}{3}\left(e_1 a_1^3\mu_1-\frac{1}{2}m_1 a_1^3\right)\pm m a_1^3\Psi(a_1,a')\right],
\end{aligned}\right\}\quad\ldots\ldots\quad(30.)$$

where

$$\chi_1(a_1)=\int_0^{a_1}\varrho\frac{d(a^5)da}{da}.$$

Hence

$$C=\frac{4}{3}\left\{S+\frac{M\Psi}{3}\right\},\quad\ldots\ldots\ldots\ldots\ldots\ldots\quad(31.)$$

making for brevity

$$S=\frac{2\pi}{5}[\chi(a')-\chi(a_1)+\chi_1(a_1),\quad\ldots\ldots\ldots\ldots\ldots\quad(32.)$$

$$\Psi=e_1\mu_1 a_1^3-\frac{1}{2}m_1 a_1^3-m\left[\frac{1}{2}-f(a')+\mu_1 a_1^3 f(a_1)-\left(\frac{1}{2}+\Psi(a_1,a')a_1^3\right]\quad\ldots\quad(33.)$$

By following a similar process we obtain

$$A=\frac{4}{3}\left\{S-\frac{M\Psi}{6}\right\},\quad B=\frac{4}{3}\left\{S-\frac{M\Psi}{6}\right\},$$

and consequently

$$\frac{2C-A-B}{C}=\frac{\Psi}{S}M,$$

neglecting quantities of the second order.

On substituting for ϱ_3 its value (art. 9.),

$$\chi(a')-\chi(a_1)=\frac{5C_2}{n_2^4}\{[2n_2^2+(n_2^2-6)q_2]\sin n_2-[2a_1^2n_2^2+(a_1^2n_2^2-6)q^2]\sin a_1n_2\};$$

and similarly, on substituting for ϱ its value in $\chi_1(a_1)$,

$$\chi_1(a_1)=\frac{5a_1\varrho_2k}{n_1^4}[2a_1^2n_1^2+(a_1^2n_1^2-6)q_1];$$

but by articles 8. and 9,

$$C_2=\frac{(\varrho_2)}{\sin n_2},\quad \varrho_2=(\varrho_2)\frac{\sin a_1n_2}{\sin n_2},\quad M=4\pi(\varrho_2)q_2n_2^2,$$

(ϱ_2) being the value of ϱ_2 at the shell's outer surface. Hence, finally,

$$S=\frac{M}{2q_2}\left\{\left[2+\left(1-\frac{6}{n_2^2}\right)q_2\right]-\left[2a_1^2+\left(a_1^2-\frac{6}{n_2^2}\right)q_2\right]\frac{\sin a_1n_2}{\sin n_2}\right.$$
$$\left.+\left[2a_1^2+\left(a_1^2-\frac{6}{n_1^2}\right)q_1\right]\frac{kn_2^2\sin a_1n_2}{n_1^2\sin n_2}\right\}.$$

But we have, by article 9,

$$k=\frac{n_1^2}{n_2^2}\frac{q_2}{q_1};$$

and on comparing equations (20.) and (23.),

$$\Psi=\frac{2e+m-\lambda}{3}; \qquad (34.)$$

hence if we make

$$\frac{2C-A-B}{C}=p',$$

we shall obtain

$$\left\{2a_1^2+\left(a_1^2-\frac{6}{n_2^2}\right)q_2-\frac{q_2}{q_1}\left[2a_1^2+\left(a_1^2-\frac{6}{n_1^2}\right)q_1\right]\right\}\sin a_1n_2=Q,$$

or

$$\left\{2a_1^2\left(1-\frac{q_2}{q_1}\right)-6\left(\frac{1}{n_2^2}-\frac{a_1^2}{n^2}\right)q_2\right\}\sin a_1n_2=Q, \qquad (35.)$$

making for brevity

$$Q=\left[2+\left(1-\frac{6}{n_2^2}\right)q_2-\frac{2}{3}\cdot\frac{2e+m-\lambda}{p'}q_2\right]\sin n_2. \qquad (36.)$$

But the value of p' is given from the phenomena of the precession of the equinoxes and the nutation of the earth's axis, and that of λ may be obtained from experiments made with the pendulum on the earth's surface, or from the inequalities in the moon's motion in longitude and latitude depending upon the form and internal constitution

of the earth. Thus an expression for finding a_1 has been obtained in which e_1 is absent, the elimination having been effected between the expressions depending on the variation of gravity at the earth's surface and the precessional phenomena.

As $C-A=C-B=\frac{2}{3}M\Psi$, it follows that the difference of the principal moments of inertia of the earth is proportional to Ψ. But from the general expression for Ψ and from Section IV., it is evident that Ψ increases as a_1 diminishes, hence we may in general conclude that *the difference between the greatest and least moment of inertia of the earth increases as the thickness of the shell increases.* This conclusion being independent of any knowledge of the absolute laws of density of the earth's interior, deserves particular attention.

VI. ON THE EXISTENCE OF A SOLID NUCLEUS WITHIN THE EARTH.

15. In the preceding investigations the earth has been supposed to solidify solely from its surface towards its centre, but it is possible to conceive how from the enormous pressure on its central strata solidification could also proceed from the centre towards the surface. If the influence of pressure in promoting solidification were sufficiently great, the earth might have solidified entirely from its centre towards its surface, according to the theory proposed by Poisson. It becomes important therefore to examine how far we are justified in adopting the theory of solidification first mentioned.

If solidification took place from the centre towards the surface alone, we should believe the earth to be now entirely solid. The forms of the solid strata, composing the spheroid in this case, would not be in general the same as if the original fluid mass had solidified from its surface towards its centre. Before the solidification of any part of the mass, it would consist, in virtue of hydrostatical laws, of a series of spheroidal strata of equal pressure. Refrigeration at the centre would proceed at an almost insensible rate from the impediments to convection adduced in article 6, Part I., and from the necessarily slow conducting power of the fluid; and the theory of solidification examined requiring the predominance of pressure over refrigeration as an agent in solidifying the mass, it must follow that the forms of the isothermal surfaces would have little influence on those of the solidified strata. For greater simplicity, I abstract, in the first instance, the effects of refrigeration in contracting the solid nucleus and the surrounding fluid.

The first small nucleus solidified would evidently be bounded by the surface of equal pressure due to its radius. No change can take place therefore in the direction of the resultant of the forces acting on any molecule of the stratum in contact with the solid nucleus, and hence on solidifying it will retain its form. The next stratum must become solid, similarly, without changing its form, and so on towards the surface. Hence the ellipticities of the strata of the solid spheroid would be the same as when it existed in a fluid state.

If the effect of refrigeration in contracting the matter of the entire spheroid be now considered, it is evident that it will tend to lessen the mean radius of the entire mass, and consequently to increase its angular velocity of rotation. The ellipticities of the strata of fluid surrounding the nucleus would be greater after every new addition to its mass than they were before that addition took place, and therefore, after the complete solidification of the entire mass, its strata would increase in ellipticity from its centre to its surface more rapidly than the strata of the original fluid mass increased in ellipticity. In this case, the variation of gravity at the surface of the earth in going from its equator to its poles, should, by the theory of the attraction of spheroids, be greater than for the same mass in a fluid state with the same ellipticity at surface. Observation however shows that, on the contrary, the variation of gravity at the earth's surface is less rapid than for the primitively fluid spheroid. The earth could not therefore have solidified in this manner required by this theory.

16. If, by the action of refrigeration and compression, solidification proceeded simultaneously from the surface towards the centre, and from the centre towards the surface, it will not be doubted but that the former process, when once commenced, must proceed far more rapidly than the latter. The temperature at the centre would be nearly constant, compared with the temperature of the surface of the fluid, from the obstacles opposed to convection and the laws of propagation of heat by conduction. Solidification at the centre will therefore be due chiefly to pressure; but if contraction accompany the change of state of the fluid matter in becoming solid, it appears from Section III. that the pressure on every stratum of the fluid will decrease with the radius of the shell's inner surface. The pressure on the solid nucleus will thus be continually diminishing, while its temperature will remain almost unchanged. The solid nucleus would therefore, instead of acquiring increased magnitude, tend to return to its original fluid state. This will evidently be true at every period of the existence of the nucleus, and hence we must conclude that this mode of solidification is incompatible with our original assumptions.

VII. THE DIRECTIONS OF THE FISSURES IN THE SHELL WHICH MIGHT BE PRODUCED BY THE ACTION OF THE PRESSURES CONSIDERED IN SECTION I.

17. If the pressure of the nucleus against any part of the shell be sufficiently intense to produce a fissure, the direction of that fissure will depend on those of the tensions resulting from the fluid pressure and on the physical structure of the shell. In the actual case of the earth it is probable that the highly crystalline structure, which we have reason to believe is characteristic of the shell, would give to some portions of it a tendency to fracture in particular directions. We have at present no precise physical reasons for thinking that this tendency should follow any general law, so that it must now be considered as only a source of irregular deviations in the directions of the fissures from those which they would have if the shell possessed an uniform cohesive strength.

Abstracting these irregular causes, an examination of the question of the directions of the fissures in a spheroidal and nearly spherical shell, composed of strata each possessing in itself an uniform cohesive strength, but differing in this respect from every other, would assist in pointing out what should be the predominant directions of the fissures, if the greater effectiveness of general over particular causes be allowed.

The problem which is here to be examined is therefore simply to find the directions of the fissures in a thin, solid and nearly spherical shell, resulting from the pressures of a mass of fluid inclosed in it, the cohesion of the shell being supposed to vary, according to some continuous law, along a vertical line from its inner to its outer surface.

If the constant pressure referred to in Section I. alone acted on the shell, the tensions at any point in one of the shell's indefinitely thin strata would be equal to each other and situated in the tangent plane at that point. The position of the maximum resultant tension would therefore be indeterminate, and consequently there would be no tendency to form a fissure in any particular direction rather than in any other. If the variable pressure be superimposed on the constant pressure, the direction of the maximum resultant tension would be determinate, and consequently also the line in which a fissure would commence. At first, if the intensity of the pressures should gradually increase, a portion of the shell at each side of the equator bounded by parallels would be subjected to tensions sufficiently great to produce fissures, while beyond these parallels the shell would remain unfractured. The portion where the tensions would be sufficiently great to produce fissures, would thus constitute a disturbed district with nearly fixed boundaries. At a point where there is a tendency towards the formation of a fissure, the direction of the maximum resultant tension will be in the direction of the tangent to the meridian; the greatest tendency to form a fissure will therefore be parallel to the equator. From the nature of the variable pressure the maximum tensions must be equal, at equal distances from the equator; hence such a fissure, when once commenced, would tend to be propagated along a parallel of latitude until the force of the tensions become sufficiently lessened by the separation of the extended portion of the shell. Similar fissures would be formed simultaneously and symmetrically at each side of the equator. As long as the tensions in the directions of the tangents to the meridians continued sufficiently great, such fissures would be formed; but, as already mentioned, their formation would tend to annul these tensions, and a new system would result, having a tendency, as may be readily deduced from Mr. Hopkins's investigations*, to produce fissures perpendicular to those previously formed. If the maximum intensity of the variable pressure be not inconsiderable compared to the constant pressure, it will follow, if the pressures continue to act with sufficient energy, that all the shell's fissures will be either parallel or perpendicular to the equator.

If, on the contrary, the constant pressure were far greater than the variable pres-

* Cambridge Philosophical Transactions, vol. vi.

sure, the maximum and minimum resultant tensions would be nearly equal, and consequently the directions of the fissures would be governed chiefly by accidental causes. Taking into consideration the causes which have been abstracted, it appears therefore that the directions of the fissures might in general have no necessary relations with that of the equator. If such a fissure commenced forming in the direction of any great circle, it would evidently continue to be propagated in the same general direction unless accidental causes should alter its course.

At some stages of the shell's existence the relation between the variable and the constant pressure might be such as to cause the maximum tension at any point to be in the direction of the tangent of the meridian, or parallel at that point, and hence, in this case at least, all simultaneously formed fissures should be parallel; the term parallel being used to mean that the tangents to the circles forming the prolongations of these fissures should all form equal angles at the points of intersection with the tangents of a great circle bisecting them.

VIII. ON THE EXISTENCE OF A ZONE OF LEAST DISTURBANCE IN THE SHELL.

18. During the process of the formation of the shell, the forces tending to fracture it, and those holding its parts together, will in general be continually varying. If the intensity of the former class of forces increased much more rapidly than that of the latter, there would be no limit to the disturbed part of the shell; if, on the contrary, the latter class of forces increased more rapidly in intensity than the former, some parts of the shell might after a certain time be comparatively undisturbed. It is even possible to conceive, if the constant pressure of the fluid against the shell be small, that in some parts of the shell the cohesive forces keeping its particles together might be always greater than the rupturing forces at these parts.

In order to form a general idea of the positions of such undisturbed portions of the shell, the variation of the effective pressure at any point in the shell must be considered. Of the two different ways in which the effective pressure could vary in going from the equator to the pole of the shell, it is evident, from Section IV., that the only one which it is necessary to consider is that of its decrease. In this case we may conclude, from article 2, that the pressure everywhere between the equator and parallel of mean pressure must be greater than that which exists at the same parallel, and that everywhere between this line and the pole the pressure will on the contrary be less. The disturbed portion of the shell must therefore be near the equator, and that line must divide it into two equal parts. In order that an undisturbed zone of the shell may exist, it is not necessary that the latter should be perfectly rigid; it may be capable of subsiding by the abstraction of a certain amount of the pressure of the fluid until its parts should rest in equilibrium. With a considerable variable pressure and a constant pressure insufficient to produce fractures in the shell, it is possible to conceive how this could occur, and hence the most general idea which can be formed of the undisturbed portion of the shell at either side of the equator, is that

it must be a zone bounded by two parallels, both of which may become coincident with the equator, or evanescent, according as the pressures are very small or very great. If it be admitted that any great disturbances in the shell must have been chiefly due to the action of such pressures, it would be useful to examine the problem of the determination of the boundaries of the zone of least disturbance. By comparing the results of our investigation with observed phenomena, we may be able to deduce some conclusions directly applicable to geology.

I proceed therefore to determine the analytical expressions for the latitude of the limiting parallel of rupture on the side of the zone nearest to the equator. Let θ_2 represent this latitude, θ_3 the latitude of a parallel in the supposed undisturbed part of the shell.

It will appear that the general equation between the rupturing and cohesive forces upon which the position of the parallel of mean pressure depends, can, when the influence of temperature is abstracted, be made to contain but two independent variables θ_2 and a_1, or that it will be of the form $F(\theta_2, a_1)=0$. The value required of θ_2 must evidently be a maximum, and hence we must determine it in general by elimination between the equations

$$F(\theta_2, a_1),\ \frac{dF}{da_1}+\frac{dF}{d\theta_2}\frac{d\theta_2}{da_1}=0,\ \text{or}\ \frac{dF}{da_1}=0;$$

the fact of the resulting value being a maximum or minimum can be determined as usual by the sign of $\frac{d^2\theta}{da_1^2}$.

19. Let two infinite planes be conceived to pass through the axis of rotation of the shell, making so small an angle with each other that the curvature of the arc of the equator intercepted between them could be considered as insensible. The ratio of the rupturing to the cohesive forces in either of the opposite portions of the shell thus intercepted, will be the same as for the whole shell under our assumed conditions. The conditions for finding the section of rupture of one of these bands will therefore be the same as those for the entire shell.

Let dL represent an element of the area of the section of rupture, l its perpendicular distance from the neutral surface, l_1 and l_2 the perpendicular distances of the neutral surface from the outer and inner surfaces of the shell, s the resistance measured by the number of units of force required to rupture a square unit of section of the material of the shell at the distance l. The meaning generally attached, in works on mechanics, to the term neutral surface, is that here attached to it, namely, the surface at which the portions of the band subjected to the rupturing forces are neither compressed nor extended.

Let, as is generally assumed, s be a function of l, and let s_1 be its value at the distance l_1.

If the deflection of the band, in a state of tension and bordering on rupture, be very small, as must undoubtedly be the case from its physical structure, the directions of

the rupturing forces will be perpendicular to the neutral surface, and therefore, from a well-known property of that surface, l_1 and l_2 will be the perpendicular distances of the centre of gravity of the section of rupture, from the outer and inner surfaces of the shell.

The force developed upon the element $d\mathrm{L}$, opposed to the compression or extension of the material of the band at the distance l_1, will be $\frac{sl}{l_1}d\mathrm{L}$, and its moment about the line formed by the intersection of the neutral surface with the section of rupture will be $\frac{s}{l_1}l^2 d\mathrm{L}$.

Let r_2 represent the radius drawn from the centre of the spheroid to the intersection of the neutral surface with the section of rupture; then the moment of the pressure P_1 between the section of rupture and the parallel of θ will be $r_2\mathrm{P}\sin(\theta-\theta_2)d\mathrm{L}'$, where θ stands for the latitude at any point of the band subjected to tension, and $d\mathrm{L}'$ an element of the area of the shell's inner surface. If we represent by b_1 the equatorial axis of the shell's inner surface, ω_1 the arc intercepted between the planes bounding the band, b_2 the equatorial axis of the neutral surface, and make $\varepsilon^2=\frac{b_1^2-a_1^2}{a_1^2}$, we shall have

$$d\mathrm{L}=\frac{\omega_1 b_1\cos\theta}{1-\varepsilon^2\sin^2\theta},\quad r=b_2\sqrt{1-\varepsilon^2\sin^2\theta}.$$

As the sum of the moments of the pressures exerted on the band at either side of the section of rupture, when the shell is in the state bordering on rupture, must be equal to the sum of the moments of the cohesive forces at the same section, we shall have

$$\frac{1}{l_1}\int sl^2 d\mathrm{L}=\int_{\theta_2}^{\theta_3} r_2\mathrm{P}_1\sin(\theta-\theta_2)d\mathrm{L}=\omega b_1^2 b_2(1-\varepsilon^2)\int_{\theta_2}^{\theta_3}\frac{\mathrm{P}_1\sin(\theta-\theta_2)\cos\theta d\theta}{(1-\varepsilon^2\sin^2\theta)^{\frac{3}{2}}}.$$

But from article 3, $\mathrm{P}_1=h(h_1+\cos^2\theta)$, making for brevity

$$h=(f-f_1)\int_0^{\pi'}\varrho a da,\quad h_1=\frac{\Pi}{h}-\frac{2}{3},$$

and remembering that here $\frac{\pi}{2}-\theta$ of that article is represented by θ. Hence

$$\frac{\int sl^2 d\mathrm{L}}{l_1\omega_1 b_1^2 b_2(1-\varepsilon^2)h}=\int_{\theta_2}^{\theta_3}\frac{(h_1+\cos^2\theta)\sin(\theta-\theta_2)\cos\theta d\theta}{(1-\varepsilon^2\sin^2\theta)\sqrt{1-\varepsilon^2\sin^2\theta}}$$

$$=h_1\cos\theta_2\int\cos\theta\sin\theta(1-\varepsilon^2\sin^2\theta)^{-\frac{3}{2}}d\theta-h_1\sin\theta_2\int\cos^2\theta(1-\varepsilon^2\sin^2\theta)^{-\frac{3}{2}}d\theta$$

$$+\cos\theta_2\int^{\theta}\cos^3\theta\sin\theta(1-\varepsilon^2\sin^2\theta)^{-\frac{3}{2}}d\theta-\sin\theta_2\int\cos^4\theta(1-\varepsilon^2\sin^2\theta)^{-\frac{3}{2}}d\theta.$$

The integrals multiplied by $\cos\theta_2$ can be easily found by ordinary methods, but the two multiplied by $\sin\theta_2$ cannot be given in a finite form. But from the nature of the problem one of the limits at least of the integrals must be an independent variable; hence the integrals multiplied by $\sin\theta_2$ can be determined only by deve-

loping the irrational factor in a converging series, and then integrating separately the resulting terms. For greater uniformity I develope in all the integrals the irrational factor, which becomes when developed

$$1+\frac{3}{2}\varepsilon^2\sin^2\theta+\frac{3}{2}\cdot\frac{5}{4}\varepsilon^4\sin^4\theta+\frac{3}{2}\cdot\frac{5}{4}\cdot\frac{7}{6}\varepsilon^6\sin^6\theta+\text{ etc.}$$

Hence the sum of the four integrals will be, neglecting terms of the order ε^4,

$$h_1\left[\cos\theta_2 f\cos\theta\sin\theta d\theta-\sin\theta_2 f\cos^2\theta d\theta\right]+\cos\theta_2 f\cos^3\theta\sin\theta d\theta-\sin\theta_2 f\cos^4\theta d\theta$$

$$-\frac{3}{2}\varepsilon^2\{h_1\left[\cos\theta_2 f\sin^2\theta\cos\theta d\theta-\sin\theta_2 f\cos^2\theta\sin^2\theta d\theta\right]+\cos\theta_2 f\cos^3\theta\sin^2\theta d\theta-\sin\theta_2 f\cos^4\theta\sin^2\theta d\theta\},$$

On taking these integrals between the limits θ_2 and θ_3, the expression becomes

$$\begin{aligned}
&h_1\left[\frac{\cos\theta_2}{4}(\cos 2\theta_3-\cos 2\theta_2)-\frac{\sin\theta_2}{2}\left[\frac{1}{2}(\sin 2\theta_3-\sin 2\theta_2)+\theta_3-\theta_2\right]+\frac{\cos\theta_2}{8}\left[\frac{1}{4}(\cos 4\theta_2-\cos\theta_3+\cos 2\theta_3-\cos 2\theta_3\right]\right.\\
&-\frac{\sin\theta_2}{4}\left[\frac{1}{3}(\sin 4\theta_3-\sin 4\theta_2)+\sin 2\theta_3-\sin 2\theta_2+\frac{3}{2}(\theta_3-\theta_2)\right]-\frac{3}{2}\varepsilon^2\left\{h_1\frac{\cos\theta_2}{8}\left[\frac{1}{4}(\cos 4\theta_3-\cos 4\theta_2)\right.\right.\\
&\left.-(\cos 2\theta_3-\cos 2\theta_3)\right]+\frac{h_1\sin\theta_2}{8}\left[\frac{1}{4}(\sin 4\theta_3-\sin 4\theta_2)-\theta_3+\theta_2\right]+\frac{\cos\theta_2}{64}\left[\frac{1}{3}(\cos 6\theta_3-\cos 6\theta_2)\right.\\
&\left.\left.-3(\cos 2\theta_3-\cos 2\theta_2)\right]+\frac{\sin\theta_2}{32}\left[\frac{1}{6}(\sin 6\theta_3-\sin 6\theta_2)+\frac{1}{2}(\sin 4\theta_3-\sin 4\theta_2)-\frac{1}{2}(\sin 2\theta_3-\sin 2\theta_2)-2(\theta_3-\theta_2)\right]\right\}\\
&=\left.\begin{aligned}
&[(a_1)\sin(\theta_3+\theta_2)\sin(\theta_3-\theta_2)+(a_2)\sin 2(\theta_3+\theta_2)\sin 2(\theta_3-\theta_2)+(a_3)\sin 3(\theta_3+\theta_2)\sin 3(\theta_3-\theta_2)]\cos\theta_2\\
&-[(b_1)\cos(\theta_3+\theta_2)\sin(\theta_3-\theta_2)+(b_2)\cos 2(\theta_3+\theta_2)\sin 2(\theta_3-\theta_2)+(b_3)\cos 3(\theta_3+\theta_2)\sin 3(\theta_3-\theta_2)]\sin\theta_2\\
&-(c_1)(\theta_3-\theta_2)\sin\theta_2.
\end{aligned}\right\}\quad\ldots\quad(37.)
\end{aligned}$$

Making for brevity

$$\left.\begin{aligned}
&(a_1)=\frac{1}{2}\left[h_1+\frac{1}{2}-\frac{3}{8}\varepsilon^2\left(h_1+\frac{3}{8}\right)\right],\quad a_2=\frac{1}{16}\left(1-\frac{3}{2}\varepsilon^2 h_1\right),\quad a_3=\frac{\varepsilon^2}{64},\\
&(b_1)=\frac{1}{2}\left[h_1+1-\frac{3\varepsilon^2}{32}\right]\qquad (b_2)=\frac{1}{16}\left[1+\frac{3}{2}\varepsilon^2\left(h_1+\frac{1}{2}\right)\right],\quad b_3=(a_3),\\
&\qquad\qquad (c_1)=\frac{1}{2}\left[h_1+\frac{3}{4}+\frac{3}{8}\varepsilon^2\left(h_1+\frac{1}{2}\right)\right].
\end{aligned}\right\}\quad\ldots\quad(38.)$$

20. I now proceed to determine the expressions for fsl^2dL. In general the cohesive strength of an unit of surface of the section of rupture may be considered proportional to the number of molecules of which it is the section, and as the density ϱ_2 must at any point be also proportional to the number of molecules at that point, it follows that

$$s=s_1\frac{\varrho_2}{(\varrho_2)}.$$

The section of rupture may evidently be considered as a trapezoid, hence

$$dL=\frac{\omega_1}{r_1}(r_2+l)\cos\theta_2 dl=\frac{\omega_1}{a_1}(1-\varepsilon^2\cos^2\theta)\cos\theta_2(a_1+l_2+l)dl=\frac{\omega_1}{a_1}(a_1+l_2+l)\cos\theta_2 dl$$

very nearly. Hence

$$fsl^2dL=\frac{\omega_1 s_1\cos\theta_2}{a_1\varrho_2}\int_{-l_2}^{+l_1}\varrho_2(a_1+l_2+l)l^2dl.$$

Let v represent the distance from the centre of the spheroid of any element of the section of rupture, v_1 the distance of the centre of gravity of that section, then

$$l_1=1-v_1,\ l_2=v_1-a_1,\ v_1=\frac{\int_{a_1}\varrho_2 v^2 dv}{\int_{a_1}\varrho_2 v dv} \quad \ldots\ldots\ldots\ldots \quad (39.)$$

But $\varrho_2=\frac{C_2 \sin a_1 n_2}{a_1}$ (art. 9.) at the distance a_1; hence, substituting its value for the distance a_1+l_2+l, we obtain

$$\int \varrho_2 (a_1+l_2+l) l^2 dl = C_2 \int l^2 \sin (a_1+l_2+l) n_2 dl.$$

Let $a_1+l_2+l=u$, and the preceding factor of C_2 becomes

$$\int u^2 \sin u n_2 du - 2(a_1+l_2)\int u \sin u n_2 du + (a_1+l_2)^2 \int \sin u n_2 du.$$

Integrating between the limits l_1 and l_2, and we have

$$\int u^2 \sin u n_2 du = \frac{1}{n_2^3}\{n_2^2 a_1^2 \cos a_1 n_2 - n_2^2 (a_1+l_1+l_2)\cos (a_1+l_1+l_2) n_2$$

$$+2(a_1+l_2+l_2) n_2 \sin (a_1+l_1+l_2) n_2 - 2 a_1 n_2 \sin a_1 n_2 + 2\cos (a_1+l_1+l_2) n_2 - 2\cos a_1 n_2\},$$

$$\int u \sin u n_2 du = \frac{1}{n_2^2}[(a_1+l_1+l_2) n_2 \cos (a_1+l_1+l_2) n_2 + a_1 n_2 \cos a_1 n_2 + \sin (a_1+l_1+l_2) n_2 - \sin a_1 n_2],$$

$$\int \sin u n_2 du = \frac{1}{n_2}[\cos a_1 n_2 - \cos (a_1+l_1+l_2) n_2].$$

But $l_1+l_2=1-a_1$; hence

$$\int_{-l_2}^{+l_1} \varrho_2 (a_1+l_2+l) l^2 dl = C_2 \left\{\frac{1}{n_2^3}[(n_2^2 a_1^2-2)\cos a_1 n_2 - (n_2^2-2)\cos n_2 + 2 n_2 (\sin n_2 - a_1 \sin a_1 n_2)]\right. \quad (40.)$$

$$\left. + \frac{2(a_1+l_2)}{n_2^2}[\sin a_1 n_2 - a_1 n_2 \cos a_1 n_2 - \sin n_2 + n_2 \cos n_2] + \frac{(a_1+l_2)^2}{n_2}(\cos a_1 n_2 - \cos n_2)\right\}.$$

To determine l_1 and l_2, I substitute for ϱ_2 its value in (39.) and integrate, then

$$v_1 = \frac{n_2 (a_1 \cos n_2 a_1 - \cos n_2) + \sin n_2 - \sin a_1 n_2}{n_2 (\cos a_1 n_2 - \cos n_2)}$$

$$l_1 = \frac{(1-a_1)\cos a_1 n_2}{2 \sin \frac{1}{2}(1+a_1) n_2 . \sin \frac{1}{2}(1-a_1) n_2} - \frac{1}{n_2}\cot \tfrac{1}{2}(1+a_1) n_2$$

$$l_2 = \frac{1}{n_2}\cot \tfrac{1}{2}(1+a_1) n_2 - \frac{(1-a_1)\cos n_2}{2 \sin \frac{1}{2}(1+a_1) n_2 . \sin \frac{1}{2}(1-a_1) n_2}.$$

If we make

$$j = \left\{(1-a_1)\cot \tfrac{1}{2}(1-a_1) n_2 - 1\right\} \cot \tfrac{1}{2}(1+a_1) n_2,$$

we shall have

$$l_1 = \frac{1-a_1+j_2}{2},\ l_2 = \frac{1-a_1-j_2}{2} \quad \ldots\ldots\ldots\ldots \quad (41.)$$

It appears, from the value of n_2 deduced by observation, that when $1-a_1$ is a small

fraction, j will be small compared with l_1 or l_2; hence in general, when the shell is thin,

$$l_1=l_2=\tfrac{1}{2}(1-a)_1.$$

21. The function in which θ_3 is to be made a minimum is by art. 19,

$$(A)-[(a_1)\sin(\theta_3+\theta_2)\sin(\theta_3-\theta_2)+(a_2)\sin 2(\theta_3+\theta_2)\sin 2(\theta_3-\theta_2)+(a_3)\sin 3(\theta_3+\theta_2)\sin 3(\theta_3-\theta_2)]$$
$$+[(b_1)\cos(\theta_3+\theta_2)\sin(\theta_3-\theta_2)+(b_2)\cos 2(\theta_3+\theta_2)\sin 2(\theta_3-\theta_2)+(b_3)\cos 3(\theta_3+\theta_2)\sin 3(\theta_3-\theta_2)]\tan\theta_2$$
$$+(c_1)(\theta_3-\theta_2)\tan\theta_2=0\ [(A)\text{ being a function of }a_1],$$

or

$$(A)-[(a_1)\sin(\theta_3+\theta_2)-(b_1)\cos(\theta_3+\theta_2)\tan\theta_2]\sin(\theta_3-\theta_2)-[(a_2)\sin 2(\theta_3+\theta_2)-(b_2)\cos 2(\theta_3+\theta_2)\tan\theta_2]\sin 2(\theta_3-\theta_2)$$
$$-(a_3)[\sin 3(\theta_3+\theta_2)-\cos 3(\theta_3+\theta_2)\tan\theta_2]+(c_1)(\theta_3-\theta_2)\tan\theta_2=0. \quad \ldots\ldots\ldots\ 42.)$$

From this expression we can deduce the following conclusions:—

1st. If θ_3 be less than $\frac{\pi}{2}$, θ_3 cannot be zero unless $(A)=0$.

2nd. If $\theta_3=\frac{\pi}{2}=\theta_2$, $\cos(\theta_3+\theta_2)=-1$, $\cos 2(\theta_3+\theta_2)=0$, $\cos 3(\theta_3+\theta_2)=-1$,

and consequently

$$(A)=(b_1)+(b_3)-(c_1)=\tfrac{1}{8}\left[1-\tfrac{3}{2}\epsilon^2\left(\tfrac{1}{2}+h_1\right)\right]$$
$$h_1=\tfrac{2}{\epsilon^2}(1-8(A))-\tfrac{1}{2}.$$

3rd. In order that in this case $\Pi=h$, we must have $(A)=\frac{1}{8}\left(1-\frac{5}{12}\epsilon^2\right)$.

From article 20,

$$(A)=\frac{4s_1\int_0^{\pi k(1-e_1)}g_2(1+a_1+2l)l^2dl}{(1-2e_1)(1+e_1)^2(1-a_1^2)a_1^2h}.$$

The value obtained for the numerator of this expression, shows that for no conceivable value of a_1 (A) can vanish; and therefore from the first of the preceding conclusions, it may be deduced that if the shell be fixed under any parallel, it must be also fixed under some other parallel at the same side of the equator and at a distance from it, depending on the cohesive strength of the material of the shell. Consequently in the case of $\theta_3<\frac{\pi}{2}$ a zone of least disturbance should exist. In general it is evident that the value of (A) $\left(\text{supposing }\theta_3=\frac{\pi}{2}\right)$, which would make θ_3 a maximum, must be extremely small; consequently, from the second and third conclusions, if $\theta_3=\frac{\pi}{2}$, or if no zone of least disturbance exist, Π must be considerably greater than h.

IX. CALCULATION OF SOME OF THE CONSTANTS CONTAINED IN THE FORMULÆ OF THE PRECEDING SECTIONS.

22. 1st. To find the numerical value of n_2 and n. It is evident that when D represents the mean density of the earth, we should have

$$\frac{1}{n_2^2}(1-n_2\cot n_2)=\frac{D}{3(\varrho_2)},$$

(ϱ_2) being the density of the shell's outer surface, or in other words, the density of the first stratum solidified from the primitive fluid mass constituting the earth. To find D, I take a mean of the results of the best experiments which have been made for its determination, namely, of the experiments of Cavendish, Baily and Reich, or respectively of the numbers 5·48, 5·68 and 5·44. The mean of these results is 5·53 nearly. The density of granite, the crystalline rock which seems to form the base of all the sedimentary formations, is evidently that which must be used for (ϱ_2). This remark is important, because in all comparisons heretofore made of the mean and surface density of the earth, the mean density of the sedimentary rocks has been erroneously taken into consideration. From a comparison of the densities of granite obtained in different countries on the authority of different geological and engineering works, I have decided that the mean density of the rock cannot be less than 2·7. Adopting this value, the above equation becomes

$$\frac{1}{n_2^2}(1-n_2\cot n_2)=\cdot 682716,$$

which is approximately satisfied by making

$$n_2=\frac{14227}{18000}\pi=142^\circ\ 16'\ 22''.$$

To find n, we have

$$\frac{n^2}{1-n\cot n}=\frac{(k)n_2^2}{1-n_2\cot n_2}=\frac{(k)}{\cdot 682716}.$$

Let $(k)=\cdot 7481$, its least value found by experiment. Then

$$\log\left(\frac{n^2}{1-n\cot n}\right)=\cdot 0397196.$$

By trials I find that when $n=154^\circ\ 25'$, the quantity at the left side of the above expression is ·0396799, and when $n=154^\circ\ 24'$, it becomes ·0398333; hence it lies between both of these values.

When $n=154^\circ\ 24'\ 30''$,

$$\log\left(\frac{n^2}{1-n\cot n}\right)=\cdot 0397196,$$

hence this value will serve for a first approximation.

When $(k)=\cdot 896$, the greatest value found by experiment, we should have

$$\log\left(\frac{n^2}{1-n\cot n}\right)=\cdot 1180679.$$

The value $n=147^\circ\ 30'$ would make this ·1188293, hence this value of n would be a little too small.

2nd. To find p' and λ. From the fourth chapter of the fourth book of the work of M DE PONTÉCOULANT, it will be perceived that

$$p=\frac{4ln}{3m^2(1+\gamma)\cos h},$$

where l represents the mean movement of the equinoxes at the time when h represents the apparent obliquity of the ecliptic, and γ the ratio of the moon's action on the earth compared to that of the sun, n and m being constants, the former depending on the earth's rotation, and the latter on the sun's mean movement. If the variation of h be referred to the plane of the ecliptic in 1800,

$$h=23^\circ\ 27'\ 55'',\ l=50''\cdot363541,$$

and also

$$n=360^\circ\cdot98561,\ m=359^\circ\ 59'\cdot37;$$

and when m is referred to the same unit of time as n,

$$m=0^\circ\cdot98561.$$

Of the three different methods by which γ can be determined, I select that depending on the phenomena of nutation. I do so because it seems that astronomers have taken much pains to determine the numerical coefficient which depends on these phenomena, and on which the value of γ depends. If we represent the coefficient of nutation by N*, we shall have

$$\frac{\gamma}{1+\gamma}=\frac{N}{13''\cdot36926},\ \text{or}\ \gamma=\frac{N}{13''\cdot36926-N}.$$

The following values have been deduced for N

By ROBINSON . . .	$9''\cdot234$
By BRINKLEY . . .	$9''\cdot25$
By LINDENAU . . .	$8''\cdot977$
By PLANA	$8''\cdot925$

the mean of which is $9''\cdot0965$, and therefore $\gamma=2\cdot128951$;

$\log(1+\gamma)=$ ·4953988	$\log l=1\cdot7021163$
$\log m=6\cdot1125773$	$\log\frac{n}{m}=2\cdot5637896$
$\log\cos h=9\cdot9625122$	$\log 4=$ ·6020600
$\log 3=$ ·4771213	$4\cdot8679659$
$7\cdot0476296$	$7\cdot0476296$
	$\log p'=\bar{3}\cdot8203363$

* PONTÉCOULANT, Théorie, &c., tome iv., Note 3, p. 654.

If we represent by λ' the coefficient of the variation of gravity at the earth's surface, found by pendulum experiments, and by λ'' the same coefficient deduced from observation of the moon's inequalities in longitude and latitude, then

$$\lambda'=\frac{5}{2}m-e', \quad \lambda''=\frac{5}{2}m-e'',$$

e' and e'' being the corresponding ellipticities of the earth's surface found by the ordinary theory. Let, in accordance with the latest calculations,

$$e'=\frac{1}{288}, \quad e''=\frac{1}{300},$$

then λ, the most probable value of the coefficient of the variation of gravity, becomes

$$\frac{5}{2}m-\frac{1}{293{\cdot}88}.$$

3rd. To find the numerical value of Q in Section V. (equation 36). From the value of n_2 just obtained,

$$1-\frac{6}{n_2^2}={\cdot}026872, \quad 2+\left(1-\frac{6}{n_2^2}\right)q_2=2{\cdot}11311.$$

If we make, in accordance with observation,

$$e=\frac{1}{300}, \quad m=\frac{1}{289},$$

and use the value of λ above obtained,

$$\frac{2}{3}\frac{2e+m-\lambda}{p'}q_2=\frac{8122.q_2}{5\times 578\times 2880p}=2{\cdot}07068,$$

$$Q={\cdot}045253\times\sin n_2={\cdot}0260259.$$

GEOLOGICAL DEDUCTIONS FROM THE FOREGOING INVESTIGATIONS.

(1.) *The Stability of the Axis of Rotation of the Earth.*

23. The conclusion arrived at in article 14, shows that if the rotation of the earth were originally stable about its axis, it would continue to rotate in the same way for ever.

The action of exterior bodies has been heretofore alone examined in considering the question of the position of the earth's axis of rotation within it. From this examination, it results that the action of such bodies would be incapable of producing any change in the position of the axis, and hence, if such a change were at all possible, it should be produced by some interior action by which the distribution of the particles composing the earth would be changed. It is admitted, that if the earth were fluid and in rotation with an angular velocity differing but little from its present angular velocity, it would rotate stably about its shorter axis. During the process of its successive solidification, it might happen that the new arrangements of the particles

* See HUMBOLDT's Kosmos, Bd. I. s. 174, and PONTÉCOULANT, tome iv. p. 486.

might be such as to disturb the rotation, not only by increasing or lessening the angular velocity, but also by changing the position of the axis. It appears however, from the article cited, that the difference of the greatest and least moment of inertia of the earth must progressively increase during the process of solidification, and hence that the stability of rotation must continually increase until it reaches its limit when the mass shall have arrived at the entirely solid state.

Thus not only does a question closely connected with geological theory seem to be definitively settled, but also the future stability of the earth's rotation appears to be completely assured.

(2.) *The Thickness of the Earth's Solid Crust.*

24. In Section II. expressions have been obtained in which the variation of gravity at the earth's surface is a function of the radius and ellipticity of the fluid nucleus supposed to exist within it, from which it will be possible to deduce the limiting values of that radius, and consequently of the thickness of the solid shell. If we refer to the general expression (20.), it will be perceived that the greater is e_1, the less the thickness of the shell; hence we would be able to obtain its greatest thickness consistent with observation, other quantities remaining unchanged, by giving to e_1 its least value. But from Section IV. the least value of e_1 is e, or the ellipticity of the surface of the shell, hence in this case $m_1=m$, and

$$\lambda=\left[\frac{5}{2}-3(f(a)-\mu_1 a_1^2 f(a_1)\right]m+2e-3\mu a_1^2 e.$$

The greatest value which a_1 can receive will depend on the limits imposed on the values of the functions depending on the earth's internal constitution; and if we give to these values favourable to a large value of a_1, e_1 must very nearly be equal to e. The above expression for λ will therefore suffice for this case by attributing the proper values to the functions $f(a)$ and $f(a_1)$. But when $e_1=e$, we have, by the expression deduced in article (5.),

$$\lambda=2e+m+\frac{3}{2}\left(\frac{5a_1^5\nu\sigma_1 m}{5\sigma_1-3}-2a_1^5 e-\frac{3m}{5\sigma-3}\right);$$

ν representing the same quantity as in article (13.), σ_1 will differ but little from σ when $e_1=e$, and with a continuous law of density of the strata of the shell; hence in a first approximation we may assume their equality. But $\lambda=\frac{5}{2}m-(e)$, (e) being the value of e, deduced by the ordinary theory from observation of the variation of gravity at the earth's surface, hence

$$a_1=\nu^{-\frac{1}{5}}\left[1-\frac{2}{3}\frac{(5\sigma-3)(e)-e\mathrm{I}}{6e-5\sigma(2e-m)}\right]^{\frac{1}{5}}.$$

To find the limiting values of a_1, we must make this a maximum or a minimum. If we suppose ν to differ but little from 1, it will be nearly constant, and therefore

$$\frac{da_1}{d\sigma}=\frac{2}{3}\left[1-\frac{2}{3}\frac{p\sigma-p'}{q'-q\sigma}\right]^{-\frac{4}{5}}\left[\frac{qp'-pq'}{(q-\sigma q)^2}\right],$$

$$\frac{d^2a_1}{d\sigma^2}=\frac{16}{15}\left\{\frac{1}{q'-\sigma q}-\frac{pq'-qp'}{3\left(1-\frac{2}{3}\frac{p\sigma-p'}{q'-q\sigma}\right)}\right\}\left(1-\frac{2}{3}\frac{p\sigma-p'}{q'-q\sigma}\right)^{-\frac{1}{2}}\frac{pq'-p'q}{(q'-\sigma q)^2};$$

putting $p=5[(e)-e]$, $p'=3[(e)-e]$, $q'=6e$, $q=5(2e-m)$.

But $1-\frac{2}{3}\frac{p\sigma-p'}{q-q\sigma}$ is always less than 1, and $q'-\sigma q$, $pq'-p'q$ are both positive and small quantities. Hence $\frac{d^2a_1}{d\sigma^2}$ is positive, and therefore the value of σ, deduced by making $\frac{da_1}{d\sigma}=0$, gives a minimum for a_1. This value is evidently $\sigma=\frac{3q'+2p'}{3q+2p}$.

On substituting this in the value of a_1, we shall have $a_1=0$: $\frac{da_1}{d\sigma}=0$ would be also satisfied by making $\sigma=\infty$, but the supposition that ν is finite and little different from 1, excludes this value. Hence a_1 must increase from its least value as σ decreases to its least value, but this is when $\sigma=1$, or when the shell is supposed to be homogeneous; consequently the greatest value of a_1 will be determined from the equation

$$a_1^2=\frac{\lambda+\frac{5}{4}m-2e}{3\left(\frac{5}{4}m-e\right)}=\frac{2}{3}+\frac{\lambda-\frac{5}{4}m}{3\left(\frac{5}{4}m-e\right)}=\frac{2}{3}+\frac{1}{3}\frac{\frac{5}{4}m-(e)}{\frac{5}{4}m-e}.$$

From Section IX. $(e)=\frac{1}{294}$ nearly; and using for e and m the values respectively $\frac{1}{300}$ and $\frac{1}{289}$, we shall obtain

$$a_1^2=\frac{1}{3}\left[2+\frac{157\times 50}{172\times 49}\right]=\cdot 97714;$$

hence $a_1=\cdot 99539$, $1-a_1=\cdot 00461$, and therefore, consistently with observation, the least thickness of the earth's crust cannot be less than 18 miles.

From Section III. it appears that as (k) diminishes the more will the constitution of the nucleus and shell be different, and consequently the least value which can be attributed to (k) from observation, may be assumed to give the least value of a_1 in equation (35.). By using this equation together with (29.), I find, after a few trials, that with $a_1=\cdot 85$, the approximate value of Q would be $\cdot 023441$. But Q increases with a_1, hence the greatest thickness of the shell cannot, on the above assumption, be greater than $\cdot 15$, or 600 miles nearly.

(3.) *The Earth's Ellipticity when entirely fluid.*

25. By the aid of equation (11.), Part I., the following values of the earth's primitive ellipticity have been obtained with different values of (k).

With $(k)=\cdot 7481$, $E=\frac{1}{302\cdot 22}$,

$(k)=\cdot 896$, $E=\frac{1}{310\cdot 74}$,

$(k)=1\cdot$, $E=\frac{1}{316}$.

The value of the earth's moment of inertia used in these calculations, was deduced by using the value of p found in Section IX., and that of λ found from the lunar inequalities. If the value of the latter quantity, found in the section referred to, were used, the resulting values of E would all be a little smaller; it may therefore be concluded, that *the earth's primitive ellipticity was less than its present ellipticity, although the difference between them may be neglected.*

(4.) *The Direction of great Lines of Elevation on the Surface of the Earth.*

26. If a zone of least disturbance existed near the parallel of mean pressure, the directions of great lines of elevation should be nearly parallel or perpendicular to the equator. Its non-existence there, which observation seems to show, proves at least that the variable pressure did not predominate over the constant pressure.

As yet, observation seems to prove that such a zone does not exist on the earth's surface; and hence from Section VIII. we must provisionally conclude that the constant pressure greatly predominated over the variable pressure, and consequently that the directions of lines of elevation must be comparatively arbitrary. Geological and geographical observations present results which are generally in accordance with these views.

(5.) *The Existence of great Friction and Pressure at the Surface of Contact of the Nucleus and Shell.*

27. The conclusions arrived at in Section IV., combined with the important result obtained by Mr. Hopkins in his second memoir on Physical Geology*, show that great friction and pressure must exist between the shell and fluid nucleus. The result alluded to, as quoted in Mr. Hopkins's third memoir, is

$$\text{“}P'-P_1=\left(1-\frac{\epsilon}{\epsilon_1}\right)\left(1-\frac{\nu}{1+\frac{h}{q^5-1}}\right)P_1,$$

where P denotes the precession of a solid homogeneous spheroid of which the ellipticity$=\epsilon_1$, that of the earth's exterior surface, and P′ the precession of the earth, supposed to consist of a heterogeneous fluid nucleus contained in a heterogeneous spheroidal shell, of which the interior and exterior ellipticities are respectively ϵ and ϵ_1, the transition being immediate from the entire solidity of the shell to the perfect fluidity of the mass.” This result was found on the hypothesis of the non-existence of friction and pressure from molecular causes at the surface of contact of the shell and nucleus; if this hypothesis were true, we should have in general, from articles 12 and 13, $P' \angle P_1$, or at least, we could not have $P' \rangle P_1$. This result is so different from that obtained by this observation, that we are entitled to assume that the motion of rotation of both shell and nucleus takes place nearly as if the mass were entirely solid. It must evidently result, that at the surface of contact of the solid and fluid, a con-

* Philosophical Transactions, 1840, p. 207.

siderable amount of friction and pressure should necessarily exist. From physical reasons, there is also a great probability of the truth of this proposition. The highly crystalline structure which must necessarily be assigned to the shell's inner surface, combined with the viscidity of the strata of the nucleus in immediate contact with it, would evidently, if sufficiently great, tend constantly to equalize the motions of both shell and nucleus, and to cause the whole to rotate as one mass. Of the truth of the fundamental assumption in article 1, I may therefore venture to hope no reasonable doubt can exist.

(6.) *The Amount of Elastic Gases given off at the Surface of the Nucleus during different Geological Epochs.*

28. If, in the strata of the nucleus, elastic gases were confined by the compression of the superior strata, it must follow from Section III. that during the solidification of the upper parts of the fluid the rest will tend to expand, and consequently to set free the confined gases. If the amount of elastic gases in a cubic unit of any stratum be proportional to the pressure to which that stratum is subjected, it must follow that the quantity evolved on the solidification of the surface stratum must be proportional to the amount of contraction of that stratum in solidifying. The expansion of the other strata, yet in a fluid state, would also tend to produce an evolution of such gases. The evolution of gases from the solidifying stratum would evidently be proportional to $(1-k)a_1^3$, and from any other stratum with the radius a_1, proportional to the density ϱ of that stratum. But it has been shown in Section III. that in general ϱ and $(1-k)$ decrease as a_1 decreases; hence we may conclude that the amount of elastic gases given off from the surface of the nucleus rapidly decreases as the thickness of the shell increases.

(7.) *The Distribution of the Waters on the Surface of the Globe.*

29. The expressions for the variation of gravity obtained in Section II., show that if the angular velocity of rotation of the earth remained unchanged, the waters on its surface would tend to accumulate towards the equator, by the gradual thickening of the shell and the consequent change in the direction of gravity.

XXIII. *On the Rolling Motion of a Cylinder.* *By the Rev.* Henry Moseley, *M.A., F.R.S., Corresponding Member of the Institute of France.*

Received March 6,—Read March 13, 1851.

THE oscillatory motion of a heterogeneous cylinder rolling on a horizontal plane has been investigated by Euler*. He has determined the pressure of the cylinder on the plane at any period of the oscillation, and the time of completing an oscillation when the arcs of oscillation are *small*.

The forms under which the cylinder enters into the composition of machinery are so various and its uses so important, that I have thought it desirable to extend this inquiry, and in the following paper I have sought to include in the discussion the case of the *continuous* rolling of the cylinder, and to determine—

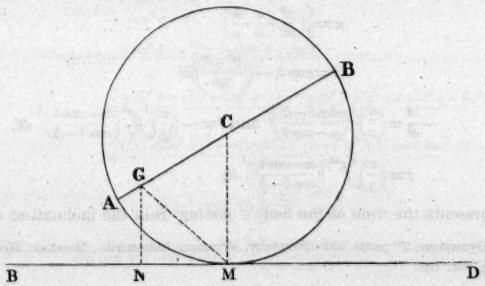
1st. The time occupied by a heterogeneous cylinder in rolling continuously through any given space.

2ndly. The time occupied in its oscillation through any given arc.

3rdly. Its pressure, when thus rolling continuously, on the horizontal plane on which it rolls.

Under the second and third heads this discussion has a practical application to the theory of the pendulum; determining the time occupied in the oscillations of a pendulum through any given arc, whether it rests on a cylindrical axis or on knife-edges, and the circumstances under which it will jump or slip on its bearings; and under the first and third, to the stability and the lateral oscillations of locomotive engines in rapid motion, whose driving-wheels are, by reason of their cranked axles, untruly balanced.

Let AMB represent the section of a heterogeneous cylinder through its centre of



* Nova Acta Acad. Petropol. 1788. "De motu oscillatorio circa axem cylindricum plano horizontali incumbentem."

gravity G and perpendicular to its axis C; and let M be its point of contact, at any time, with the horizontal plane BD on which it is rolling. Assume

a =AC, h=CG, θ=ACM.

W=weight of cylinder. Wk^2=momentum of inertia of the cylinder about an axis passing through G and parallel to the axis of the cylinder.

ω=given value of the angular velocity $\left(\frac{d\theta}{dt}\right)$ when θ has the given value θ_1.

θ_1=given value of θ when the angular velocity has the given value ω.

l=given value of GM corresponding to the value θ_1 of θ.

Then $W(k^2+\overline{GM}^2)=W(k^2+a^2+h^2-2ah\cos\theta)$=moment of inertia about M. Since moreover the cylinder may be considered to be in the act of revolving about the point M by which it is in contact with the plane, one-half of its *vis viva* is represented by the formula

$$\frac{1}{2}\frac{W}{g}(k^2+a^2-2ah\cos\theta+h^2)\left(\frac{d\theta}{dt}\right)^2,$$

and one-half of the *vis viva* acquired by it in rolling through the angle $\theta_1-\theta$, by

$$\frac{1}{2}\frac{W}{g}\left\{(k^2+a^2-2ah\cos\theta+h^2)\left(\frac{d\theta}{dt}\right)^2-(k^2+l^2)\omega^2\right\}.$$

But the vertical descent of the centre of gravity while the cylinder is passing from the one position into the other, is represented by

$$h(\cos\theta-\cos\theta_1).$$

Therefore, by the principle of *vis viva**,

$$\frac{1}{2}\frac{W}{g}\left\{(k^2+a^2-2ah\cos\theta+h^2)\left(\frac{d\theta}{dt}\right)^2-(k^2+l^2)\omega^2\right\}=Wh(\cos\theta-\cos\theta_1),$$

whence we obtain

$$\left(\frac{d\theta}{dt}\right)^2=\frac{2gh(\cos\theta-\cos\theta_1)+(k^2+l^2)\omega^2}{k^2+a^2-2ah\cos\theta+h^2}$$

$$=\left(\frac{g}{a}\right)\frac{\cos\theta-\left(\cos\theta_1-\frac{k^2+l^2}{2gh}\omega^2\right)}{\frac{1}{2}\left(\frac{k^2}{ah}+\frac{a}{h}+\frac{h}{a}\right)-\cos\theta} \quad \ldots\ldots\ldots\ldots (1.)$$

Let

$$\alpha=\frac{1}{2}\left(\frac{k^2}{ah}+\frac{a}{h}+\frac{h}{a}\right) \quad \ldots\ldots\ldots\ldots (2.)$$

$$\beta=\cos\theta_1-\left(\frac{k^2+l^2}{2gh}\right)\omega^2 \quad \ldots\ldots\ldots\ldots (3.)$$

$$\therefore\ -\frac{d\theta}{dt}=\left(\frac{g}{a}\right)^{\frac{1}{2}}\left(\frac{\cos\theta-\beta}{\alpha-\cos\theta}\right)^{\frac{1}{2}} \text{ and } t=-\left(\frac{a}{g}\right)^{\frac{1}{2}}\int_{\theta_1}^{\theta}\left(\frac{\alpha-\cos\theta}{\cos\theta-\beta}\right)^{\frac{1}{2}}d\theta,$$

$$\therefore\ t=\left(\frac{a}{g}\right)^{\frac{1}{2}}\int_{\theta}^{\theta_1}\left(\frac{\alpha-\cos\theta}{\cos\theta-\beta}\right)^{\frac{1}{2}}d\theta, \quad \ldots\ldots\ldots\ldots (4.)$$

where t represents the time of the body's passing from the inclination θ_1 to zero.

* Poisson, Dynamique, 2^{me} partie, 565; Poncelet, Mécanique Industrielle; Moseley, Mechanical Principles of Engineering, Art. 129.

Now let it be observed that in this function $\alpha > \beta$ so long as a is less than g, since

$$k^2+l^2 > -(k^2+l^2)\omega^2, \text{ or } k^2+a^2-2ah\cos\theta_1+h^2 > -(k^2+l^2)\omega^2,$$

and $$\therefore\ k^2+a^2+h^2 > 2ah\cos\theta_1-(k^2+l^2)\omega^2,$$

and $$\frac{1}{2}\left(\frac{k^2}{ah}+\frac{a}{h}+\frac{h}{a}\right) > \cos\theta_1-\frac{k^2+l^2}{2ah}\cdot\omega^2.$$

Let $$\frac{1+\alpha}{1+\beta}=p^2,\quad -\frac{1-\alpha}{1-\beta}=q^2,\quad \frac{\alpha-\cos\theta}{\cos\theta-\beta}=q^2\sec^2\varphi.$$

Then when $\theta=0$, $\quad q^2\sec^2\varphi=\frac{\alpha-1}{1-\beta}=q^2,\ \therefore\ \sec\varphi=1$ and $\varphi=0$.

When $\theta=\theta_1$ let $\varphi=\varphi_1$,

$$\therefore\ q^2\sec^2\varphi_1=\frac{\alpha-\cos\theta_1}{\cos\theta_1-\beta}=\frac{\frac{1}{2}\left(\frac{k^2}{ah}+\frac{h}{a}+\frac{a}{h}\right)-\cos\theta_1}{\left(\frac{k^2+l^2}{2gh}\right)\omega^2}=\left(\frac{g}{a}\right)\frac{k^2+a^2+h^2-2ah\cos\theta_1}{(k^2+l^2)\omega^2}=\frac{g}{a\omega^2};$$

also

$$q^2=-\frac{1-\alpha}{1-\beta}=-\frac{1-\frac{1}{2}\left(\frac{k^2}{ah}+\frac{a}{h}+\frac{h}{a}\right)}{(1-\cos\theta_1)+\left(\frac{k^2+l^2}{2gh}\right)\omega^2}=\left(\frac{g}{a}\right)\frac{k^2+(a-h)^2}{2gh\ \mathrm{vers}\,\theta_1+(k^2+l^2)\omega^2},$$

$$\therefore\ \sec^2\varphi_1=\frac{(k^2+l^2)\omega^2+2gh\ \mathrm{vers}\,\theta_1}{\{k^2+(a-h)^2\}\omega^2}, \qquad (5.)$$

Now $$\int_0^{\theta_1}\left(\frac{\alpha-\cos\theta}{\cos\theta-\beta}\right)^{\frac{1}{2}}d\theta=\int_0^{\varphi_1}\left(\frac{\alpha-\cos\theta}{\cos\theta-\beta}\right)^{\frac{1}{2}}\frac{d\theta}{d\varphi}d\varphi.$$

And since $$\frac{\alpha-\cos\theta}{\cos\theta-\beta}=q^2\sec^2\varphi,$$

$$\therefore\ \frac{2\cos\theta-(\alpha+\beta)}{(\alpha-\beta)}=\frac{\cos^2\varphi-q^2}{\cos^2\varphi+q^2},$$

$$\therefore\ 2\cos\theta=\frac{(\alpha+\beta)(\cos^2\varphi+q^2)+(\alpha-\beta)(\cos^2\varphi-q^2)}{\cos^2\varphi+q^2}$$

$$\cos\theta=\frac{\alpha\cos^2\varphi+\beta q^2}{\cos^2\varphi+q^2} \qquad (6.)$$

$$\sin^2\theta=\frac{(\cos^2\varphi+q^2)^2-(\alpha\cos^2\varphi+\beta q^2)^2}{(\cos^2\varphi+q^2)^2}$$

$$=\frac{(\cos^2\varphi+q^2-\alpha\cos^2\varphi-\beta q^2)(\cos^2\varphi+q^2+\alpha\cos^2\varphi+\beta q^2)}{(\cos^2\varphi+q^2)^2}$$

$$=\frac{\{(1-\beta)q^2+(1-\alpha)\cos^2\varphi\}\{(1+\beta)q^2+(1+\alpha)\cos^2\varphi\}}{(q^2+\cos^2\varphi)^2}$$

$$=(1-\beta^2)q^2\cdot\sin^2\varphi\cdot\frac{q^2+p^2\cos^2\varphi}{(q^2+\cos^2\varphi)^2};$$

$$\therefore\ \sin\theta=q(1-\beta^2)^{\frac{1}{2}}\sin\varphi\cdot\frac{(q^2+p^2\cos^2\varphi)^{\frac{1}{2}}}{q^2+\cos^2\varphi}. \qquad (7.)$$

Now

$$\frac{d\theta}{d\phi}=\frac{d\theta}{d\cos\theta}\cdot\frac{d\cos\theta}{d\cos\phi}\cdot\frac{d\cos\phi}{d\phi}=\frac{\sin\phi}{\sin\theta}\cdot\frac{d\cos\theta}{d\cos\phi}. \quad \ldots\ldots\ldots\ldots \quad (8.)$$

Also by equation (6.),

$$\frac{d\cos\theta}{d\cos\phi}=\frac{2\alpha(q^2+\cos^2\phi)\cos\phi-2(\alpha\cos^2\phi+\beta q^2)\cos\phi}{(q^2+\cos^2\phi)^2}=\frac{2(\alpha-\beta)q^2\cos\phi}{(q^2+\cos^2\phi)^2};$$

$\therefore$ by equations (7.) and (8.),

$$\frac{d\theta}{d\phi}=\frac{2(\alpha-\beta)q^2}{(1-\beta^2)^{\frac{1}{2}}q}\cdot\frac{q^2+\cos^2\phi}{(q^2+p^2\cos^2\phi)^{\frac{1}{2}}}\cdot\frac{\cos\phi}{(q^2+\cos^2\phi)^2}=\frac{2(\alpha-\beta)q}{(1-\beta^2)^{\frac{1}{2}}}\frac{\cos\phi}{(q^2+\cos^2\phi)(q^2+p^2\cos^2\phi)^{\frac{1}{2}}},$$

$$\therefore\left(\frac{\alpha-\cos\theta}{\cos\theta-\beta}\right)^{\frac{1}{2}}\frac{d\theta}{d\phi}=\frac{2(\alpha-\beta)q^2}{(1-\beta^2)^{\frac{1}{2}}}\cdot\left\{\frac{1}{(q^2+\cos^2\phi)(q^2+p^2\cos^2\phi)^{\frac{1}{2}}}\right\}$$

$$=\frac{2(\alpha-\beta)q^2}{(1-\beta^2)^{\frac{1}{2}}}\cdot\left\{\frac{1}{(q^2+1-\sin^2\phi)(q^2+p^2-p^2\sin^2\phi)^{\frac{1}{2}}}\right\}$$

$$=\frac{2(\alpha-\beta)q^2}{(1-\beta^2)^{\frac{1}{2}}(p^2+q^2)^{\frac{1}{2}}(1+q^2)}\left\{\frac{1}{\left(1-\frac{1}{1+q^2}\sin^2\phi\right)\left(1-\frac{p^2}{p^2+q^2}\sin^2\phi\right)^{\frac{1}{2}}}\right\}$$

$$=\frac{2(\alpha-\beta)q^2}{(1-\beta^2)^{\frac{1}{2}}(p^2+q^2)^{\frac{1}{2}}(1+q^2)}\left\{\frac{1}{(1-n\sin^2\phi)(1-c^2\sin^2\phi)^{\frac{1}{2}}}\right\}.$$

If

$$n=\frac{1}{1+q^2}=\frac{1}{1-\frac{1-\alpha}{1-\beta}}=\frac{1-\beta}{\alpha-\beta}, \quad \ldots\ldots\ldots\ldots \quad (9.)$$

and

$$c^2=\frac{p^2}{p^2+q^2}=\frac{\frac{1+\alpha}{1+\beta}}{\frac{1+\alpha}{1+\beta}-\frac{1-\alpha}{1-\beta}}=\frac{(1+\alpha)(1-\beta)}{2(\alpha-\beta)}, \quad \ldots\ldots\ldots\ldots \quad (10.)$$

$$\therefore\int_0^{\phi_1}\left(\frac{\alpha-\cos\theta}{\cos\theta-\beta}\right)^{\frac{1}{2}}\frac{d\theta}{d\phi}d\phi=\frac{2(\alpha-\beta)q^2}{(1-\beta^2)^{\frac{1}{2}}(p^2+q^2)^{\frac{1}{2}}(1+q^2)}\int_0^{\phi_1}\frac{d\phi}{(1-n\sin^2\phi)(1-c^2\sin^2\phi)^{\frac{1}{2}}}$$

$$=\frac{2(\alpha-\beta)q^2}{(1-\beta^2)^{\frac{1}{2}}(p^2+q^2)^{\frac{1}{2}}(1+q^2)}\Pi\left(-nc\phi_1\right), \quad \ldots\ldots \quad (11.)$$

where $\Pi(-nc\phi_1)$ is that elliptic function of the third order whose parameter is $-n$ and modulus c.

Now

$$\frac{1}{(p^2+q^2)^{\frac{1}{2}}}=\frac{1}{\left\{\left(\frac{1+\alpha}{1+\beta}\right)-\left(\frac{1-\alpha}{1-\beta}\right)\right\}^{\frac{1}{2}}}=\sqrt{\frac{1-\beta^2}{2(\alpha-\beta)}}$$

$$\frac{q^2}{1+q^2}=\frac{-\left(\frac{1-\alpha}{1-\beta}\right)}{1-\left(\frac{1-\alpha}{1-\beta}\right)}=\left(\frac{\alpha-1}{\alpha-\beta}\right),$$

* I cannot find that this function has before been integrated, except in the case in which θ is exceedingly small.

$$\therefore \frac{2(\alpha-\beta)q^2}{(1-\beta^2)^{\frac{1}{2}}(p^2+q^2)^{\frac{1}{2}}(1+q^2)} = \frac{2(\alpha-1)}{\sqrt{2(\alpha-\beta)}} = \frac{\left(\frac{k^2}{ah}+\frac{a}{h}+\frac{h}{a}\right)-2}{\sqrt{\left(\frac{k^2}{ah}+\frac{a}{h}+\frac{h}{a}\right)-2\cos\vartheta_1+\frac{(k^2+l^2)\omega^2}{gh}}}$$

$$= \frac{k^2+(a-h)^2}{\sqrt{ah(k^2+l^2)\left(1+\frac{a\omega^2}{g}\right)}}, \quad \ldots\ldots\ldots\ldots \quad (12.)$$

$\therefore$ by equations 11 and 4,

$$t = \frac{k^2+(a-h)^2}{\sqrt{gh(k^2+l^2)\left(1+\frac{a\omega^2}{g}\right)}} \cdot \Pi(-nc\varphi_1), \quad \ldots\ldots\ldots\ldots \quad (13.)$$

where (9.) (2.) (3.)

$$n = \frac{1-\beta}{\alpha-\beta} = \frac{1-\cos\vartheta_1+\frac{k^2+l^2}{2gh}\omega^2}{\frac{1}{2}\left(\frac{k^2}{ah}+\frac{a}{h}+\frac{h}{a}\right)-\cos\vartheta_1+\frac{k^2+l^2}{2gh}\omega^2} = a\frac{2h\,\mathrm{vers}\,\vartheta_1+\frac{k^2+l^2}{g}\omega^2}{(k^2+l^2)\left(1+\frac{a\omega^2}{g}\right)}, \quad \ldots\ldots \quad (14.)$$

and (10.) (2.) (3.)

$$c^2 = \frac{(1+\alpha)(1-\beta)}{2(\alpha-\beta)} = \frac{\left\{\frac{1}{2}\left(\frac{k^2}{ah}+\frac{h}{a}+\frac{a}{h}\right)+1\right\}\left\{\mathrm{vers}\,\vartheta_1+\frac{k^2+l^2}{2gh}\omega^2\right\}ah}{(k^2+l^2)\left(1+\frac{a\omega^2}{g}\right)}$$

$$= \frac{\left\{k^2+(a+h)^2\right\}\left\{\mathrm{vers}\,\vartheta_1+\frac{k^2+l^2}{2gh}\omega^2\right\}}{2(k^2+l^2)\left(1+\frac{a\omega^2}{g}\right)} \quad \ldots\ldots\ldots\ldots \quad (15.)$$

The value of $\Pi(-nc\varphi_1)$ being determinable by known methods (LEGENDRE, Fonctions Elliptiques, vol. i. chap. xxiii.), the time of rolling is given by equation 13.

In the case in which the rolling motion is not continuous but oscillatory, we have $\omega=0$; and therefore (equation 5.) $\varphi_1=\frac{\pi}{2}$; $\Pi(-nc\varphi_1)$ becomes therefore in this case a complete function.

To express the value of this complete elliptic function of the third order in terms of functions of the first and second orders, let

$$\sin^2\psi = \frac{n}{c^2} = \frac{2}{1+\alpha} = \frac{2ah}{k^2+(a+h)^2}. \quad \ldots\ldots\ldots\ldots \quad (16.)$$

Then*

$$\Pi\left(-nc\tfrac{\pi}{2}\right) = \mathrm{F}\left(c\tfrac{\pi}{2}\right)+\frac{\tan\psi}{(1-c^2\sin^2\psi)^{\frac{1}{2}}}\left\{\mathrm{F}\left(c\tfrac{\pi}{2}\right).\mathrm{E}(c\psi)-\mathrm{E}\left(c\tfrac{\pi}{2}\right).\mathrm{F}(c\psi)\right\},$$

Representing therefore the time of a semi-oscillation by t_1,

$$t_1 = \frac{k^2+(a-h)^2}{\sqrt{gh(k^2+l^2)}}\left\{\mathrm{F}\left(c\tfrac{\pi}{2}\right)+\frac{\tan\psi}{1-c^2\sin^2\psi}\left\{\mathrm{F}\left(c\tfrac{\pi}{2}\right)\mathrm{E}(c\psi)-\mathrm{E}\left(c\tfrac{\pi}{2}\right)\mathrm{F}(c\psi)\right\}\right\}, \quad \ldots \quad (17.)$$

where (15.)
$$c^2 = \frac{k^2+(a+h)^2}{2(k^2+l^2)}\,\mathrm{vers}\,\vartheta_1. \quad \ldots\ldots\ldots\ldots \quad (18.)$$

* LEGENDRE, Calcul des Fonctions Elliptiques, vol. i. chap. xxiii. Art. 116.

Since the values of elliptic functions of the first and second orders, having given amplitudes and moduli, are given by the tables of LEGENDRE, it follows that the value of t is given by this formula for all possible values of c and ψ.

If the angle of oscillation θ_1 be very small c is very small, so that its square may be neglected in comparison with unity. In this case

$$\mathrm{F}c\psi=\mathrm{E}c\psi=\psi \text{ and } \mathrm{F}c\tfrac{\pi}{2}=\mathrm{E}c\tfrac{\pi}{2}=\tfrac{\pi}{2},$$

$$\therefore\ \mathrm{F}c\tfrac{\pi}{2}\mathrm{E}c\psi-\mathrm{E}c\tfrac{\pi}{2}\mathrm{F}c\psi=0.$$

For small oscillations therefore

$$t=\frac{k^2+(a-h)^2}{\sqrt{gh(k^2+l^2)}}\cdot\frac{\pi}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19.)$$

If the pendulum oscillate on knife-edges $a=0$, $l=h$, and we obtain the well-known theorem of LEGENDRE (Fonctions Elliptiques, vol. i. chap. viii.)

$$t=\sqrt{\frac{k^2+h^2}{gh}}\cdot\mathrm{F}\left(c\tfrac{\pi}{2}\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20.)$$

where (18.) $$c^2=\frac{1}{2}\operatorname{vers}\theta_1=\sin^2\frac{\theta_1}{2},$$

$$\therefore\ c=\sin\frac{1}{2}\theta_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21.)$$

In the case of the small oscillations of a pendulum resting on knife-edge, equation 20. becomes

$$t=\sqrt{\frac{(k^2+h^2)}{gh}}\cdot\frac{\pi}{2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22.)$$

which is the well-known formula applicable to that case.

If the pendulum be one which for small arcs beats seconds (21.),

$$1=\sqrt{\frac{k^2+h^2}{gh}}\cdot\pi,$$

$$\therefore\ (20.)\ 2t=\frac{2\mathrm{F}\left(c\frac{\pi}{2}\right)}{\pi}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23.)$$

by which equation the time of the oscillation *through any arc*, of a pendulum which oscillates through a *small* arc in one second, may be determined. I have caused the following Table to be calculated from it.

Table of the Time occupied in oscillating through every two degrees of a complete circle, by a Pendulum which oscillates through a small arc in one second.

Arc of oscillation on each side of the vertical in degrees.	Logarithm of $F\left(c\frac{\pi}{2}\right)$ from the tables of LEGENDRE.	Logarithm of $\frac{2}{\pi}.F\left(c\frac{\pi}{2}\right)$.	Time of one complete oscillation in seconds.	Arc of oscillation on each side of the vertical in degrees.	Logarithm of $F\left(c\frac{\pi}{2}\right)$ from the tables of LEGENDRE.	Logarithm of $\frac{2}{\pi}.F\left(c\frac{\pi}{2}\right)$.	Time of one complete oscillation in seconds.
2	0·1961529	0·0000330	1·0001	92	0·2716435	0·0755336	1·1899
4	0·1962521	0·0001322	1·0003	94	0·2752672	0·0791473	1·2000
6	0·1964176	0·0002977	1·0006	96	0·2790010	0·0828811	1·2123
8	0·1966493	0·0005294	1·0012	98	0·2828480	0·0867281	1·2210
10	0·1969473	0·0008274	1·0019	100	0·2868113	0·090614	1·2322
12	0·1973118	0·0011919	1·0027	102	0·2908945	0·0947746	1·2439
14	0·1977430	0·0016231	1·0037	104	0·2951011	0·0989812	1·2560
16	0·1982408	0·0021209	1·0049	106	0·2994353	0·1033154	1·2686
18	0·1988057	0·0026858	1·0052	108	0·3039012	0·1077813	1·2817
20	0·1994377	0·0033178	1·0077	110	0·3085036	0·1123837	1·2953
22	0·2001372	0·0040173	1·0100	112	0·3132474	0·1171275	1·3099
24	0·2009044	0·0047845	1·0110	114	0·3181380	0·1220181	1·3249
26	0·2017396	0·0056197	1·0130	116	0·3231814	0·1270615	1·3400
28	0·2026431	0·0065232	1·0151	118	0·3283839	0·1322640	1·3560
30	0·2036153	0·0074954	1·0174	120	0·3337526	0·1376327	1·3729
32	0·2045494	0·0084295	1·0196	122	0·3392950	0·1431751	1·3905
34	0·2057675	0·0096476	1·0224	124	0·3450196	0·1488997	1·4089
36	0·2069483	0·0108284	1·0252	126	0·3509356	0·1548157	1·4283
38	0·2081996	0·0120797	1·0290	128	0·3570532	0·1609333	1·4486
40	0·2095219	0·0134020	1·0314	130	0·3633838	0·1672639	1·4698
42	0·2109158	0·0147959	1·0347	132	0·3699399	0·1738200	1·4922
44	0·2123818	0·0162619	1·0381	134	0·3767357	0·1806158	1·5157
46	0·2139206	0·0178006	1·0418	136	0·3837869	0·1876670	1·5405
48	0·2155329	0·0194130	1·0457	138	0·3911115	0·1949916	1·5667
50	0·2172193	0·0210994	1·0500	140	0·3987297	0·2026098	1·5944
52	0·2189808	0·0228609	1·0540	142	0·4066647	0·2105448	1·6238
54	0·2208180	0·0246981	1·0585	144	0·4149432	0·2188233	1·6551
56	0·2227319	0·0266120	1·0632	146	0·4235961	0·2274762	1·6884
58	0·2247233	0·0286034	1·0681	148	0·4326595	0·2365396	1·7240
60	0·2267932	0·0306733	1·0732	150	0·4421759	0·2460560	1·7622
62	0·2289427	0·0328228	1·0785	152	0·4521963	0·2560764	1·8032
64	0·2311728	0·0350528	1·0840	154	0·4627819	0·2666620	1·8478
66	0·2334846	0·0373647	1·0898	156	0·4740076	0·2778877	1·8963
68	0·2358794	0·0397595	1·0959	158	0·4859666	0·2898467	1·9491
70	0·2383585	0·0422386	1·1021	160	0·4987770	0·3026571	2·0075
72	0·2409232	0·0448033	1·1087	162	0·5125914	0·3164715	2·0724
74	0·2435750	0·0474551	1·1154	164	0·5276128	0·3314929	2·1453
76	0·2463154	0·0501955	1·1225	166	0·5441204	0·3480005	2·2285
78	0·2491459	0·0530260	1·1298	168	0·5625136	0·3663937	2·3248
80	0·2520684	0·0559485	1·1375	170	0·5833962	0·3872763	2·4393
82	0·2550846	0·0589647	1·1454	172	0·6077506	0·4116307	2·5801
84	0·2581965	0·0620766	1·1536	174	0·6373550	0·4412351	2·7621
86	0·2614060	0·0652861	1·1622	176	0·6760772	0·479907	3·0193
88	0·2647155	0·0685956	1·1711	178	0·7351923	0·5390724	3·4600
90	0·2681272	0·0720073	1·1802	180	Infinite.	Infinite.	Infinite.

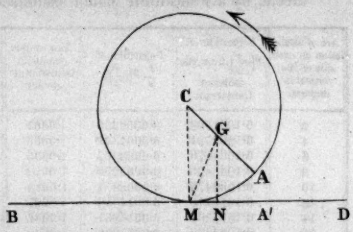
The pressure of the cylinder on its point of contact with the plane on which it rolls.

Let A′ be the point where the point A of the cylinder was in contact with the plane.

Let A′N$=x$, NG$=y$.

$-$X$=$horizontal pressure on M in direction A′M.

Y$=$vertical pressure on M in direction MC.



Since the centre of gravity G moves as it would do if, the whole mass being collected there, all the impressed forces were applied to it, we have, by the principle of d'ALEMBERT,

$$\left.\begin{aligned}\frac{W}{g}\frac{d^2x}{dt^2}&=-X\\ \frac{W}{g}\frac{d^2y}{dt^2}&=Y-W\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (28.)$$

But since CA$=a$, CG$=h$, MCA$=\theta$,

$$\therefore\quad x=a\theta-h\sin\theta,$$

$$y=a-h\cos\theta;$$

$$\therefore\quad \frac{dx}{dt}=(a-h\cos\theta)\frac{d\theta}{dt},$$

$$\frac{dy}{dt}=h\sin\theta\frac{d\theta}{dt}$$

$$\therefore\quad \left.\begin{aligned}\frac{d^2x}{dt^2}&=h\sin\theta\left(\frac{d\theta}{dt}\right)^2+(a-h\cos\theta)\frac{d^2\theta}{dt^2}\\ \frac{d^2y}{dt^2}&=h\cos\theta\left(\frac{d\theta}{dt}\right)^2+h\sin\theta\frac{d^2\theta}{dt^2}\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (29.)$$

Assume $\left(\frac{d\theta}{dt}\right)^2=M$, $\left(\frac{d^2\theta}{dt^2}\right)=-N$,

$\therefore$ by equation (29.),

$$\frac{d^2x}{dt^2}=Mh\sin\theta-N(a-h\cos\theta)$$

$$\frac{d^2y}{dt^2}=Mh\cos\theta-Nh\sin\theta;$$

by equation (28.),

$$\left.\begin{aligned}X&=\frac{Wh}{g}\left\{-M\sin\theta+N\left(\frac{a}{h}-\cos\theta\right)\right\}\\ Y&=W+\frac{Wh}{g}\left\{M\cos\theta-N\sin\theta\right\}.\end{aligned}\right\} \quad \ldots\ldots\ldots \quad (30.)$$

But by equation (1.), substituting $-\theta$ and $-\theta_1$ for θ and θ_1,

$$M=\left(\frac{d\theta}{dt}\right)^2=\frac{2gh(\cos\theta-\cos\theta_1)+(k^2+l^2)\omega^2}{k^2+a^2-2ah\cos\theta+h^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31.)$$

$$=\left(\frac{g}{a}\right)\frac{2ah(\cos\theta-\cos\theta_1)+(k^2+l^2)\frac{a\omega^2}{g}}{k^2+a^2-2ah\cos\theta+h^2}$$

$$=\left(\frac{g}{a}\right)\frac{k^2+a^2+h^2-2ah\cos\theta_1+(k^2+l^2)\frac{a\omega^2}{g}-(k^2+a^2+h^2-2ah\cos\theta)}{k^2+a^2-2ah\cos\theta+h^2},$$

$$\therefore \quad M=\left(\frac{g}{a}\right)\left\{\frac{(k^2+l^2)\left(1+\frac{a\omega^2}{g}\right)}{k^2+a^2+h^2-2ah\cos\theta}-1\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32.)$$

Observing that $a^2+h^2-2ah\cos\theta_1=l^2$.

Differentiating this equation and dividing by $\left(\frac{d\theta}{dt}\right)$,

$$-N=\left(\frac{d^2\theta}{dt^2}\right)=-\frac{gh(k^2+l^2)\left(1+\frac{a\omega^2}{g}\right)\sin\theta}{(k^2+a^2+h^2-2ah\cos\theta)^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33.)$$

Substituting these values of M and N in equation (30.), and reducing,

$$X=\frac{Wh\sin\theta}{a}\left\{1-\frac{(k^2+l^2)(k^2+h^2-ah\cos\theta)(g+a\omega^2)}{g(k^2+a^2+h^2-2ah\cos\theta)^2}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34.)$$

$$Y=\frac{Wh}{a}\left\{\left(\frac{a}{h}-\cos\theta\right)-\frac{(k^2+l^2)(g+a\omega^2)\{ah\cos^2\theta-(k^2+a^2+h^2)\cos\theta+ah\}}{g(k^2+a^2+h^2-2ah\cos\theta)^2}\right\} \quad . \quad . \quad . \quad (35.)$$

The rotation of a body about a cylindrical axis of small diameter.

Assuming $a=0$ in equations (31.), (33.), and $\theta_1=0$, we have

$$M=\frac{2gh(\cos\theta-1)}{k^2+h^2}+\omega^2 \quad N=\frac{gh\sin\theta}{k^2+h^2}.$$

Therefore, by equation (30.),

$$X=\frac{Wh}{g}\left\{\frac{gh(2-3\cos\theta)}{k^2+h^2}-\omega^2\right\}\sin\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (40.)$$

$$Y=W+\frac{Wh}{g}\left\{\frac{gh(3\cos^2\theta-2\cos\theta-1)}{k^2+h^2}+\omega^2\cos\theta\right\} \quad . \quad . \quad . \quad . \quad . \quad (41.)$$

The last equation may be placed under the form

$$Y=W+\frac{3Wh^2}{k^2+h^2}\left\{\left\{\cos\theta+\frac{1}{3}\left(\frac{k^2+h^2}{2gh}\omega^2-1\right)\right\}^2-\frac{1}{9}\left(\frac{k^2+h^2}{2gh}\omega^2-1\right)^2-\frac{1}{3}\right\}.$$

If $\frac{1}{3}\left(\frac{k^2+h^2}{2gh}\omega^2-1\right)$ be numerically less than unity, whether it be positive or negative, there will be some value of θ between 0 and π for which this expression will be equalled, with an opposite sign, by $\cos\theta$, and for which the first term under the

bracket in the value of Y will vanish. This corresponds to a minimum value of Y represented by the formula

$$Y=W-\frac{Wk^2}{k^2+h^2}\left\{\frac{1}{3}\left(\frac{k^2+h^2}{2gh}\omega^2-1\right)^2+1\right\}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (42.)$$

But if $\frac{1}{3}\left(\frac{k^2+h^2}{2gh}\omega^2-1\right)$ be numerically greater than unity, then the minimum of Y will be attained when $\theta=\pi$, and when

$$Y=W-\frac{Wh}{g}\left\{\omega^2-\frac{4gh}{k^2+h^2}\right\}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (43.)$$

The jump of an axis.

If Y be negative in any position of the body, the axis will obviously jump from its bearings, unless it be retained by some mechanical expedient not taken account of in this calculation. But if Y be negative in any position, it must be negative in that in which its value is a minimum. If a jump take place at all, therefore, it will take place when Y is a minimum; and whether it will take place or not, is determined by finding whether the minimum value of Y is negative. If therefore the expression (42.) or (43.) be negative, the axis will jump in the corresponding case. An axis of infinitely small diameter, such as we have here supposed, becomes a fixed axis; and the pressure upon a fixed axis, supposed to turn in cylindrical bearings *without friction*, is the same whatever may be its diameter; equations (40.) and (41.) determine therefore that pressure, and equation (42.) or (43.) determines the vertical strain upon the collar when the tendency of the axis to jump from its bearings is the greatest.

The jump of a rolling cylinder.

Whether a jump will or will not take place, has been shown to be determined by finding whether the minimum value of Y be negative or not.

Substituting α for $\frac{1}{2}\left(\frac{k^2}{ah}+\frac{h}{a}+\frac{a}{h}\right)$ and reducing, equation (35.) becomes

$$Y=W\left(1-\frac{h}{a}\cos\theta\right)-\frac{W(k^2+l^2)(g+a\omega^2)}{4ga^2}\left\{\frac{\cos^2\theta-2\alpha\cos\theta+1}{(\alpha-\cos\theta)^2}\right\},$$

or

$$Y=W\left(1-\frac{h}{a}\cos\theta\right)-\frac{W(k^2+l^2)(g+a\omega^2)}{4ga^2}\left\{1-\frac{\alpha^2-1}{(\alpha-\cos\theta)^2}\right\}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44.)$$

$$\therefore \quad \frac{dY}{d\theta}=W\left\{\frac{h}{a}-\frac{(k^2+l^2)(g+a\omega^2)(\alpha^2-1)}{2ga^2(\alpha-\cos\theta)^3}\right\}\sin\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45.)$$

$$\frac{d^2Y}{d\theta^2}=W\left\{\frac{h}{a}-\frac{(k^2+l^2)(g+a\omega^2)(\alpha^2-1)}{2ga^2(\alpha-\cos\theta)^3}\right\}\cos\theta+\frac{3(k^2+l^2)(g+a\omega^2)(\alpha^2-1)}{2ga^2(\alpha-\cos\theta)^4}\sin^2\theta,$$

$\therefore \quad \frac{dY}{di}=0$, 1st, when $\frac{h}{a}-\frac{(k^2+l^2)(g+a\omega^2)(\alpha^2-1)}{2ga^2(\alpha-\cos\theta)^3}=0$, 2ndly, when $\theta=\pi$, 3rdly, when $\theta=0$.

The first condition evidently yields a positive value of $\frac{d^2Y}{d\theta^2}$, since it causes the first term of the preceding equation to vanish; and the second term is essentially positive, α being always greater than unity.

If, therefore, the first condition be possible, or if there be any value of θ which satisfies it, that value corresponds to a position of minimum pressure. Solving in respect to $\cos\theta$, we obtain

$$\alpha - \sqrt[3]{\frac{(k^2+l^2)(g+a\omega^2)(\alpha^2-1)}{2gah}} = \cos\theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (46.)$$

The first condition will therefore yield a position of minimum pressure, if

$$\alpha - \sqrt[3]{\frac{(k^2+l^2)(g+a\omega^2)(\alpha^2-1)}{2gah}} \begin{matrix} > -1 \\ < +1, \end{matrix} \text{ or if } \sqrt[3]{\frac{(k^2+l^2)(g+a\omega^2)(\alpha^2-1)}{2gah}} \begin{matrix} < (\alpha+1) \\ > (\alpha-1), \end{matrix}$$

or if

$$\left.\begin{matrix} \frac{(k^2+l^2)(g+a\omega^2)(\alpha^2-1)}{2gah} \begin{matrix} < (\alpha+1)^3 \\ > (\alpha-1)^3, \end{matrix} \text{ or if } \frac{(k^2+l^2)(g+a\omega^2)(\alpha-1)}{2gah(\alpha+1)^2} < 1 \\ \text{and } \frac{(k^2+l^2)(g+a\omega^2)(\alpha+1)}{2gah(\alpha-1)^2} > 1 \end{matrix}\right\}, \quad . \quad . \quad . \quad . \quad (47.)$$

or if

$$g+a\omega^2 < \frac{2gah(\alpha+1)^2}{(k^2+l^2)(\alpha-1)}, \text{ or } \omega^2 < \frac{2gh(\alpha+1)^2}{(k^2+l^2)(\alpha-1)} - \frac{g}{a},$$

and

$$g+a\omega^2 > \frac{2gah(\alpha-1)^2}{(k^2+l^2)(\alpha+1)}, \text{ or } \omega^2 > \frac{2gh(\alpha-1)^2}{(k^2+l^2)(\alpha+1)} - \frac{g}{a};$$

whence, substituting for α and reducing, we obtain finally, the conditions

$$\omega^2 < \left(\frac{g}{a}\right)\frac{\{k^2+(a+h)^2\}^2}{(k^2+l^2)\{k^2+(a-h)^2\}} - \left(\frac{g}{a}\right) \text{ and } \omega^2 > \left(\frac{g}{a}\right)\frac{\{k^2+(a-h)^2\}^2}{(k^2+l^2)\{k^2+(a+h)^2\}} - \left(\frac{g}{a}\right) \quad . \quad (48.)$$

Of these inequalities the second always obtains, because

$$\{k^2+(a-h)^2\}^2 < (k^2+l^2)\{k^2+(a+h)^2\},$$

whatever be the values of k, a and h. And the first is always possible, since

$$\{k^2+(a+h)^2\}^2 > (k^2+l^2)\{k^2+(a-h)^2\}.$$

If the *first* obtain, there are two corresponding positions of CA on either of the vertical, determined by equation (46.), in which the pressure Y of the cylinder upon the plane is a minimum.

Substituting the other two values (π and 0) of θ which cause $\frac{dY}{dx}$ to vanish, in the value of $\frac{d^2Y}{d\theta^2}$ we obtain the values

$$-\left\{\frac{h}{a} - \frac{(k^2+l^2)(g+a\omega^2)(\alpha-1)}{2ga^2(\alpha+1)^2}\right\} \text{ and } \left\{\frac{h}{a} - \frac{(k^2+l^2)(g+a\omega^2)(\alpha+1)}{2ga^2(\alpha-1)^2}\right\},$$

or

$$-\frac{h}{a}\left\{1 - \frac{(k^2+l^2)(g+a\omega^2)(\alpha-1)}{2gha(\alpha+1)^2}\right\} \text{ and } \frac{h}{a}\left\{1 - \frac{(k^2+l^2)(g+a\omega^2)(\alpha+1)}{2gha(\alpha-1)^2}\right\}, \quad . \quad . \quad . \quad . \quad (49.)$$

which expressions are both negative if the inequalities (47.) obtain. The same conditions which yield minimum values of Y in two corresponding oblique positions of CA, yield, therefore, maximum values in the two vertical positions; so that if the inequalities (48.) obtain, there are two positions of maximum and two of minimum pressure.

Substituting the value of $\cos\theta$ (equation 46) in equation (44.), and reducing, we obtain for the minimum value of Y in the case in which the inequalities (48.) obtain,

$$Y=\frac{W}{4a^3}\left\{2(a^3-k^3-h^3)-(k^3+l^3)\left(1+\frac{a\omega^2}{g}\right)+3\sqrt[3]{(k^3+l^3)\{k^3+(a+h)^3\}\{k^3+(a-h)^3\}\left(1+\frac{a\omega^2}{g}\right)}\right\}.$$

If this expression be negative the cylinder will jump.

In the case in which $\omega=0$, which is that of a *pendulum* having a cylindrical axis of finite diameter, it becomes

$$Y=\frac{W}{4a^3}\left\{2a^3-2h^3-3k^3-l^3+3\sqrt[3]{(k^3+l^3)\{k^3+(a+h)^3\}\{k^3+(a-h)^3\}}\right\}^{*} \quad . \quad . \quad . \quad (50.)$$

If the first of the inequalities (48.) do not obtain, no position of minimum pressure corresponds to equation (46.); and the inequalities (47.) do not obtain, so that the values (49.) of $\frac{d^2Y}{d\theta^2}$, given respectively by the substitution of π and 0 for θ, are no longer both negative, but the second only. In this case the value π of θ is that, therefore, which corresponds to a position of minimum pressure, which minimum pressure is determined by substituting π for θ in equation (35.), and is represented by

$$Y=W\left(1+\frac{h}{a}\right)-\frac{W(k^3+l^3)(g+a\omega^2)}{4ga^3}\left\{1-\frac{a^3-1}{(a+1)^3}\right\}=\frac{W}{a}\left\{a+h-\frac{(k^3+l^3)(g+a\omega^2)}{2ga(a+1)}\right\}=\frac{W}{a}\left\{a+h-\frac{h(k^3+l^3)(g+a\omega^2)}{g\{k^3+(a+h)^3\}}\right\}$$

$$Y=\frac{W}{a}\left\{a+h-\frac{h\left\{k^3+(a+h)^3-4ah\cos^2\frac{1}{2}\theta_1\right\}(g+a\omega^2)}{g\{k^3+(a+h)^3\}}\right\}=\frac{W}{a}\left\{a+h-\frac{h}{g}\left\{1-\frac{4ah\cos^2\frac{1}{2}\theta_1}{k^3+(a+h)^3}\right\}(g+a\omega^2)\right\},$$

$$\therefore\ Y=W\left\{1-\frac{h\omega^2}{g}+\frac{4h^3\left(1+\frac{a\omega^2}{g}\right)\cos^2\frac{1}{2}\theta_1}{k^3+(a+h)^3}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (51.)$$

The cylinder will jump if this expression be negative, that is, if

$$\frac{h\omega^2}{g}>1+\frac{4h^3\left(1+\frac{a\omega^2}{g}\right)\cos^2\frac{1}{2}\theta_1}{k^3+(a+h)^3},\ \text{or if}\ \frac{\omega^2h}{g}\left\{1-\frac{4h\cos^2\frac{1}{2}\theta_1}{k^3+(a+h)^3}\right\}>1+\frac{4h^3\cos^2\frac{1}{2}\theta_1}{k^3+(a+h)^3};$$

or, substituting and reducing, if

$$\omega^2>\frac{g}{h}\left\{1+\frac{4h(a+h)\cos^2\frac{1}{2}\theta_1}{k^3+l^3}\right\}.$$

If the angular velocity ω be assumed to be that acquired in the highest position of

* When the pendulum oscillates on knife-edges $a=0$, and this expression assumes the form of a vanishing fraction, whose value may be determined by the known rules. See the next article.

the centre of gravity, $\theta_1=\pi$, and $\cos\frac{1}{2}\theta_1=0$. In this case, therefore, (equation 51.)

$$Y=W\left(1-\frac{h\omega^2}{g}\right); \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (52.)$$

and there will be a jump if $\omega^2>\frac{g}{h}$. . . . . (53.)

The Pendulum oscillating on knife-edges.

In this case a is evanescent, and $\omega=0$. Equations (31.) and (33.) become, therefore,

$$M=\frac{2gh\,(\cos\theta-\cos\theta_1)}{k^2+h^2} \text{ and } N=\frac{gh\sin\theta}{k^2+h^2}.$$

Substituting these values of M and N in equation (30.),

$$X=\frac{Wh^2}{k^2+h^2}\left\{-2\,(\cos\theta-\cos\theta_1)\sin\theta-\cos\theta\sin\theta\right\}, Y=W+\frac{Wh^2}{k^2+h^2}\left\{(\cos\theta-\cos\theta_1)\cos\theta-\sin^2\theta\right\};$$

$$\therefore \quad X=\frac{Wh^2}{k^2+h^2}(2\cos\theta_1-3\cos\theta)\sin\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (54.)$$

$$Y=\frac{Wh^2}{k^2+h^2}\left(3\cos^2\theta-2\cos\theta\cos\theta_1+\frac{k^2}{h^2}\right). \quad . \quad . \quad . \quad . \quad . \quad (55.)$$

Y is a minimum when $\cos\theta=\frac{1}{3}\cos\theta_1$, in which case

$$Y=\frac{Wh^2}{k^2+h^2}\left(\frac{k^2}{h^2}-\frac{1}{3}\cos\theta_1\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (56.)$$

There will therefore be a jump of the pendulum upon its bearings at each oscillation, if the amplitude θ_1 of the oscillation be such, that

$$\frac{1}{3}\cos\theta_1>\frac{k^2}{h^2}, \text{ or } \cos^2\theta_1>\frac{3k^2}{h^2}.$$

The jump of the falsely-balanced Carriage-wheel.

The theory of the falsely-balanced carriage-wheel differs from that of the rolling cylinder,—1st, in that the inertia of the carriage applied at its axle influences the acceleration produced by the weight of the wheel, as its centre of gravity descends or ascends in rolling; and 2ndly, in that the wheel is retained in contact with the plane by the weight of the carriage. The first cause may be neglected, because the displacement of the centre of gravity is always in the carriage-wheel very small, and because the angular velocity is, compared with it, very great.

If W_1 represent that portion of the weight of the carriage which must be overcome in order that the wheel may jump (which weight is supposed to be borne by the plane), and if Y_1 be taken to represent the pressure upon the plane, then (equation 52.)

$$Y_1=W_1+Y=W_1+W\left(1-\frac{h\omega^2}{g}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (57.)$$

In order that there may be a jump, this expression must be negative,

or

$$\frac{Wh\omega^2}{g} > W + W_1 \text{ or } \omega^2 > \frac{g}{h}\left(1+\frac{W_1}{W}\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (58.)$$

or

$$h > \frac{g}{\omega^2}\left(1+\frac{W_1}{W}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (59.)$$

The Driving-Wheel of a Locomotive Engine.

The attention of engineers was some years since directed to the effects which might result from the false balancing of a wheel by accidents on railways, which appeared to be occasioned by a tendency to jump in the driving-wheels of the engines. The cranked axle in all cases destroys the balance of the driving-wheel unless a counterpoise be applied; at that time there was no counterpoise, and the axle was so cranked as to displace the centre of gravity more than it does now. Mr. George Heaton, of Birmingham, appears to have been principally instrumental in causing the danger of this false balancing of the driving-wheels to be understood. By means of an ingenious apparatus*, which enabled him to roll a falsely-balanced wheel round the circumference of a table with any given velocity, and to make any required displacement of the centre of gravity, he showed the tendency to jump, produced even by a very small displacement, to be so great, as to leave no doubt on the minds of practical men as to the danger of such displacement in the case of locomotive engines, and a counterbalance is now, I believe, always applied. To determine what is the degree of accuracy required in such a counterpoise, I have calculated from the preceding formulæ that displacement of the centre of gravity of a driving-wheel of a locomotive engine, which is necessary to cause it to jump at the high velocities not unfrequently attained at some parts of the journey of an express train; from such information as I have been able to obtain as to the dimensions of such wheels, and their weights, and those of the engines†. The weight of a pair of driving-wheels, six feet in diameter, with a cranked axle, varies, I am told, from 2½ to 3 tons; and that of an engine on the London and Birmingham Railway, when filled with water, from 20 to 25 tons. If n represent the number of miles per hour at which the engine is travelling, it may be shown by a simple calculation, that the angular velocity, in feet, of a six-feet wheel is represented by $\frac{22n}{45}$, or by $\frac{1}{2}n$ very nearly. In this case we have, therefore,—since W represents the

* This apparatus is exhibited by Professor Cowper in his lectures on machinery at King's College. It has also been placed by Col. Morin among the apparatus of the Conservatoir des Arts et Métiers at Paris.

† I have not included in this calculation the inertia of the crank rods, of the slide gearing, or of the piston and piston rods. The effect of these is to increase the tendency to jump produced by the displacement of the centre of gravity of the wheel; and the like effect is due to the thrust upon the piston rod. The discussion of these subjects does not belong to my present paper.

weight of a single wheel and its portion of the axle, and W_1 represents the weight, exclusive of the driving-wheels, which must be raised that either side of the engine may jump*, that is, half the weight of the engine exclusive of the driving-wheels,—$W=1\frac{1}{4}$ to $1\frac{1}{2}$ tons, $W_1=8\frac{3}{4}$ to $11\frac{1}{4}$ tons, $\omega=\frac{1}{2}n$, $g=32{\cdot}19084$; whence I have made the following calculations from formula (59.).

Weight of the engine in tons, including the driving-wheels.	Weight of a pair of wheels with cranked axle, in tons.	Formula (59.) reduced, $h>\frac{128{\cdot}76\left(1+\frac{W_1}{W}\right)}{n^2}$	Displacement of the centre of gravity of a six-feet driving-wheel, which will cause a jump of the wheel on the rail. Rate of travelling in miles per hour.		
			50.	60.	70.
20	2·5	$\frac{1030{\cdot}08}{n^2}$	·4128	·2867	·2106
	3	$\frac{858{\cdot}4}{n^2}$	·3434	·2384	·1751
25	2·5	$\frac{1287{\cdot}6}{n^2}$	·5150	·3576	·2628
	3	$\frac{1073}{n^2}$	·4292	·2908	·2189

It appears, by formula (59.), that the displacement of the centre of gravity necessary to produce a jump at any given speed, is not dependent on the actual weight of the engine or the wheels, but on the ratio of their weights; and, from the above Table, that when the weight of the engine and wheels is $6\frac{2}{3}$ times that of the driving-wheels, a displacement of $2\frac{3}{4}$ inches in the centre of gravity is enough to create a jump when the train is travelling at sixty miles an hour, or of 2 inches when it is travelling at seventy miles; this displacement varying inversely as the square of the velocity is less, other things being the same, as the square of the diameter of the wheel is less; for the radius of the wheel being represented by a, the angular velocity is represented by $\omega=\frac{22n}{15a}$, and substituting this value, formula (59.) becomes

$$h>\left(\frac{15}{22}\right)^2\frac{g a^2\left(1+\frac{W_1}{W}\right)}{n^2}.$$

If the weight W of the wheel be supposed to vary as the square of its diameter and be represented by μa^2, this formula will become

$$h>\left(\frac{15}{22}\right)^2\frac{g\left(a^2+\frac{W_1}{\mu}\right)}{n^2},$$

* It will be observed, that the cranks being placed on the axle at right angles to one another, when the centre of gravity on the one side is in a favourable position for jumping, it is in an unfavourable position on the other side, so that it can only jump on one side at once, and the efforts on the two sides alternate.

still showing the displacement of the centre of gravity necessary to produce a jump to diminish with the diameter of the wheel. These conclusions are opposed to the use of light engines and small driving-wheels; and they show the necessity of a careful attention to the true balancing of the wheels of the carriages as well as the driving-wheels of the engine. It does not follow that every jump of the wheel would be high enough to lift the edge of the flange off the rail; the determination of the height of the jump involves an independent investigation. Every jump nevertheless creates an oscillation of the springs, which oscillation will not of necessity be completed when the jump returns; but as the jumps are made alternately on opposite sides of the engine, it is probable that they may, and that after a time they will, so synchronize with the times of the oscillations, as that the amplitude of each oscillation shall be increased by every jump, and a rocking motion be communicated to the engine attended with danger.

Whilst every jump does not necessarily cause the wheel to run off the rail, it nevertheless causes it to *slip* upon it, for before the wheel jumps it is clear that it must have ceased to have any hold upon the rail or any friction.

The Slip of the Wheel.

If f be taken to represent the coefficient of friction between the surface of the wheel and that of the rail, the actual friction in any position of the wheel will be represented by $Y_1 f$. But the friction which it is necessary the rail should supply, in order that the rolling of the wheel may be maintained, is X. It is a condition therefore necessary to the wheel *not slipping* that

$$Y_1 f > X, \text{ or } f > \frac{X}{Y_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (60.)$$

If, therefore, taking the maximum value of $\frac{X}{Y_1}$ in any revolution, we find that f exceeds it, it is certain that the wheel cannot have slipped in that revolution; whilst if, on the other hand, f falls short of it, it must have slipped*. The positions between which the slipping will take place continually, are determined by solving, in respect to $\cos \theta_1$, the equation

$$f = \frac{X}{Y_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (61.)$$

The application of these principles to the slip of the carriage-wheel is rendered less difficult by the fact, that the value of h is always in that case so small, as compared with the values of k and a, that $\frac{h}{a}$ may be neglected in formulæ (34.) and (35.), as compared with unity. Those equations then become

$$X = \frac{Wh \sin \theta}{a} \left\{ 1 - \frac{k^2 (g + a\omega^2)}{g(k^2 + a^2)} \right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (62.)$$

* Of course, the slipping, in the case of the driving-wheels of a locomotive, is diminished by the fact that whilst one wheel is not biting upon the rail the other is.

and

$$Y=\frac{Wh}{a}\left\{\frac{a}{h}-\cos\theta+\frac{(g+a\omega^2)\cos\theta}{g}\right\}=W\left\{1+\frac{h\omega^2\cos\theta}{g}\right\},$$

whence we obtain

$$Y_1=W_1+W\left\{1+\frac{h\omega^2\cos\theta}{g}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (63.)$$

and

$$\frac{X}{Y_1}=\frac{\frac{Wh}{a}\left\{1-\frac{k^2(g+a\omega^2)}{g(k^2+a^2)}\right\}\sin\theta}{W_1+W\left\{1+\frac{h\omega^2\cos\theta}{g}\right\}}=\frac{\frac{g}{a\omega^2}\left\{1-\frac{k^2(g+a\omega^2)}{g(k^2+a^2)}\right\}\sin\theta}{\left(1+\frac{W_1}{W}\right)\frac{g}{h\omega^2}+\cos\theta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (64.)$$

Assume

$$\beta=\left(1+\frac{W_1}{W}\right)\frac{g}{h\omega^2} \text{ and } u=\frac{\sin\theta}{\beta+\cos\theta}$$

$$\therefore \quad \frac{du}{d\theta}=\frac{1+\beta\cos\theta}{(\beta+\cos\theta)^2} \quad \frac{d^2u}{d\theta^2}=\frac{\{-\beta(\beta+\cos\theta)+2(1+\beta\cos\theta)\}\sin\theta}{(\beta+\cos\theta)^3}.$$

Now if $\beta>1$, there will be some value of θ for which $\frac{1}{\beta}+\cos\theta=0$, and therefore $1+\beta\cos\theta=0$; and since for this value of θ, $\frac{du}{d\theta}=0$, and $\frac{d^2u}{d\theta^2}=-\frac{\beta^2}{(\beta^2-1)^{\frac{3}{2}}}$, it follows that it corresponds to a maximum value of u, and therefore of $\frac{X}{Y}$.

But if $\beta<1$, then there is some value of $\cos\theta$ for which $\beta+\cos\theta=0$, and therefore for which $u=$infinity, which value corresponds therefore in this case to the maximum of $\frac{X}{Y}$.

Thus then it appears that according as

$$\beta \text{ or } \left(1+\frac{W_1}{W}\right)\frac{g}{h\omega^2}\lessgtr 1 \text{ or } \omega^2\gtrless\frac{g}{h}\left(1+\frac{W_1}{W}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (65.)$$

the maximum value of $\frac{X}{Y_1}$ is attained when $\cos\theta=-\beta$ or $=-\frac{1}{\beta}$; that is, when

$$\cos\theta=-\frac{g}{h\omega^2}\left(1+\frac{W_1}{W}\right) \text{ or } =-\frac{h\omega^2}{g\left(\frac{W_1}{W}+1\right)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (66.)$$

In the one case the maximum value of $\frac{X}{Y_1}$ will be infinity, ... (67.)

and in the other case it will be represented by the formula

$$\frac{X}{Y_1}=\frac{g\left\{1-\frac{k^2(g+a\omega^2)}{g(k^2+a^2)}\right\}}{\left\{g^2\left(1+\frac{W_1}{W}\right)^2-h^2\omega^4\right\}^{\frac{1}{2}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (68.)$$

In the first case, *i. e.* when $\beta<1$, the wheel will slip every time that it revolves, whatever may be the value of f. In the second case, or when $\beta>1$, it will slip if f do not exceed the number represented by formula (68.). The conditions (65.) are obviously

the same with those (59.) which determine whether there be a jump or not, which agrees with an observation in the preceding article, to the effect, that as the wheel must cease to bite upon the rail before it can jump, it must always slip before it can jump. When the conditions of slipping obtain, one of the wheels always biting when the other is slipping, and the slips of the two wheels alternating, it is evident that the engine will be impelled forwards, at certain periods of each revolution, by one wheel only, and at others, by the other wheel only; and that this is true irrespective of the action of the two pistons on the crank, and would be true if the steam were thrown off. Such alternate propulsions on the two sides of the train cannot but communicate alternate oscillations to the buffer-springs, the intervals between which will not be the same as those between the propulsions; but they may so synchronize with a series of propulsions as that the amplitude of each oscillation may be increased by them until the train attains that fish-tail motion with which railway travellers are familiar. It is obvious that the results shown here to follow from a displacement of the centres of gravity of the driving-wheels, cannot fail also to be produced by the alternate action of the connecting rods at the most favourable driving points of the crank and at the dead points*, and that the operation of these two causes may tend to neutralize or may exaggerate one onother. It is not the object of this paper to discuss the question under this point of view.

* A slip of the wheel may thus be, and probably is, produced at each revolution.

Wandsworth, Feb. 28, 1851.

XXIV. *Observations upon the Anatomy and Physiology of* Salpa *and* Pyrosoma.

By Thomas Henry Huxley, *Assistant-Surgeon R.N.*
(*late of H.M.S. "Rattlesnake"*). *Communicated by Prof.* Edward Forbes, *F.R.S.*

Received February 26,—Read March 27, 1851.

1. THE *Salpæ*, those strange gelatinous animals, through masses of which the voyager in the great ocean sometimes sails day after day, have been the subject of great controversy since the time of the publication of the celebrated work of Chamisso, 'De Animalibus quibusdam è classe Vermium Linnæana.'

In this work were set forth, for the first time, the singular phenomena presented by the reproductive processes of these animals,—phenomena so strange, and so utterly unlike anything then known to occur in the whole province of zoology, that Chamisso's admirably clear and truthful account was received with almost as much distrust as if he had announced the existence of a veritable Peter Schlemihl.

In later days an opposite fate has fallen upon the statements in question. They have been made the keystone of a revived* theory, and the phenomena presented by the *Salpæ* have been cited as "glaring instances" of a law governing the vast majority of the lower invertebrata—the law of the "Alternation of Generations."

2. There appeared then to be two main points to be kept in view in examining the *Salpæ*:—1st. Are the statements made by Chamisso correct? and 2ndly, if they be correct, how far is the "alternation theory" a just and sufficient generalization of the phenomena?

3. These questions, however, could not be entered upon without a thorough preliminary study of the structure of the *Salpæ*, the opportunities for which are granted but to few.

Such opportunities were afforded to the writer of the present paper at Cape York, in November 1849: for a time the sea was absolutely crowded with *Salpæ*, in all states of growth, and of a size very convenient for examination. At subsequent periods the writer had occasion repeatedly to verify the results at which he had arrived, and to find strong analogical confirmation in the structure of *Pyrosoma* and other allied genera†.

* Not new, see (70.).

† Those who are acquainted with the nature of the service on which H.M.S. 'Rattlesnake' was engaged, will readily comprehend that the author's investigations were almost *necessarily* original, and independent of anything going on in Europe.

It is the more necessary to state this, as it will be seen, in the historical part of this paper, that M. Krohn,

4. It is proposed in the following pages to consider—

I. The structure of the species of *Salpa* examined.

II. The structure of *Pyrosoma*.

III. The homology of structure of *Salpa* and *Pyrosoma*, and of these with the ordinary Ascidians.

IV. The history of our knowledge of the *Salpæ*.

I. *The Structure of* Salpa.

5. Before entering upon this question, there is a point of some importance to be determined, as to the upper and lower surfaces, the anterior and posterior extremities of the *Salpæ*. Observation will not decide this apparently simple matter, for as the writer has frequently seen, they swim indifferently with either end forward and with either side uppermost; and the determinations of authors are most contradictory.

Throughout the present paper, that side on which the heart is placed will be considered as the dorsal side; that on which the ganglion and auditory vesicle are placed, as the ventral side. That extremity to which the mouth is turned will again be considered as the anterior extremity, the opposite as the posterior. Such a view of the case appears to be more harmonious with the determinations of corresponding parts in other animals than any other. In all the invertebrata the mouth end is always considered as the anterior, the heart side as the dorsal side.

6. The two so-called species of *Salpa* examined were the *S. democratica* of Forskahl, *spinosa* of Otto, and the *S. mucronata* of Forskahl, *pyramidalis* of Quoy and Gaimard. They are described by Sars as *S. spinosa* and *S. mucronata*, or rather as *S. spinosa*, *proles solitaria*, and *S. spinosa*, *proles gregaria*. This however begs the question, as to the truth of Chamisso's theory, and I shall therefore prefer to name the two forms I observed simply *Salpa* A and *Salpa* B.

At Cape York, and only there, these two forms were always obtained together. They were of about the same size, but so totally distinct in appearance, that, had they belonged to any other genus, they would have been justly regarded as separate species.

7. *Salpa* A (Plate XV. fig. 1).—The body is gelatinous, transparent and colourless, except the nucleus (*i*), which has a deep reddish-brown tint. It has a general square prismatic shape, and is abruptly truncated and somewhat convex at each extremity. The posterior extremity is provided with eight hornlike processes, which project backwards. Two of these are short and hook-like, placed one before the other in the median line at the posterior part of the superior surface. On the upper part of the

in the Ann. des Sciences for 1846, has completely anticipated the chief results arrived at, a fact of which the author was totally unaware until his arrival in England in the end of 1850.

Still, as M. Krohn gives merely his conclusions without details or figures, his promised memoir not having appeared (so far as the writer of the present paper is aware), it is hoped that this anticipation will, by showing that perfectly independent observers arrive at the same result, rather tend to increase than to diminish any weight that may be attached to the present researches.

lateral surfaces there is, on each side, a short process. From about midway between the upper and lower edges of this surface, a long, conical process curves upwards and backwards; these processes are distinguished from the others by containing a cæcal process of the system of sinuses in their base (*y*).

Close to the lower edge of the lateral surfaces there is another short process like the uppermost one.

The respiratory apertures are wide and provided with valvular lips. The posterior (*b*) is narrower, and has the valvular lip more marked.

The ganglion (*d*) is less than one-fourth of the length of the body distant from the anterior respiratory aperture.

The otolithes are four in number, hemispherical, and with a dark, blackish brown coloured spot on their external surface, Plate XVI. fig. 5.

The endostyle (*c*) is nearly half the length of the body (reaches as far as the sixth muscular band, counting from before backwards).

The outer surface of the integument is everywhere covered with minute asperities, like little prickles.

The muscular bands (*k*) are seven in number, and, with the exception of the anterior and posterior, completely encircle the body of the animal. This form was always free and solitary*.

8. *Salpa* B, Plate XV. fig. 2, on the other hand, is thus characterized. The body is subovoid, smaller at the posterior extremity than at the anterior (*a*); the former ends in a point, the latter in a small square facet.

The sides are flattened into several irregular facets, and the upper and lower edges are sometimes somewhat carinated. The apertures are similar in general structure to those of the form A, and the anterior and posterior extremities project considerably beyond them.

The ganglion (*d*) is placed at about one-fourth of the length of the body from the anterior extremity. The otolithes resembled those of A.

The endostyle (*c*) is not nearly equal in length to half the body; it does not extend so far back as to the third muscular band.

The outer surface of the integument is smooth.

The muscular bands (*k*) are five in number, and none of them encircle the body of the animal, the dorsal extremities being always separated by a considerable interval. This form, when young, was sometimes found in chains; the adults were always separate.

These forms, it will be observed, are widely different, and the difference is as great between the youngest forms of each as between the adults, so that they are not derived from one another by any species of metamorphosis, properly so called.

Whatever be their external differences, however, their internal organization is so similar that the same description applies to both.

* The statements of Meyen (*op. cit.*) to the contrary are certainly erroneous.

9. The *Salpa*, then, may be considered as a hollow cylinder, consisting of two tunics, an external and an internal (α, β), the former (α) forming the mantle, the latter (β) the wall of the respiratory cavity. These tunics are continuous with one another at the respiratory apertures, but elsewhere they are separated by a more or less wide space.

In very young *Salpæ* this space is like the cavity of a serous sac, but in the older forms it becomes broken up into smaller channels by the adhesion of the inner and outer tunics to one another at various places, and so constitutes a system of sinuses; it may be conveniently called the "sinus system."

10. Running obliquely from behind forwards and downwards, a thickish column or band (e) crosses the respiratory cavity; it is hollow, and its cavity opens above and below into the sinus system. This is the "gill."

It presents an edge anteriorly and superiorly, and on each side of this, the lateral surfaces are beset with a series of small, oval, ciliated spaces. In this species the gill has but a single grand sinus running through it, and presents no appearance of vascular ramifications. The name gill has been applied to this structure somewhat too exclusively, as there can be little doubt that the whole respiratory cavity performs the branchial function. It is proposed, therefore, to call it the *hypopharyngeal* band, on the supposition that the proper respiratory cavity of the Ascidians answers to an enlarged pharynx.

11. The muscular bands (k) are closely adherent to the inner tunic; they are composed of flattened fibrils, about $\frac{1}{1900}$th of an inch in diameter, which are very distinctly transversely striated, the striæ being about $\frac{1}{7000}$th of an inch apart. The bands appear to possess no sarcolemma.

12. The intestinal canal (Plate XV. figs. 5 and 6) commences by a wide somewhat quadrangular mouth (r) opening into a flattened œsophagus, and placed at the re-entering angle formed by the hypopharyngeal band and the upper wall of the respiratory cavity. The intestine passes backwards, then becomes suddenly bent upwards upon itself, and curving slightly to the right, terminates in a wide flattened anus, close above and to the right side of the mouth (s).

A wide cæcal sac (t), given off on the left side of the intestine and bending upwards and to the right side, constitutes the stomach.

13. There is a very peculiar appendage to the intestinal canal, hitherto, it is believed, quite undescribed, and consisting of a system of delicate, transparent, colourless tubes, with clear contents, arising by a single stem from the upper part of the stomachal cæcum, and thence ramifying over the surface of the intestine (5, 6, u), on what may be called the rectum, that is, the terminal portion of the intestine; it forms a sort of expansion of parallel anastomosing vessels, which all terminate at the same distance from the anus anteriorly, and from the bend of the intestine posteriorly, either by uniting with one another or in small pyriform cæca, Plate XV. figs. 5 and 6.

Do these represent a hepatic organ, or are they not more probably a sort of rudi-

mentary lacteal system, a means of straining off the nutritive juices from the stomach into the blood by which these vessels are bathed?

The intestine is connected with the parietes of the sinus in which it lies by innumerable delicate short threads, like a fine areolar tissue.

14. In *Salpa* A, the only other organ contained in the circum-visceral sinus, besides the intestine and "system of tubes," is a mass of clear cells (*æ*), rendered polygonal by mutual pressure, and placed at the upper and back part of the sinus; to this body the name of "elæoblast" has been given by KROHN. It has by some authors been confounded with a liver, an organ to which it certainly has no analogy whatever. The elæoblast is much larger and more conspicuous in the young than in the adult *Salpæ*, and frequently, but not always, its cells contain an oily matter.

There would seem to be no clue either to the homology or to the function of this elæoblast. Without hazarding a conjecture, it may be remarked, as a curious fact, that these animals, so remarkable for possessing in the fœtal state a true though rudimentary placental circulation, possess an organ which in structure and duration somewhat calls to mind the thymus gland.

15. The nervous system consists of a single subspherical ganglion (*d*), situated in the space between the inner and outer tunics, just where the anterior and lower extremity of the hypopharyngeal band joins the ventral paries. It gives off two delicate branches forward to the "languet" (16.), and a great many in all directions to the parietes of the body. There were no branches traceable specially to the mouth or towards the œsophagus.

A delicate but strong vesicle attached to the anterior and lower surface of the ganglion, and containing four subhemispherical calcareous bodies, with black pigment spots on their outer surface, evidently represents the auditory vesicle and its otolithes in the gasteropod and acephalous Mollusca: and a conical depression in the outer tunic leading towards this auditory vesicle, would appear to be intended to bring it into closer relation with the surrounding medium, Plate XVI. fig. 5.

16. There would appear to be yet another organ of special sense, composed of the "languet" (*f*) and the "ciliated fossa" (*w*), called by ESCHRICHT the "längliches organ."

The "languet" (Plate XVI. fig. 5) is a long tongue-shaped or conical process, fixed by its base to the ventral surface of the respiratory cavity where this is joined by the anterior extremity of the gill, and for the rest of its extent floating freely in the respiratory cavity: it is curved so as to be convex anteriorly and concave posteriorly, and its anterior surface is marked by a shallow vertical groove; at the base this groove is wider, and where it becomes continuous with the surface of the respiratory cavity, it presents a narrow median slit, which leads into a small purse-shaped cavity, flattened from side to side and richly ciliated within, Plate XVI. fig. 5 *w*.

The posterior contour of this ciliated fossa is formed by a delicate thickened band or filament, much more distinct in some other species than in the present.

It would appear probable that the languet and the ciliated fossa subserve in some manner the performance of the gustatory function.

17. From each side of the base of the languet a narrow "ciliated band" (*x*) runs upwards, until it meets with its fellow of the opposite side, the two thus encircling the anterior aperture of the respiratory cavity.

18. The dorsal wall of the respiratory cavity is marked by two longitudinal folds, running from before backwards to the mouth. These are the dorsal folds of Savigny and others; but there is an organ to which the name of "Endostyle" may be given (*c*), very distinct from these, and yet which has been invariably confounded with them, consisting of a long tubular filament, with very thick strongly refracting walls, Plate XV. fig. 4 *c*. This body lies in the dorsal sinus; its anterior extremity is slightly curved downwards, somewhat pointed, and looks stronger and more developed than the posterior extremity, which is paler, more delicate and truncated. By its ventral surface this "endostyle" is attached to a ridge of the inner tunic, which rises up into the dorsal sinus.

19. It has been stated that the circulatory system consists, not of vessels with distinct parietes, but of more or less irregular sinuses. However irregular in form, the position of several of these is very constant. There is a *dorsal sinus* running along the dorsal surface and enclosing the internal shell; there is a *ventral sinus* opposite to this and containing the ganglion; there are *lateral sinuses* connecting these. Then there is the sinus in which the intestine and generative organs lie, the *peri-intestinal sinus*, and, finally, the sinus which, connecting the dorsal and ventral system of sinuses, traverses the gill and constitutes the *branchial sinus*.

These sinuses all communicate together round the œsophagus, and above and in front of this, the heart (*g*) is developed. The heart lying obliquely at the posterior extremity of the dorsal sinus, is not tubular, as it has been described; it forms not more than three-fifths of a tube; nor is it correct to say that it lies in a pericardium. Its true nature will be best conceived by supposing the inner surface of a sinus to have become developed for about three-fifths of its circumference into a free muscular membrane, Plate XV. fig. 9.

This membrane is exceedingly delicate, and is composed of a single layer of flat striated muscular fibrils.

20. The direction of the circulation depends entirely upon the order of contraction of the muscular fibrils of the heart. If they contract successively from behind forwards, the blood is forced in that direction; after a certain number of such contractions, they all become simultaneously, as it were, paralysed for a short period, and then they begin to contract again, but in the inverse order, and of course with an opposite effect upon the direction of the circulation.

The blood, in its alternate flux and reflux, bathes all the internal organs—the intestine, the endostyle, the brain and the generative organs, the corpuscles finding their way as they best may among the interstices. When the force of the heart

diminishes, they frequently accumulate around the intestine in consequence of becoming entangled among the meshes of the areolar tissue (13.) connecting the intestine with the parietes.

21. So far, the structure of the two forms A and B has been identical; but in proceeding to examine the reproductive organs, it will be necessary to treat of each separately.

The form A is always found to possess a connected series of young forms, the so-called *Salpa* chain, encircling its visceral nucleus; the form B, on the other hand, never possesses the *Salpa* chain, but generally contains a solitary fœtus, pendent from the upper and posterior part of its respiratory cavity. It is clear therefore that in each of these forms reproduction takes place. But is the mode of reproduction in each case similar or different? Are both, processes of gemmation, or processes of sexual reproduction, or is one process of the one description, the other of the other description? To come at the solution of this question, it will be necessary to know first, the nature and relations of the chain of young in A, then the nature and relations of the solitary fœtus in B, and, finally, to trace back the development of both to their first origin.

22. *Salpa chain of* A (Plate XV. fig. 1 *h*. Plate XVI. fig. 1. Plate XV. fig. 9). The chain is formed of a double series of fœtuses, commencing on the right side of the nucleus, curving under it, then turning upwards and over it to the right side, and finally terminating in the middle line by a free extremity midway between the two long posterior horns.

The chain is enclosed in a proper cavity, hollowed out in the substance of the outer tunic, and this sometimes opens externally opposite the free extremity of the chain, Plate XV. fig. 9.

23. The fœtuses do not form a chain by mere apposition; they are all attached by pairs to one side of a cylindrical double-walled tube, which is connected, at its anterior or proximal extremity, with the system of sinuses of the parent, to the right of the heart. The tube is in fact merely a diverticulum of the sinus system, Plate XV. fig. 9, and the blood contained in the sinuses passes freely into it. It is divided by a partition (*y*) into two canals, which are distinct for the whole length of the tube, except at its very extremity, where they communicate just as the two scalæ of the cochlea do; and it thence happens, that in the living animal, a constant current passes upon one side of the partition and down on the other, the direction of the two currents being generally, but not always, reversed with the reversal of the general circulation.

If the fœtuses be traced back upon this tube, it will be found that towards the proximal end of the tube they lose their distinctive form and become mere buds, processes of its wall, Plate XV. fig. 9. Itmay thence be conveniently termed the "gemmiferous tube."

24. The proximal extremity of the gemmiferous tube is simply transversely striated,

Plate XV. fig. 9; further on, two elevations become apparent on either side of the median line in each of these striæ. These elevations are rudiments, the inner, of the nucleus, the outer, of the ganglion of a fœtal *Salpa*. Still more towards the distal end of the tube, the young *Salpæ* are much larger in proportion to the tube; the internal organs become marked, the heart becomes visible by its contractions, and the body itself, although the respiratory apertures are as yet only marked out, not open, contracts occasionally. Finally, the otolithes make their appearance, the body becomes larger relatively to the nucleus and ganglion, and the respiratory orifices open, Plate XVI. figs. 1, 2.

25. The cavity of the gemmiferous tube communicates with the dorsal sinus system of the fœtus. Apparently the inner canal communicates by two canals, a wider and a narrower (Plate XVI. fig. 1), with the anterior portion of the dorsal sinuses of the fœtus, and the outer canal communicates with the middle of the dorsal sinuses of the fœtus. However this may be, it is quite easy to watch the blood-corpuscles of the parent making their way from the gemmiferous tube into and out of the sinus system of the fœtuses. The writer has seen one of the large blood-corpuscles of the parent entangled in the heart (which was not more than $\frac{1}{500}$th of an inch long) of a very young fœtus.

It is not exactly true that *a gradual* series in the development of the fœtuses is to be traced along the gemmiferous tube. The tube is rather marked out into sharply-defined lengths (generally three in number), in each of which the fœtuses are nearly at the same stage of growth, while they are much further developed than in the "length" on the proximal side, much less advanced than in the "length" on the distal side.

26. In this species the young *Salpæ* thus produced were extruded, when fully developed, from the aperture mentioned in (22.); but it rarely happened that even two or three adhered together, and they never formed the remarkable free-swimming chain of other species. Generally they were found solitary, presenting only on their lateral faces traces of their former adhesion. Those which were connected adhered together in a single series, the left antero-lateral extremity of the one being applied to the right postero-lateral extremity of the other; and when they became free the traces of the connection were visible as angular processes of the sinus system.

It is not correct to say that the *Salpa* chains have organs of attachment. At first they are attached by the whole length of their lateral faces, the sinus system of one being continuous by a wide channel with the sinus system of the other; but as they grow these communicating channels become more and more narrowed until they are mere points of connection; all communication then ceases, and the *Salpæ* become free from one another and move about independently.

27. Having thus determined the nature and relations of the *Salpa* chain, it remains only to be said, that the young when freed, have a sub-ovoid, posteriorly-pointed form, five muscular bands, facetted sides, and in short are identical in form, and ultimately

in size, with the form described as *Salpa* B. One-half therefore of Chamisso's theory is clearly correct; *the solitary* Salpa (Salpa A) *produces the aggregate form* (Salpa B); and we may add, that this takes place by a *process of gemmation from the walls of a tube in free communication with the circulatory system of the parent.*

28. *Solitary Fœtus of Salpa* B.—Whilst this form still forms part of the chain or is but just freed, it is sure to contain a solitary fœtus; and frequently one may be found in it when it has attained its full size, but as often not.

When the solitary fœtus exists, it hangs freely in the respiratory cavity (Plate XV. figs. 4, 8) by means of a pedicle attached to the upper and posterior part of its wall, on the left side of the mouth of the parent. In its youngest and most rudimentary state it is a somewhat conical papilla (Plate XV. fig. 7) or bulging of the inner tunic, consisting of an inner oval or pyriform cellular mass, enveloped in a delicate transparent membrane, which appears to be a continuation of the inner tunic.

As development proceeds the inner mass becomes divided into two portions, a larger turned towards the respiratory cavity, and which projects more and more into it, and a smaller subspherical, turned towards and lying in the cavity of the sinus, and bathed by the parental blood.

29. The whole mass goes on enlarging, but the former portion faster than the latter. The former becomes somewhat ovate, with its long diameter in the same direction as the long diameter of the parent, and gradually assumes the form of a *Salpa*. The muscular bands, the gill, the ganglion and its otolithic sac become distinct, and eventually the heart is obviously seen pulsating close behind the pedicle of attachment, Plate XVI. fig. 6.

In the meanwhile the smaller subspherical mass has undergone a remarkable change. It has likewise become thrust from the sinus towards the respiratory cavity, so that it no longer lies in the former, but is situated in the thick pedicle of the young *Salpa*.

It has furthermore become hollow, and contains two perfectly distinct cavities or sacs; of these the outer is concave and cup-shaped and envelopes the inner, which is subspherical, Plate XVI. fig. 6 *m*. Now the outer sac is in free communication by a narrower neck, divided into two channels by a partition, with the dorsal sinus of the fœtus; and the inner sac is in equally free communication by a neck similarly divided, with a short sinus arising immediately behind the heart; and as there is no communication between the two sacs, it follows that the current of blood in each is perfectly distinct from and independent of, that in the other. A more beautiful sight indeed can hardly be offered to the eye of the microscopic observer than the circulation in this organ. The blood-corpuscles of the parent may be readily traced entering the inner sac on one side of the partition, coursing round it, and finally re-entering the parental circulation on the other side of the partition; while the fœtal blood-corpuscles, of a different size from those of the parent, enter the outer sac, circulate round it at a different rate, and leave it to enter into the general circulation in the dorsal sinus.

More obvious still does the independence of the two circulations become when the circulation of either mother or fœtus is reversed.

30. Whether this body perform the function or not, it can hardly be wrong to give it the name of a placenta. It is identical in structure with a single villus contained in a single venous cell of the mammalian placenta, except that in the Salpian placenta the villus belongs to the parent, the cell to the fœtus; the reverse obtaining in the Mammalia.

As the young *Salpa* increases in size, the placenta, ceasing to grow, becomes proportionately smaller, until the pedicle gradually narrowing the communication with the parent ceases and the fœtus becomes free, Plate XVI. fig. 3. The remains of the placenta are traceable for some time as a small diverticulum of the dorsal sinus of the young *Salpa*, Plate XVI. fig. 3 *m*.

31. The latter as it grows nowise resembles its parent. It has a prismatic form, has seven muscular bands, and developes processes from its posterior extremity. It becomes indeed perfectly similar to the form which has been described as *Salpa* A.

It thence appears that the other half of Chamisso's theory is also perfectly true, viz. *the aggregate form of* Salpa (Salpa B) *produces the solitary form* (*Salpa* A), and the circulatory system of the fœtus in this case is connected with that of the parent, *not immediately, but by means of a very distinct and well-developed placenta.*

Here is one very clear distinction between the two processes of reproduction. Are there any other differences? To answer this question we must proceed to trace back both processes to their origin.

32. It has been seen that the young *Salpæ* B are developed by a process of gemmation from the gemmiferous tube of *Salpa* A. Whence comes the tube itself?

The smaller the individual of the form A examined, the shorter is the gemmiferous tube, and the less developed the buds upon it. In individuals just free, or about to be free, it is a very short cylindrical tube, arising on the right side and just in front of the heart, and curving downwards and backwards, Plate XVI. figs 3, 3 *a*.

In still smaller attached specimens it appears as a very short, somewhat conical process (imperfectly divided by a partition) of the dorsal sinus, close to the heart; its walls are smooth, and the blood-corpuscles are easily seen passing up one side and down the other of the partition, Plate XVI. fig. 4.

It is clear therefore that the gemmiferous tube is nothing more than a stolon, containing a diverticulum of the circulatory system of the parent, and the whole process of reproduction as it is manifested in *Salpa* A is one of gemmation. *Salpa* B *is a bud of Salpa* A.

33. Following the same course of investigation with regard to the young *Salpa* A (which it has been seen is produced from *Salpa* B), it is found, that in *Salpæ* B, which are either still adherent to the gemmiferous tube or just set free, there is no protuberance of the inner tunic into the respiratory cavity; but where this afterwards exists, a pedicle of greater or less length is attached, and running backwards, carries

at its extremity an oval cellular mass, Plate XVI. fig. 8. This hangs suspended by its pedicle in the cavity of the sinus, and is freely bathed by the blood. In one specimen the length of the pedicle was $\frac{1}{150}$th of an inch, the long diameter of the oval body about $\frac{1}{600}$th of an inch.

In still younger forms of the *Salpa* B., and indeed as soon as the separate organs are distinguishable, the outer tunic bulges slightly in the middle line behind the outline of the posterior aperture and beneath the nucleus, Plate XVI. figs. 1, 2; this protuberance is caused by the presence of a spherical body (*q*) about $\frac{1}{1000}$th of an inch in diameter, containing a clear vesicle $\frac{1}{1700}$th of an inch in diameter, which again frequently contained a round opake spot or nucleus about $\frac{1}{8000}$th of an inch in diameter; the latter sometimes appeared as a thick-walled vesicle. This is plainly an ovum; a narrow pedicle (*q'*) is attached to its upper extremity and runs upwards, curving slightly forwards to the same point as in the preceding forms.

It would appear then that the *Salpa* B developes a single ovum, which is at first placed in the median line in the ventral sinus; that partly by the increase in size of the body, and partly in consequence of a shortening of its pedicle which acts as a gubernaculum, it becomes drawn from this position upwards and to the left side; and that in the meanwhile, probably in consequence of fecundation, it becomes altered in structure, and precisely similar to and identical with the cellular mass which has been seen to form the rudiment of the young *Salpa* A, Plate XVI. fig. 7. In this case *the* Salpa A *would be a true embryo developed by a process of sexual generation.*

34. Sexual generation however presupposes a male fecundating organ, and this is found in *Salpa* B as a ramified body, hitherto generally called a liver (*p*), Plate XV. figs. 6 and 7, closely surrounding the intestinal canal with a network, solid in the younger form, but in the older tubular, with very thin walls, and containing a vast number of pale-greenish circular cells, from $\frac{1}{3000}$th to $\frac{1}{1700}$th of an inch in diameter; and besides these detached spermatozoa, with very thin tails and long narrow heads, about $\frac{1}{1600}$th of an inch in length. The testis had no visible excretory organ, but such might well escape notice.

Nothing at all resembling this body is found in the form A; its contents sufficiently demonstrate its real nature, and its existence on the other hand is strong confirmatory evidence, if any be needed, that the pediculate body described above is a true ovum.

One curious circumstance needs to be remarked; the testis does not develope *pari passu* with the ovum and attain its full development at the same time, as might be imagined. The testis is always behind the ovum in its progress, and does not, indeed, seem to have attained its full development until the latter has become freed from the parent.

Without carefully tracing the form B through all its stages, it might readily be supposed to be always male; in fact, fully-grown specimens, while they always possess

a well-developed testis, rarely contain any embryo, this being generally set free when the parent is about half or two-thirds grown. The careful observer will, however, be always able to detect a trace of its former attachment, in a sort of cicatrix, left at the corresponding part of the respiratory chamber.

35. It is not clear by what channel fecundation takes place, whether each *Salpa* B impregnates its own ovum by discharging the contents of its own testis into the circulatory fluid, which would be a procedure altogether anomalous; or whether, on the other hand, impregnation do not rather take place from without, a presumption which is strengthened by analogy, and by the fact, that the testis does not seem to attain maturity early enough to fecundate its own ovum. The spermatic fluid may have access to the ovum by the gubernaculum becoming hollow and tubular, as will be seen to be the case in the *Pyrosomata*, and indications of such an occurrence have occasionally manifested themselves.

36. To recapitulate.—The form A (*Salpa solitaria*) produces a stolon, from which, by gemmation, arises the form B (*Salpa gregata*). This contains a testis and a single ovum attached by a pedicle or "gubernaculum" to the wall of the respiratory chamber. Fecundation takes place in a manner not yet clearly ascertained, and the "gubernaculum" shortens until the ovum is brought into close contact with the respiratory wall or inner tunic. The latter then protrudes into the respiratory canal, enveloping the ovum in a close sac; the ovum becomes developed into an embryo, which is connected by a genuine placenta with its parent, and ultimately assuming the form of *Salpa* A becomes detached and free.

37. While Chamisso's formula, then, expressed the truth with regard to the generation of the *Salpæ*, it did not express the whole truth.

True it is, that the *Salpa solitaria* always produces the *Salpa gregata*, and the *Salpa gregata* the *Salpa solitaria*; but it is most important to remember that the word "produce" here means something very different in the one case, from what it means in the other. In the *Salpa solitaria* the thing produced is *a bud*; in the *Salpa gregata a true embryo*. There is no "alternation of generations," if by generation sexual generation be meant; but there is an *alternation of true sexual generation with the altogether distinct process of gemmation*.

It would be irrelevant to discuss here the wide question of the "alternation of generations" in all its bearings; but the writer may be permitted to express his belief, founded upon many observations upon the Polypes, Acalephæ, &c., that the phenomena classed under this name are always of the same nature as in the *Salpæ*; that under no circumstances are two forms alternately developed by *sexual generation*; but that wherever the so-called "alternation of generations" occurs it is *an alternation of generation with gemmation*.

38. Using the terminology of insect metamorphosis, as Chamisso has done (70.), the larva never produces the imago by sexual generation, the imago again producing the larva by sexual generation. But a pseud-imago, which is indeed nothing more,

homologically, *than a highly individualized generative organ*, is developed from the larva, ova are produced by it, and from these the larva again is developed; the whole process differing from that common to animals in general, in nothing but the independence and apparent individuality of the generative organ.

39. It cannot be too carefully borne in mind that zoological individuality is very different from metaphysical individuality, and that the whole question of the propriety of the "alternation theory" as a means of colligating the facts (for at best it can be nothing more) turns upon the nature and amount of this difference.

If the true definition of the zoological individual be (as the writer believes it to be) "the sum of the phenomena successively manifested by, and proceeding from, a single ovum, whether these phenomena be invariably collocated in one point of space or distributed over many," then there is no essential difference between the reproductive processes in the higher and lower animals, and the alternation theory becomes unnecessary.

In accordance with this definition, neither the form A, nor the form B would be a zoological individual; not either of their forms, but both together, answer to the "individual" among the higher animals.

In strictness both *Salpa* B and *Salpa* A are only parts of individuals,—are organs; but as we are unaccustomed to associate so much independence and completeness of organization with a mere organ, to give them such a name would sound paradoxical. It is proposed therefore to call them, and all pseudo-individual form resembling them, "zoöids," bearing in mind always that while the distinction between zoöid and individual is real, and founded upon the surest zoological basis,—a fact of development,—that between zoöid and organ is purely conventional, and established for the sake of convenience merely*.

40. In the *Salpæ*, then, the parent and the offspring are not dissimilar, but the individual is composed of two zoöids.

In *Cyanea*, the individual is composed of two "zoöids," a medusiform and a polypiform zoöid.

In the *Trematoda* there are frequently three "zoöid" forms to the individual.

In the Aphidæ the sum of from nine to eleven "zoöids" composes the individual, the great number of zoöid forms in this case being simply an instance of that "irrelative repetition" of parts so common among the lower animals.

A similar irrelative repetition exists among the so-called "compound" animals, the Polypes and compound Ascidians; and consistently with the present theory we must call a *Sertularia* or a *Pyrosoma*, for instance, not an aggregation of individuals

* For a further consideration of this subject the author begs to refer to Dr. Carpenter's "Principles of Physiology," in which the whole question of individuality in plants and animals is treated in a very clear and masterly manner; to Mr. Thwaites's papers in the Annals of Natural History; and to an attempt to apply the principles advocated in the text to the metamorphosis of the Echinoderms in a Report by himself.—*Annals*, July 1851.

into a common mass, but an individual which is developed into a greater or less number of zoöid forms, which in the present case remain united.

Thus the stem and branches of the polypidom in *Sertularia* are "organs," the ovarian vesicles are "organs," the polypes are "zoöids;" the sum of the organs and zoöids constitutes the individual.

If the separate polypes be individuals, what is the polypidom which exists before them, and therefore cannot be derived from them?

It seems startling to assert that a *Salpa* of the form A with some fifty or sixty of the form B which proceed from it, constitutes but one zoological individual; still more to aver this of some millions of Aphides all proceeding indirectly from one ovum; but these difficulties have reference merely to our ordinary notion of individuality, and involve us in no self-contradictions and inconsistencies such as seem inherent in any other view of the case.

Section II.—*The Anatomy of* Pyrosoma.

41. This genus, first established and very imperfectly described by Peron*, received elaborate investigation from Lesueur† and from Savigny‡, who very carefully described every part of its organization with the exception of the generative organs, and one or two other points of minor importance.

Subsequently M. Milne-Edwards§ showed that the nature of the circulation was the same in it as in the other Ascidians.

Of the three species distinguished by Lesueur the present appears most closely to resemble the *P. atlanticum*.

42. The only specimen of this remarkable animal which the writer has had an opportunity of examining in the fresh state, was procured on the night of the 15th of June 1850, in about 45°·85 S. lat. and 110°·30 W. long. The sky was clear but moonless, and the sea calm; and a more beautiful sight can hardly be imagined than that presented from the decks of the ship as she drifted, hour after hour, through this shoal of miniature pillars of fire gleaming out of the dark sea, with an ever-waning, ever-brightening, soft bluish light, as far as the eye could reach on every side. The *Pyrosomata* floated deep, and it was with difficulty that some were procured for examination and placed in a bucketfull of sea-water. The phosphorescence was intermittent, periods of darkness alternating with periods of brilliancy. The light commenced in one spot, apparently on the body of one of the "zoöids," and gradually spread from this as a centre in all directions; then the whole was lighted up; it remained brilliant for a few seconds, and then gradually faded and died away, until the whole mass was dark again. Friction at any point induces the light at that point, and from thence the phosphorescence spreads over the whole, while the

* Annales du Museum, 1804.
† Journal de Physique, 1815.
‡ Mem. sur les Animaux sans Vertèbres.
§ Comptes Rendus, 1840.

creature is quite freshly taken; afterwards, the illumination arising from friction is only local.

43. So far as could be observed the *Pyrosoma* had no power of locomotion; any such power arising from a contraction of the hollow cylinder is out of the question, as its substance is cartilaginous and non-contractile. Any one who does not examine these animals quite closely may be readily deceived on this point; for the alternate fading and brightening of the phosphorescent light gives rise to the impression that the creature recedes from and approaches the eye; and viewing them from the deck of a ship only, it is difficult to imagine that they do not really move with some rapidity*.

44. The *Pyrosoma* may be described as a hollow cylinder, solid and hard to the touch, closed and rounded at one end, open at the other. A narrow lip projects inwards at the open extremity; it has been called a membranous diaphragm, but in the specimen examined it was certainly cartilaginous and immoveable, like the rest of the animal. The thickness of the wall of the cylinder was about two-fifths of an inch; its diameter was about 1 inch and a half; its length was about 10 inches. The outer surface of the cylinder was covered with a multitude of small projections, and close to them opened small circular apertures. The inner surface of the cylinder was uneven but not rough, and was similarly pierced with circular apertures.

The wall of the cylinder consists of a vast number of minute Ascidian "zoöids" lying perpendicular to the axis of the tube, and united together by a common cartilaginous basis; and the small circular apertures correspond respectively, the outer to the anterior aperture of the *Salpa*, the inner to the posterior aperture.

Each aperture is provided with a small dentated membranous valve, Plate XVII. fig. 1.

45. In each zoöid there is at one point a ganglion (*d*), with a mass of deep red otolithes. As in *Salpa*, this must be called the ventral side; the opposite is the dorsal side, and contains (as in *Salpa*) an endostyle, Plate XVII. figs. 1, 2 *c*. The ganglionic or ventral surfaces of all the polypes are turned the same way, and towards the open end of the cylinder.

By far the greater part of the space occupied by each zoöid is taken up by the respiratory cavity. This is elliptical, and compressed laterally. It is lined by the proper branchial network, hereafter to be described (*v*), and communicates freely by means of the apertures in the branchial network with the post-branchial or anal cavity, which, as before stated, opens into the interior of the cylinder.

46. The viscera lie behind the branchiæ. They consist of the digestive canal, heart, and generative organs.

* My observations upon the power of locomotion of *Pyrosoma* were very imperfect, as I was anxious rather to attend to the more interesting points of structure. Certainly the cylinder does not contract as a whole, but it is very possible that the zöoids do, and so move by the reaction of the forced-out water against the closed end of the cylinder.

The intestine (*r*, *s*, *t*) commences by a wide mouth with thick lips, at the posterior, ventral extremity of the respiratory cavity. The œsophagus (*r*) runs back, and then upwards to terminate in the wide subquadrilateral stomach (*t*). A narrow pylorus communicates with the intestine, which passes at first upwards and forwards, and then suddenly becoming bent upon itself, runs downwards and to the right side, to end in the wide flattened anus (*r*).

The œsophagus is dotted over with branched carmine pigment-cells; and similar cells are frequently seen upon the intestine just beyond the pylorus.

47. A tubular axis (*u*) arises from the stomach, and branches out on the rectum into a system of tubes as in *Salpa*; but the ramifications are less numerous and less regular, with wider meshes than in the latter. The tubes are less transparent and have more the appearance of solid fibres, and finally they terminate towards the anus in wide globular cæca.

The stem of this system is about $\frac{1}{1100}$th of an inch in diameter.

48. Each zoöid is composed of two tunics, an outer (α), confluent with the general cartilaginous basis, and an inner (β), continuous with the outer at its anterior and posterior extremities, and adherent to it antero-laterally, in two oval spots, one on each side, which, when examined by the microscope, appeared to consist of nothing more than an aggregation of clear circular cells about $\frac{1}{800}$th of an inch in diameter. (δ). These were considered by Savigny to be the ovaria, but they have not the appropriate structure, and it will be seen that the ova are formed elsewhere.

In all the rest of their extent the inner and outer tunics are separated by a very obvious space. This is one large vascular sinus, and the viscera lie in it and are bathed by the blood which fills it.

The heart (*g*) is placed on the dorsal side just behind the posterior extremity of the internal shell. In structure it perfectly resembles that of *Salpa*, and its contractions are reversed in a similar manner. No distinct vessels were to be traced in these animals.

49. The endostyle (*c*) resembles that of *Salpa* in its structure. It is as long as the branchial chamber, and lies in the dorsal sinus, supported by a projecting ridge of the inner tunic. On each side of it below, there is a longitudinal thickening, which readily gives rise to the appearance of four dorsal bands or "undulated vessels," described by Savigny.

50. The branchiæ (*v*) are symmetrical, one on each side, and are composed of a network formed by longitudinal and vertical bars or laminæ.

The vertical bars are outside; the longitudinal bars are at equal distances along their inner surface, and are attached at the point of intersection.

The vertical bars are attached to the inner tunic at their upper and lower extremities; for the rest of their extent they are free.

The longitudinal bars (*v*, fig. 3) are rather laminæ, flattened horizontally, slightly thickened at their free edges, and beset along the upper surface of these

edges with small teeth; they project anteriorly and posteriorly beyond the vertical bars.

Both systems of bars appeared to be tubular, although no corpuscles were seen moving in them, and the edges of the vertical sinus were thickly covered with long cilia, moving in opposite directions on the opposite sides.

51. The dorsal edges of the two branchiæ were separated by a space containing the thickened dorsal folds already mentioned, and this is continuous posteriorly with a band which connects the mouth with the dorsal surface of the respiratory cavity, and allows the water to pass back on each side of it to the post-branchial cavity.

52. Anteriorly on the ventral side is an organ (fig. 10 *w*) analogous to the "ciliated fossa" of the *Salpæ*, and behind this a series of tongue-shaped eminences (fig. 1 *f*) projects into the respiratory cavity, analogous to the "languet" of the *Salpæ*.

The "ciliated fossa" is compressed laterally, and placed upon the upper surface of a protuberance, formed by the ventral wall of the respiratory cavity in the middle line. On each side a flattened ciliated band (fig. 10 *x*) runs up on the respiratory wall in front of the anterior edge of the branchiæ, and meets above with its fellow of the opposite side.

The "languets" are altogether eight in number. They extend in a longitudinal series between the ciliated fossa and the mouth. They are all slightly excavated and ciliated anteriorly.

53. Immediately beneath the ciliated fossa, and in the midst of the ventral sinus, lies the ganglion. This is about $\frac{1}{140}$th of an inch long, somewhat egg-shaped, with its large end forwards. Its posterior extremity is in contact with a mass of deep red otolithes, fig. 10 *d*.

A small nerve runs from the ganglion to the lateral ciliated band. Five or six branches are distributed to the anterior aperture, and two principal branches run backwards to the posterior aperture, giving off branches to the mouth in their course.

54. The *Pyrosomata* are hermaphrodite.

The testis (*p*) is the so-called "hepatic organ" of Lesueur, Savigny and Peron. It consists of ten, twelve, or more cæca, connected by their posterior extremities, and here joining a central duct, which opens by a papilla at the upper and posterior part of the respiratory cavity. The spermatic sacs lie loosely in a dilatation of the vascular sinus, and are bathed freely by the blood.

Each sac is delicate and thin-walled, about $\frac{1}{300}$th of an inch in diameter, and very variable in length. In adult specimens the distal or anterior end of each sac is filled with a pale cellular mass. Towards the attached end this becomes darker and more distinctly granulous, and the filiform bodies of masses of spermatozoa are plainly perceived.

The spermatozoa have narrow elongated heads and very long delicate tails.

55. There cannot be said to be any ovary properly so called. But to the left, and

rather in front of the testis (fig. 5), there could always be found more or less decided traces of one or more ova.

Commonly there was a single ovum (figs. 4–6), measuring about $\frac{1}{250}$th of an inch in diameter, with a clear germinal vesicle $\frac{1}{600}$th of an inch in diameter, and a vesicular thick-walled germinal spot $\frac{1}{1800}$th of an inch in diameter.

The ovum is inclosed in a strong transparent sac, continuous with a pedicle or gubernaculum (fig. 6 *q'*), which runs to the upper and posterior part of the inner tunic on the left side, and there terminates in a papilla, like that of the vas deferens, projecting into the post-branchial cavity.

In young specimens, when the ovum is small and the yelk pale, this gubernaculum frequently appears to be solid; but in fully-grown specimens, when the ovum has its full size, and the yelk is darker and granulous, it presents the appearance of a wide tube, especially at its upper part. And here there was frequently an appearance of dark striæ and moving granules, prompting the belief that spermatozoa had travelled thus far.

In one instance (fig. 6) the sac of the ovum was empty and the gubernaculum or duct widely distended. The appearance of spermatozoa in the duct was here very strong, fig. 5.

None of the compound ova described by Savigny were present in the specimens of *Pyrosoma* examined.

56. The young polypes described by Savigny as existing between the fully-formed ones, in all stages of development, are formed by gemmation, Plate XVII. fig. 7 *q*.

A diverticulum of the dorsal sinus of the parent is formed just above the heart; the extremity of this diverticulum thickens and enlarges, and assumes the form of a single zoöid. For a long time a vascular connection is maintained between it and the parent, by means of a duct, in which there seemed to be traces of a longitudinal partition, as in the gemmiferous tube of *Salpa*. Ultimately the connection appears to cease, and the two polypes live on independently.

It is to be remarked, that while in *Salpa* the ventral side of the young bud is first marked out, and the communication of the parent with the young is thence on the dorsal side of the fœtus, in *Pyrosoma* the dorsal side is first developed, and the communicating canal opens on the ventral side of the young.

57. The ovum or ova, for there are sometimes two or three, are perceptible very early in the young polype produced by gemmation, and are then situated in the middle line posteriorly.

58. The muscular system is best seen in a young specimen (fig. 8 *k*). Two very delicate bands encircle the inner tunic anterior to the ganglion. From the posterior extremity of the ganglion two strong bands arise, which diverge for about half the distance between the ganglion and the mouth. Here they divide into two branches, one of which continues the original direction, while the other meets its fellow just behind the mouth. The former, as it leaves the under surface to become lateral, is

much increased in size, and eventually terminates at a short distance from the generative glands, forming on each side the band of which SAVIGNY speaks as passing towards the liver.

Midway between the ganglion and the point of division, the diverging bands give off each a thin band, which runs to the lateral oval cellular masses.

SECTION III.—*The Homology of Structure of* Salpa *and* Pyrosoma, *and of these with the ordinary Ascidians.*

59. It seems to have been pretty generally admitted by naturalists, that the *Tunicata* are susceptible of division into two great classes, the Monochitonida and Dichitonida, characterized by certain differences in the structure of the branchiæ and in the degree of adhesion of the inner and outer tunics.

Of the two species whose structure has been described, *Salpa and Pyrosoma*, the former was placed among the Monochitonida (or "those having the inner sac adherent throughout to the outer tunic"), while the latter was reckoned among the Dichitonida (or those "whose inner sac is adherent to the outer tunic, at its two orifices, only").

Now there is an ambiguity which must be noticed here at starting, as it is one which has caused much confusion, and must, unless cleared up, cause our conception of the real structure of the Ascidians to be very indistinct. Authors speak of the greater or less adherence of the outer and inner sacs, and consider the "outer sac" of the ordinary Ascidian to be homologous with the outer tunic of the *Salpa*. The "inner sac," again, is with them homologous with the inner tunic of the *Salpa*. But it is not so; every Ascidian, as M. MILNE-EDWARDS has clearly shown in *Clavelina*, consists of three tunics: an outer, the *test*; a middle, which is here called outer *tunic*; and an inner, the inner *tunic*. The inner tunic of the *Salpa* answers to the inner tunic of *Clavelina*, but its outer tunic answers to the test and the outer *tunic* together (90.)*.

However, with regard to the two genera in question, whatever be the nature of the two membranes of which they are composed, there is absolutely no distinction whatever to be drawn between them. The inner membrane is just as much or as little adherent to the outer in *Pyrosoma* as in *Salpa*. In each case the wide sinuses between the two membranes form the sole vascular system.

60. It may be said that there is an essential difference between *Salpa* and *Pyro-*

* This essential difference between the test and the two *tunics* of the Ascidians has its origin in the embryo. The tunics are formed by the ordinary process of development, while the test having a totally different chemical composition, is in a manner secreted round, and envelopes the whole embryo.

There seems to be a certain independence in the mode of growth of the embryo and that of the test, the former lying at first quite free in the latter; and it appears to depend entirely upon the relative rates of growth of the two whether the resulting Ascidian shall be Monochitonidous or Dichitonidous.

The test of the Ascidian composed of cellulose is every way homologous with the test of the Mollusk composed of carbonate of lime.

soma in the structure of the branchiæ. A little consideration, however, will show that this is merely a difference in degree.

Savigny has shown that in certain *Salpæ* there is an upper division of the "gill," an "epipharyngeal band" (to carry out the nomenclature adopted at (10.)), as well as a hypopharyngeal band.

Now in the genus *Doliolum* (88.)* this epipharyngeal band has attained a much greater development (though the mouth still remains at the upper part of the cavity), and like the hypopharyngeal band carries a number of ciliated branchial bars. These bars have a direction more or less parallel to those carried by the hypopharyngeal band, and hence there appear to be two branchiæ, an upper and a lower†.

But in *Pyrosoma* the mouth is on the ventral side of the animal; the epipharyngeal band, developed in proportion to the distance of the mouth from its normal position, takes a direction at right angles to the axis, and thence comes to carry the branchial bars belonging to it parallel to the axis; while the hypopharyngeal band carrying its branchial bars as before, the two sets cross and produce the branchial network.

The line between the Monochitonida and Dichitonida then can certainly not be drawn between the *Salpæ* and *Pyrosomata*.

61. The *Pyrosomata*, in the main, have the closest similarity in structure to the Botryllidæ and other compound Ascidians; but in these latter, the separation between the test and the outer tunic becomes more and more marked, until it attains its greatest amount in the Clavelinidæ, Cynthiæ, &c.

Now does this separation furnish a character of any value or importance, systematically? Surely not, for the value of a character depends upon the number of differences of which it is a mark; and this is the mark of none.

Savigny observed the close resemblance between *Botryllus* and *Pyrosoma*, which yet differ in this character.

Clavelina and *Perophora* are acknowledged to be closely allied genera, and yet in the former the test and outer tunic are separated to their utmost extent; in the latter‡ they are as closely united as in any *Salpa*.

In the Cynthiæ the test and tunics are generally very distinct; but in *Cynthia ampulloides*, judging by the descriptions of Van Beneden, they are confounded together§.

Again, in the *Salpa vaginata* of Chamisso, the test makes its appearance as a

* And it may be added in the genus *Anchinia*, described in Wiegmann's Archiv for 1833, which seems to be a most interesting transition form between the *Salpæ* and *Doliolum*, if indeed it be not the young form of *Doliolum caudatum* itself.

† See also on the homology of the branchial organs of the *Salpæ* and ordinary *Tunicata*, M. Milne-Edwards, Sur les Ascidies composées, p. 55.

‡ See the very beautiful figures and descriptions of Lister in the Philosophical Transactions for 1834.

§ Mém. de l'Acad. Roy. de Bruxelles, tome xx.

separate structure, and the cavity in which the gemmiferous tube lies in all the *Salpæ*, and through which it makes its way to the exterior, seems to represent the normal separation of the test and tunics.

The homology of the cellulose test of the Ascidian with the calcareous test of the Mollusk has already been adverted to; and it would seem that the separation of the former from the tunics, or its confusion with it, is of as little value as a character among the *Tunicata*, as the imbedding of the shell in the mantle in *Limax* would be in separating it from *Helix*, whose shell is distinct from the mantle.

62. It would appear, indeed, that in no natural family is it less possible to draw any very broad line of demarcation among the various members than in the *Tunicata*.

Tracing them one by one, we find that all the organs of the *Salpæ* have their homologues among the other Ascidians; the various genera passing one into the other by almost imperceptible gradations.

Even the connection of the fœtus with the parent by a placenta, a feature apparently so unique in the *Salpæ*, seems to be not without its analogue in the Didemnidæ*.

The actual fact of a placental circulation indeed has not been observed, but it may be surmised, as M. Milne-Edwards† has observed the ova to be developed within a diverticulum of the vascular system of the parent.

The peculiarly formed heart, the circulation without distinct vessels, and the reversal of its direction are common to all *Tunicata*.

63. In all the *Tunicata*, again, it would seem that the first bend of the intestine (whatever its subsequent course) is dorsal, *i. e.* to the side opposite the ganglion, and almost always to the right side. *Doliolum*, however, seems to be a sinistral *Tunicate*.

What has been described in the present paper as the "Tubular System" was found by Lister (Philosophical Transactions, 1834) in *Perophora*, and described by him as "transparent vessels that may be supposed lacteals."

In *Chelyosoma* there is a mass of otolithes and a fossa, seemingly analogous to the "ciliated fossa."

The "languet" of the *Salpa* has its homologues in *Pyrosoma*, *Chelyosoma* and *Clavelina*, and is represented by smaller tentacular filaments in *Cynthia*, *Diazona*, *Synoicum* and *Polyclinum*.

64. All the *Tunicata* are hermaphrodite; and from the small size of the only efferent

* The only remaining important difference of *Salpa* from its congeners consists in the *Salpa* larvæ being tailless, while, as the beautiful researches of M. Milne-Edwards have shown, the other Ascidian larvæ have tails. This exception, however, is singularly paralleled among the Amphibia. The larvæ of the ordinary Amphibia have, as is well known, deciduous tails like the ordinary Ascidians. *In the genus* Pipa, *however, which carries its young in cells upon its back, the larva is tailless.* (Leuckart, Ueber Metamorphose, &c. Siebold and Kölliker's Zeitschrift für Wissenschaftliche Zoologie, 1851.) Such a very striking analogy needs no comment.

† Obs. sur les Ascidies composées, p. 23.

generative duct in the Botryllidæ, it would seem that the ova make their exit in the same way as in the *Salpæ*.

All the *Tunicata* possess the power of gemmation, and the buds are always formed as in *Salpa* (though not always with the same regularity), from a diverticulum of the sinus system.

65. With regard to the "endostyle," which appears hitherto to have been strangely confounded with the dorsal folds of the inner tunic, the writer can speak positively as to its existence in the *Salpæ*, *Pyrosomata* and certain Botryllidæ. In all these species it is figured by Cuvier, Chamisso, Savigny and Milne-Edwards*, but not described; and as a precisely similar structure is figured by Savigny and others in the solitary Ascidians, it is not perhaps too much to assume that the endostyle exists in them also. Should such be the case, we should be furnished with a new and very remarkable distinctive character of the *Tunicata*†.

The "elæoblast" of the *Salpæ* appears to be represented in the solitary Ascidians by the calcareous mass in contact with the heart, described by Van Beneden in *Cynthia ampulloides*.

66. The simple epipharyngeal and hypopharyngeal bands of the *Salpa* have been traced through their first degree of complication in *Doliolum* to *Pyrosoma* and *Botryllus*, where they form a true branchial sac, differing from that of the simple Ascidians only in the number and size of its meshes.

On the other hand, in *Pelonaia* the hypopharyngeal band itself has disappeared. It is a *Salpa* in which the oral and cloacal orifices have approximated while the "gill" has become obliterated‡.

In the strange form *Appendicularia* (79.), the simplification is carried a step further, for there is but one orifice, the oral. The anus opens on the dorsal surface, and a long appendage is added in the same position as that of *Boltenia*, but instead of being a long pedicle of attachment, it is a free and energetically moving fin.

67. To sum up what has been said, it appears that the *Salpæ* are not, as has been generally supposed, an aberrant form of the *Tunicata*, but rather that they are connected by insensible gradations with the other forms of the group; neither is there any circumstance in their two modes of multiplication at all at variance with what takes place in other genera of the family.

The distinction between monochitonidous and dichitonidous *Tunicata* cannot be kept up in its present sense, for the proper inner and outer tunics are equally adherent in all *Tunicata*; and as expressing the degree of adherence of the test to the

* Lister, in his description of *Perophora*, *loc. cit.*, figures the endostyle and says, "along the middle of the back is a vertical compound stripe, *d* (fig. 4), that seemed to me cartilaginous."

† Since the above was written the author has had the satisfaction of finding both the endostyle and the ciliated sac, in a small, very transparent *Cynthia* (—— ? sp.) obtained at Felixstow, on the Suffolk coast.

‡ *Chelyosoma* would appear to resemble *Pelonaia* in the absence of any distinct branchial sac; but Eschricht's figures are not very clear.

outer tunic, the distinction is of no value, systematically, as the character may vary greatly in the same or closely allied genera.

Section IV.—*History of our Knowledge of the* Salpæ.

68. Forskahl, the Danish naturalist, founded the genus *Salpa* upon certain animals taken in the Mediterranean. No less than eleven species are described and figured by him (and with remarkable clearness and accuracy) in his "Descriptiones Animalium," a work which he unfortunately did not live to see published, but which made its appearance in the year 1775.

The following is his definition of the genus: "Salpa corpore libero, gelatinoso, oblongo, utroque apice aperto; intus vacuo; intestino obliquo variat: *a*) nucleo globoso, opaco, juxta anum *b*) nucleo nullo sed linea dorsali opaco.

"Nomen mutuatum a Σάλπα, pisce a Græcis cognito et huic vermi additum ob similitudinem formæ cum tubo canoro. Animal plerumque gregarium; mira cohærens symmetria motum corporis per systolem et diastolem, siphonica arte perficiens."

Browne*, who appears to have been unacquainted with Forskahl's work, gave the name of *Thalia* to some *Salpæ*, which he describes and figures in the rudest manner.

Bosc seems to have been the first to suspect the identity of these two genera, a suspicion which was converted into a certainty by the researches of Cuvier, who not only disentangled the nomenclature of the genus from the confusion into which it had fallen, but gave the first accurate idea of the anatomy of the *Salpæ*, and first announced their true zoological relations.

69. Much was added piecemeal to the foundation thus laid by Cuvier, by subsequent authors. Meyen and Milne-Edwards described the nervous system, Kuhl and Von Hasselt, Eschscholtz and Milne-Edwards, announced the singular nature of the circulation. Cuvier and Chamisso hinted, and Meyen described, the placental connexion of the solitary fœtus with the parent; Eschricht and Sars declared the proximate nature and mode of origin of the *Salpa* chain.

70. Chamisso again founded the theory of the "alternation of generations," using that very phrase† to express the peculiarities accurately observed by him in the mode

* Natural History of Jamaica, 1785.

† Justice seems to have been hardly done to Chamisso as the first promulgator of the theory of the "alternation of generations." He says at p. 10, "Qua seposita (Salpâ bicorni) *alternationem generationum* legem esse ut posuimus genericum, omnibus communem speciebus, observationibus innititur;" and at p. 3, "Talis speciei metamorphosis generationibus in Salpis duabus successivis perficitur, *forma per generationes* (*nequaquam in prole seu individuo*) *mutata*. Verum enimvero qua lege proles Salparum, ut animal ab ovo, imago a larva, inter se differunt, parum elucet." And in his interesting "Reise um die Erde," Chamisso shows still more clearly his distinct conception of the theory by the remarkable phrase, "Es ist als gebäre die Raupe den Schmetterling und der Schmetterling hinwiederum die Raupe." "It is as if the Caterpillar brought forth the Butterfly, and the Butterfly the Caterpillar."

Subsequent writers seem not to have done much more in reality than oring new cases under the law here so

of multiplication of the *Salpæ*, but the nature and existence of the sexual organs remained undetermined until KROHN and STEENSTRUP in 1846 discovered the male organs; subsequently KROHN made out the true ovaries also.

Finally, a most accurate account (to which indeed the present memoir can be considered only as confirmatory independent testimony) of the whole course of development and reproduction of the *Salpæ* was given by KROHN in the Annales des Sciences for 1846.

71. Without undertaking the somewhat unprofitable task of giving a detailed historical account of all that has been written upon the *Salpæ*, it may be of interest to notice, with a view to reconcile, a few of the more important discrepancies among the statements of the chief investigators. And first:

Of the sides and ends of the Salpæ.—On so simple a matter as this, almost every writer has different views. CUVIER calls the ganglionic surface ventral, the opposite dorsal, the nuclear end anterior, the opposite posterior. SAVIGNY appears to follow him. CHAMISSO follows CUVIER as to the anterior and posterior ends, but reverses the dorsal and ventral sides.

MM. QUOY and GAIMARD give the ganglionic end as anterior, the nuclear as posterior, the nuclear side as ventral, the ganglionic as dorsal.

MEYEN gives the same determination. ESCHRICHT considers the nuclear end to be posterior, the ganglion side ventral; as also SARS.

M. MILNE-EDWARDS seems to follow CHAMISSO. It is much to be wished that some uniform nomenclature could be adopted. The reasons for the terms used in the present paper have already been given (5.).

72. *The Nervous System.*—The nervous system was denied by CUVIER altogether. SAVIGNY describes the ganglion, without recognizing its true nature, as the "tubercule qui dans les Ascidies est contigu au gros ganglion."

CHAMISSO describes what appears to be the thickened edge of the "ciliated fossa," and states that ESCHSCHOLTZ considered it to be a nerve (*op. cit.* p. 5).

QUOY and GAIMARD describe the ganglion, but omit all mention of the auditory sac.

MEYEN claims the discovery of the true nervous system; but although he figures it pretty accurately, he omits all mention of the otolithic sac, and seems after all in doubt whether it may not be a respiratory organ; and it was reserved for M. MILNE-EDWARDS to give the first satisfactory account of these structures.

Both ESCHRICHT and SARS subsequently omit to describe the auditory sac.

clearly expressed, and they do not always seem to have kept so clearly in mind the modest renunciation of any claim on the part of the theory to be an *explanation* of the facts, contained in the last paragraph of the former quotation.

Finally, it must not be forgotten, that though CHAMISSO was the first promulgator of the "alternation," he expressly (with a candour impossible to be too much commended) gives the credit of the conception to his companion ESCHSCHOLTZ, "generationis Salparum primus et perspicax fuit indagator amicissimus ESCHSCHOLTZ," p. 9, and again in the preface to the second fasciculus of his observations.

73. *The "Ciliated Sac" and "Languet."*—Cuvier refers to this organ in *Salpa Filesii* as "l'anneau irregulière qui la termine (la branchie) en arrière."

It has already been mentioned that this organ is mentioned by Chamisso as a problematical nervous apparatus. Quoy and Gaimard described its thickened rib as a vessel, adding, "nous dirons un vaisseau parceque nous croyons avoir vu le sang circuler dans son intérieur," apparently mistaking the ciliary motion for a circulation.

Meyen calls it the "Respirations-ring," and says that it is a respiratory organ. He first described the cilia, but denies the existence of any aperture leading into the organ.

In *S. mucronata*, not perceiving that he has to do with the very same organ under a different form, he describes the "ciliated sac" as a testis. The languet he calls simply "Haken."

Eschricht gives it and the languet together, the name of "das längliche organ," and considers it as a tactile organ analogous to the pulps of bivalves. He rightly describes the nervous cords connecting it with the ganglion.

Sars confirms Eschricht's view, and considers the organ as analogous to the tentacles of the Ascidians, which, however, cannot be the case, as in many Ascidians (e. g. *Clavelina*) the tentacles and the "languets" co-exist*.

M. Milne-Edwards figures the "ciliated fossa" as the "fossette prébranchiale" in the plates of the last (commemorative) edition of Cuvier's 'Règne Animal.'

74. *The Structure of the Heart.*—Eschricht and Sars describe the heart to be composed of a series of vesicles, which is certainly a mistake, arising from the fact that the heart always presents two or three constrictions, so as to appear almost moniliform.

75. *Tubular System.*—This is figured by MM. Quoy and Gaimard (pl. 88, fig. 12) in *Salpa pinnata*, under the name of "vaisseaux mésenteriques." Is it to this structure to which M. Krohn refers, when he says that "the meshes or lamellæ of the elæoblast are traversed by numerous vessels opening into two trunks, which apparently form the attachment between this organ and the visceral nucleus?"

Perhaps also it is to this system that Eschricht refers when he speaks of the intestine as beset with "stalked granules."

76. *The Gemmiferous Tube.*—Cuvier, Savigny, Chamisso and Quoy and Gaimard consider this structure as more or less partaking of the nature of an ovary.

Meyen mistakes it for a liver in *Salpa democratica*. Eschricht describes the process of development of the young from the gemmiferous tube and their connection with it very carefully; but he does not seem to consider it as mere gemmation.

He calls the organ "Keim-röhre," germ-tube, and considers it as a "quite new form" of propagative organ. From the mode of expression in the following para-

* In the *Cynthia* examined by the author (see note (65.), the "ciliated sac" was seated upon a tubercle, but there was no "languet."

graph he evidently thinks the propagation here to be in some manner sexual. "Die Eintheilung in Strecken deren jede Fötus von einerlei Ausbildung enthält, deutet allzu bestimmt auf verschiedene Befruchtungen hin, als das hier nicht eine wiederholte Geburt in längeren Zwischenzeiten anzunehmen wäre."

Sars conjectures that the solitary fœtuses arise by a process of sexual generation, but does not state very clearly what he considers to be the nature of the production of the associated forms.

77. *The Placenta.*—Cuvier speaks of finding a fœtus attached to the parent by a pedicle; and referring to a figure, he says, "Ce corps rond (evidently the placenta) seroit-il un organe servant uniquement pendant le temps de la gestation pour établir l'union entre la mère et son petit et qui s'effaceroit ensuite?"

Chamisso calls the pedicle of attachment "pediculus umbilicalis;" the placenta, "globulus opacus."

Meyen was the first to give this structure the name of placenta, and his account of it is so very clear and precise, that it is wonderful it should have been subsequently forgotten or overlooked. He says, "Wir haben bei ganz jungen Individuen den Verlauf der Blut-bewegung selbst bei 200-maliger Vergrösserung beobachten können. Der Muttertheil der Placenta hat nur wenige Gefässen um so mehr aber der Fötus-theil, in dem sich ein ausserordentliches Convolut von Gefässen befindet, das sich in einem Stämme endigt, der sich in das grosse Bauchgefäss ganz in der Nähe des Hergens ergiesst. Ein unmittelbares uebergehen der Blutgefässe ausdem Muttertheil in dem Fötus-theil haben wir nicht sehen können. Hat der Fötus die hinlängliche Ausbildung im Leibe der Mutter erreicht, so verwächst das grosse Blut-gefäss und die Placenta fällt ab."—P. 400.

78. After what has been stated concerning the development of the two forms of the *Salpæ*, it would be useless to enter upon the consideration of the various theories propounded since the time of Chamisso (such as that of Eschricht for instance) to account for the phenomena they present.

It may be sufficient to say, that it is now quite certain that the *Salpæ* never unite after being once separated, and that they do not produce successive broods of a different form.

Much remains to be done with regard to the minute process of development of the young forms of both kinds, and to this difficult inquiry it is to be hoped that future observers will address themselves*.

In order to avoid the necessity of incessant references to the text, a list of the works consulted and alluded to, is here subjoined in chronological order.

Salpa.—Forskahl. Descriptiones Animalium, 1775.

Browne. Natural History of Jamaica, 1785.

* An essential service to zoology will be rendered by any one who will revise and critically compare the species of the *Salpæ*. At present, they are in a most unedifying state of hopeless confusion.

Cuvier. Mémoires du Museum, 1804.
Savigny. Mémoires sur les Animaux sans Vertèbres, 1816.
Quoy and Gaimard. Freycinet's Voyage, 1817–20.
Chamisso. De Animalibus quibusdam e classe Vermium Linnæana. Fasc. i. 1819.
Chamisso. Fasciculus secundus, Nova Acta Acad. Nat. Cur. tom. x. 1821.
Kuhl and Von Hasselt. Annales des Sciences Naturelles, t. iii. 1824.
Eschscholtz. Isis, 1824.
Quoy and Gaimard. D'Urville's Voyage, 1826–29.
Meyen. Beiträge zur Zoologie, Nova Acta Nat. Cur. t. xvi. 1832.
Milne-Edwards. Annales des Sciences Naturelles, 1840.
Krohn. Ueber die Mannlichen Zeugungs Organe der Ascidien u. Salpen, Frorief's Notizen, 1841.
Eschricht. Isis, 1843–44.
Sars. Fauna Littoralis Norvegiæ, 1846.
Steenstrup. Ueber das Vorkommen d'Hermaphrodismus, 1846.
Krohn. Annales des Sciences Naturelles, 1846.

The Ascidians in general.—Lister. Philosophical Transactions, 1834.
Milne-Edwards. Sur les Ascidies composées, 1841.
Van Beneden. Mémoires de l'Académie Royale de Bruxelles, t. xx.
Forbes and Goodsir. Edinburgh New Philosophical Journal, 1841.
Rupert Jones. Art. *Tunicata*, Cyclopædia of Anatomy and Physiology.

Pyrosoma.—Peron. Annales du Museum, 1804.
Lesueur. Journal de Physique, 1815.
Savigny. *Op. cit.*
Milne-Edwards. Comptes Rendus, 1840.
Milne-Edwards. Annales des Sciences, t. xiii.

XXV. *Remarks upon* Appendicularia *and* Doliolum, *two genera of the* Tunicata.

By Thomas Henry Huxley, *Assistant-Surgeon R.N.*
(*late of H.M.S. "Rattlesnake"*). *Communicated by Prof.* Edward Forbes, *F.R.S.*

Received February 26,—Read March 27, 1851.

79. The genus *Appendicularia* was first formed by Chamisso from an animal found by him near Behring's Straits, and thus described: "Corpus gelatinosum, subovoideum, vix quartam pollicis partem æquans, punctis rubescentibus (interaneis) transparentibus. Appendix gelatinosa cestoidea, rubro marginata corpore duplo vel triplo longior. Motu flexuoso natationi inserviens. Motus animalis vividus." And he adds, "genus ultra recognoscendum, generi Cestum (Les.) forsitan affine." The specific name "flagellum" was conferred upon the animal, and it was figured (P. XXXI.), though very indifferently*.

Ten years afterwards (in 1828) Mertens, voyaging in the same regions, rediscovered this animal, and he subsequently published a long account of it† under the name of *Oikopleura Chamissonis*.

The only other notice of the genus (so far as I am aware) is that given by MM. Quoy and Gaimard‡. It was observed in immense masses off Algoa Bay, South Africa, and was called by them *Fritillaria*, until they afterwards became acquainted with the descriptions of Chamisso and Mertens. Recognising as they do the priority of discovery of the former, they yet adopt the name conferred by the latter, and, without any very just reason, give to the specimens observed by themselves a new specific name, *O. bifurcata*.

Vast numbers of the species observed by myself were found on the coast of New Guinea and in the Southern Pacific. The differences separating this from the species observed by Mertens are not to my mind sufficient to form the basis of any specific distinction, and as the description given by MM. Quoy and Gaimard, and by Chamisso, are too imperfect to establish any certain distinguishing characters in their case either, I shall consider that only one species has been observed. And as I can see no reason for the construction of a new and by no means euphonious name by Mertens, I shall retain both the generic and specific names given by Chamisso, *Appendicularia flagellum* (Chamisso), Syn. *Oikopleura Chamissonis* (Mertens), *Oikopleura bifurcata* (Quoy and Gaimard).

* De Animalibus quibusdam et classe Vermium, Fasc. Secundus. Nova Acta Acad. Cur. tom. x. 1821.

† In the Mémoires de l'Acad. Imp. de St. Pétersbourg, 1831.

‡ Zoology of the Astrolabe, vol. iv. p. 304.

80. The animal has an ovoid or flask like body, Plate XVIII. fig. 1, one-sixth to one-fourth of an inch in length, to which is attached a long curved lanceolate appendage or tail, by whose powerful vibratory motions it is rapidly propelled through the water.

The body frequently appears wrinkled and crumpled externally, and its upper, smaller extremity has rarely clear and well-defined edges. The lower part of the body is frequently separated from the upper by a slight cleft or constriction, fig. 4, and it is here that MERTENS places the orifice of the mouth, supposing indeed that the upper part of the body plays the part of a maxilla!

81. The smaller extremity of the animal is perforated by a wide aperture (*d*) which leads into a chamber, which occupies the greater part of the body, and at the bottom of this chamber is the mouth. The chamber answers to the respiratory cavity of the *Tunicata*, and is lined by an inner tunic distinct from the outer; the space between these, as in the *Salpæ*, being occupied by the sinus system.

On the side to which the caudal appendage is attached, an endostyle (*c*), altogether similar to that of the *Salpæ*, lies between the inner and outer tunics; and opposite to this, or on the ventral side, close to the respiratory aperture, there is a nervous ganglion, to which is attached a very distinct spherical auditory sac, containing a single, also spherical, otolithe. The sac is about $\frac{1}{300}$th of an inch in diameter. The otolithe about $\frac{1}{800}$th, figs. 1, 2, 4 *a*.

Anteriorly, a nerve is given off from the ganglion (*a*) which becomes lost about the parietes of the respiratory aperture; another large trunk passes backwards (*b*) over the left side of the œsophagus, and between the lobes of the stomach, until it reaches the appendage, along the axis of which it runs, giving off filaments in its course, fig. 2. I did not observe anything resembling the "languet" of the *Salpæ*; but MERTENS describes two leaf-like laminæ existing, one on each side of a "semicylindrical" body, which seems to be the nervous ganglion.

82. There is no proper branchia; but that organ seems to be represented by a richly ciliated band or fold (*e*) of the inner tunic, which extends from the opening of the mouth forwards, along the ventral surface of the respiratory cavity, to nearly as far as the ganglion; when it divides into two branches, one of which passes up on each side, so as to encircle the cavity (*f*). This circlet evidently represents the "ciliated band" of *Salpa*.

The mouth (*g*) is wide, and situated at the posterior part of the ventral paries of the respiratory chamber. The œsophagus (*h*) short, and slightly curved, opens into a wide stomach (*i*) curved transversely, so as to present two lobes posteriorly.

Between the two lobes, posteriorly, the intestine (*k*) commences, and passing upwards (or forwards) terminates on the dorsal surface just in front of the insertion of the caudal appendage (*l*).

The heart lies behind, between the lobes of the stomach. I saw no corpuscles, and the incessant jerking motion of the attached end of the caudal appendage rendered it very difficult to make quite sure even of the heart's existence.

83. MERTENS describes a vascular system, consisting of an aortic vessel, which runs forwards on the dorsal surface, and of a principal vein of a red colour, which passes to the ventral surface, and there divides into three branches, one of which runs forwards to the "semicylindrical body" (ganglion?), and the other two pass to the dorsal region.

A circular canal communicating with the aortic vessel exists, he says, on each side of the anus, and is connected with the ventral vessel by means of a vessel, through which no corpuscles were seen to pass.

I have seen nothing of this vascular system. The caudal appendage (A) is attached or rather inserted into the body on the dorsal surface just behind the anus. It consists of a long, apparently structureless, transparent, central axis (*m*), rounded at the attached, and pointed at the free end. This axis is enveloped in a layer (*o*) of longitudinal, striped, muscular fibres; which form the chief substance, in addition to a layer of polygonal epithelium cells, of the broad alary expansion on each side of the axis.

I did not observe the lateral canal containing air, described by MERTENS.

84. The only unequivocal generative organ I found in *Appendicularia* was a testis (*p*), consisting of a mass of cells developed behind and below the stomach, enlarging so much in full-grown specimens as to press this completely out of place.

In young specimens the testis is greenish, and contains nothing but small pale circular cells; but in adults it assumes a deep orange red-colour, caused by presence of multitudes of spermatozoa, whose development from the circular cells may be readily traced.

This orange-red mass, or rather masses, for there are two in juxtaposition, is described by MERTENS as the "Samen-behälter" or vesiculæ seminales. He describes them as making their exit, bodily, from the animal, and then becoming diffused in the surrounding water. This circumstance, indeed, appears to have furnished his principal reason for believing these bodies to be what the name indicates.

The spermatozoa have elongated and pointed heads about $\frac{1}{5000}$th of an inch in length, and excessively long and delicate filiform tails.

MERTENS describes as an ovary, two granulous masses, which he says lie close to the vesiculæ seminales, and have two ducts, which unite and open into this "ovisac."

This appears to me to be nothing more than the granulous greenish mass of cells and undeveloped spermatozoa, which exists in the testis at the same time as the orange-red mass of fully developed spermatozoa.

I saw nothing of any ducts, nor do I know what the "ovisac" can be, unless it be a further development of an organ which I found in two specimens (fig. 3 *q*), consisting of two oval finely granulous masses, about $\frac{1}{300}$th of an inch in diameter, attached, one on each side of the middle line, to the dorsal parietes of the respiratory cavity, and projecting freely into it.

MERTENS' "ovisac" has about the same position as these bodies, and he says he

saw "living animals" proceed from it, the part being afterwards evidently collapsed.

Unfortunately, however, he does not appear to have noticed the endostyle, whence confusion might readily arise; nor does he give the slightest hint as to the nature of the "living animals" which he saw come forth.

85. Still less am I able to give any explanation of the extraordinary envelope or "House" to which, according to MERTENS, each *Appendicularia* is attached in its normal condition. I have seen many hundred specimens of this animal, and have never observed any trace of this structure; and I have had them in vessels for some hours, but this organ has never been developed, although MERTENS assures us that it is frequently re-formed, after being lost, in half an hour.

At the same time it is quite impossible to imagine, that an account so elaborate and detailed, can be otherwise than fundamentally true, and therefore, as MERTENS' paper is not very accessible, I will add his account of the matter, trusting that further researches may clear up the point.

The formation of the envelope or "Haus" commences by the development of a lamina from the "semicylindrical organ" (ganglion?). This, as it grows, protrudes through the opening at the apex of the animal (respiratory aperture). Its corners then become bent backwards and inwards, and thus a sort of horn is formed on each side, the small end of which is turned towards the apex of the animal, while its mouth looks backwards, downwards and outwards.

At the same time two other horns are developed upwards (the animal is supposed to have its small end downwards), one on each side. These are smaller and more convoluted than the others.

This four-horned structure consists of a very regular network of vessels, in which, at the time of the development of the organ, a very evident circulation is visible; the blood-corpuscles streaming from the attached end of the organ. "The clearness with which the circulation was perceptible, together with the great abundance of vessels and the large extent over which they were spread, were circumstances which led me (says MERTENS) to believe this truly enigmatical structure to be an organ, whose function was the decarbonization of the blood. The ease with which the animal becomes separated from this organ is no objection to this view; the necessity there seems to exist for the reproduction of the latter rather confirming my opinion."

It is highly desirable that more information should be gained about this extraordinary respiratory organ, which, if it exist, will not only be quite *sui generis* in its class, but in all animated nature. And in a physiological point of view, the development of a vascular network, many times larger than the animal from which it proceeds, in the course of half an hour, will be a fact equally unique and startling.

86. As to the zoological relations of *Appendicularia*, its discoverer, as we have seen, considers that "it may possibly be allied to *Cestum*," a conjecture in which no one can possibly coincide.

Mertens, on the other hand, says, "The relation of this animal with the *Pteropoda* is unmistakeable; if the *Oikopleura* possessed two tail-like appendages, every one would recognize in them the wings of the *Pteropoda*;" and he proceeds to draw, what seems to me, a very forced comparison between *Oikopleura* and *Clio*.

I do not think that any one who has read the preceding pages will be at all disposed to agree with Mertens either.

87. For my own part, I think there can be no doubt that the animal is one of the *Tunicata*. The whole organization of the creature, its wide respiratory sac, its nervous system, its endostyle, all lead to this view.

In two circumstances, however, it differs widely from all *Tunicata* hitherto known. The first of them is, that there is only one aperture, the respiratory, the anus opening on the dorsum; and secondly, that there is a long caudal appendage.

As to the first difference, it may be observed, that, in the genus *Pelonaia*, an undoubted Ascidian, there are indeed two apertures, but there is no separation into respiratory and cloacal chambers. Suppose that in *Pelonaia* the cloacal aperture ceased to exist, and that the rectum, instead of bending down to the ventral side of the animal, continued in its first direction and opened externally, we should have such an arrangement as exists in *Appendicularia*.

With regard to the second difference, I would remark, that it is just the existence of this caudal appendage which makes this form so exceedingly interesting.

It has been long known that all the Ascidians commence their existence as larvæ, swimming freely by the aid of a long caudate appendage; and as in all great natural groups some forms are found which typify, in their adult condition, the larval state of the higher forms of the group, so does *Appendicularia* typify, in its adult form, the larval state of the Ascidians.

Appendicularia, then, may be considered to be the lowest form of the *Tunicata*; connected, on the one hand, with the *Salpæ*, and on the other with *Pelonaia*, it forms another member of the hypothetical group so remarkably and prophetically indicated by Mr. MacLeay, and serves to complete the circle of the *Tunicata*.

88. *Doliolum*.—This name was given by Otto* to a free-swimming gelatinous case, altogether structureless, of which a single example was found by him in the Gulf of Naples. Its nature is altogether unknown, for it is hardly justifiable, in the face of Otto's words, "Die Ränder sind aber völlig glatt ohne alle Spur von Zerreissung, nirgend sieht man inwendig Rauhigkeiten wo die Eingeweide angessessen haben könnten und die äussere Haut geht ohne Unterbrechung in die innere über," to assume with MM. Quoy and Gaimard, that it is only a *Biphore* whose intestines have been destroyed by a parasitic *Phronima*.

Furthermore, Otto states that the animal moved by a "worm-like contraction of its walls," which by no means describes the mode of contraction of the *Salpæ*, with which animals he was perfectly acquainted, and with a mutilated specimen of which,

* Nova Acta Acad. Curiosorum, t. xi. pars secunda, pp. 313 and 314.

he expressly states he might, except for its peculiar motion, have confounded the form he describes*.

MM. Quoy and Gaimard†, altogether denying the existence of Otto's genus as a distinct form, appropriated his name for two species of tunicate animals observed by them at Amboyna and Vanikoro, and which they justly recognized as being very closely allied to the *Salpæ*.

Of these two species, *Doliolum denticulatum* and *Doliolum caudatum*‡, the former is the only one with which I have met.

MM. Quoy and Gaimard give only the following short description:—

"Its form is nearly that of the vessel from which we have derived its generic name, that is to say, it is enlarged in the middle and narrowed at its two extremities where the openings are situated. The anterior opening is somewhat projecting and denticulated like a crown. Eight circles in relief surround the body at nearly equal distances. They have rather a polygonal than a circular form, and are probably vessels. In the interior the branchia is visible, divided into two portions, which have their oblique lamellæ upon a central vessel, as in the Pectinibranchiata. Near the union of the two divisions posteriorly is the heart, and between them (?) a vessel, the aorta, ascends; not far from the heart is a transparent nucleus. This is all that the vivacity of the mollusk, which bounded like an arrow through the water, allowed us to make out of its organization."

Although I cannot think that MM. Quoy and Gaimard have done well in appropriating Otto's name to an animal confessedly different from that which he describes, it will perhaps cause least confusion to follow their example.

The specimens which I examined were taken in the South Pacific, a little to the northward of Sydney, N.S.W., between Sydney and New Zealand, and in considerable numbers just at the entrance of the Bay of Islands.

89. *Doliolum denticulatum*, figs. 5, 6.—A small transparent body, varying in length from one-sixth to one-third of an inch, and looking very much like a barrel open at each end, which swims by contracting its whole body, and forcing the water out at one or the other extremity.

The apertures are considerably less in diameter than the central cavity. The anterior (*d*) is produced into a sort of tube, with about twelve rounded dentations,

* Prof. E. Forbes informs me that a body answering precisely to Otto's description, was found by him, occurring in considerable numbers, on the coast of Scotland, and was eventually discovered to be nothing more than the detached siphonic tubes of *Solenocurtis strigillatus*.

† Voyage de l'Astrolabe. Zoologie, t. iii. part. 2. p. 599.

‡ Little more than a description of the outward form is given by MM. Quoy and Gaimard of the *Doliolum caudatum*, but it strikingly agrees in everything with what one of the associated forms of the singular genus *Anchinia* might be supposed to become if set free; unfortunately, the description of the latter genus itself is very scanty. See note (60.)

Has the *Doliolum denticulatum* itself been ever an attached form? From certain appearances (90.) this appears very possible.

which are turned inwards. The base of the tube is surrounded by a thickened muscular rim.

The posterior extremity is similarly produced into a short tube with a thickened base, but the tube looks outwards, and its walls are very delicate, and consist of fine fibres like those of the fin of *Sagitta*.

90. The body of the animal consists of two tunics, an inner and an outer*, which surround a wide central respiratory cavity.

Six muscular bands (*t*), pretty nearly equidistant, gird the inner tunic.

In some specimens a sort of shrivelled tubular process projects on the dorsal surface posteriorly between the two last muscular bands. Is this the remains of an earlier pedicle of attachment?

91. A tubular endostyle (*c*) lies in the dorsal sinus between the first and third muscular bands.

In the ventral sinus a round ganglion (*a*) lies just in front of the third muscular band. It gives off several long nerves, four of which are especially remarkable, and run diagonally to the anterior and posterior apertures. There is no auditory sac nor otolithes.

92. The branchiæ divide the respiratory cavity into an anterior and a posterior chamber. They are formed by the epipharyngeal and hypopharyngeal (*x*) bands which stretch aross the respiratory cavity, supporting on each side a number of tubular bars (*y*). In the upper and lower division of the branchiæ, these bars are adherent to the walls of the respiratory cavity, *i. e.* to the inner tunic, and there their canals open into the lateral sinuses; but in the middle part of the branchiæ their extremities unite and form loops without adhering to the inner tunic, merely lying against it. There is a free passage for the water between the bars, and on each side of the central supporting bands.

The edges of the bars are richly ciliated, and the cilia of their opposite sides move in opposite directions.

Although it appeared quite certain that the canals of the bars communicated with the sinus system, yet no blood-corpuscles could be traced into them.

The branchial bars did not extend so far forward above as below. In the former case they reach as far as the second muscular band only, in the latter, beyond the first; seen from above or below, the branchia appeared as an oval plate, with a clear space down its middle and transverse bars on each side.

93. The mouth (*g*) opens in the middle of the upper division of the gill, just anterior to the fourth muscular band; a narrow œsophagus (*h*) leads from it into a two-lobed stomach (*i*); from this a narrow intestine passes, and bending a little upwards and then downwards and to the left side, terminates in a papillary (*l*) anus. Just at its bend

* The outer tunic, which I consider as homologous with the test and outer tunic of the Ascidian fused together, was found by MM. Löwig and Kölliker to contain cellulose, whence they concluded the Ascidian nature of the animal, a deduction strikingly confirmed by anatomical investigation.

the intestine gives attachment to three or four small cæca (*s*), which appear to represent a liver, and a system of transparent anastomosing tubules, similar to that described in *Salpa*, arises from the stomach and envelopes the intestine in a network.

The heart (*r*) lies above and in front of the mouth. In structure it resembles that of *Salpa*. There are no vessels of any kind, the blood-corpuscles making their way at random among the viscera. No reversal of the circulation was observed in this Ascidian.

94. All the specimens examined possessed only the male generative apparatus, in the shape of a long tubular* testis (*p*), placed on the right side and below, and opening posteriorly into the respiratory chamber by a papillary elevation (*p'*) just before the penultimate muscular band.

The testis lies quite freely in the sinus, and is bathed by the blood (fig. 7).

When most fully developed the testis nearly equals the body in length; but in young specimens it may be not more than one-half to one-third that size.

The young testis is a delicate sac, containing a mass of circular cells, about $\frac{1}{3800}$th of an inch in diameter, of a pale greenish colour, and flattened.

As development proceeds, these cells assume a redder tint, and become perfect spermatozoa, with elongated cylindrical heads $\frac{1}{2500}$th of an inch in length, and very delicate, long filiform tails.

95. There is a small cavity (*u*) resembling the ciliated fossa of the *Salpæ*, seated upon the anterior face of the singular process of the ventral paries of the respiratory cavity.

This process lies anterior to the first muscular band; it is somewhat conical and excavated behind. The two lips of the excavation are thickened and ciliated, and the right lip is continuous on the left side with a ciliated band, which runs up parallel with the first muscular band, passes over to the right side, and running down, becomes eventually lost in the right portion of the base of the conical process.

This would seem to be a rudimentary languet. A number of small granular masses were always to be seen attached to the inner tunic close to the posterior aperture.

The structure of the branchiæ of this Ascidian, the position of the two orifices, and the structure of the testis, all indicate a position for *Doliolum* intermediate between *Salpa* and *Pyrosoma*.

Its apparent unisexuality very likely arises from the ova being developed, and leaving the parent in a younger state than any I examined. I have elsewhere mentioned the liability to deception in *Salpa* from a similar cause.

Note.—Since writing the above I have found a short notice of *Appendicularia* in Müller's Archiv for 1846†, under the name of *Vexillaria flabellum*. The de-

* Very similar to that of *Salpa cristata*, described as an hepatic organ by Meyen.

† Bericht über einige neue Thierformen der Nordsee.

scriber, Prof. Müller, confesses that he does not know in what division of the animal kingdom to place this creature; and his account of its structure is not a little vague, including little more than its mere external appearance. He does not seem to have observed anything corresponding to the "Haus" of Otto.

Description of the Plates.

PLATE XV. *Salpa*.

Fig. 1. *Salpa* A.
Fig. 2. *Salpa* B.
Fig. 3. Dorsal view of *Salpa* A, the muscular bands being omitted.
Fig. 4. Dorsal view of *Salpa* B, the muscular bands being omitted.
Fig. 5. Intestinal canal of *Salpa* A.
Fig. 6. Intestinal canal of *Salpa* B.
Fig. 7. Dorsal view of extremity of *Salpa* B.
Fig. 8. Part of the respiratory chamber of *Salpa* B, showing the fœtus suspended. View from above.
Fig. 9. Connection of the gemmiferous tube with the heart.

PLATE XVI. *Salpa*.

Fig. 1. Young gemmæ (*Salpa* B) attached to the gemmiferous tube.
Fig. 2. Nuclear end of one of these, showing the ovum and its pedicle or gubernaculum.
Fig. 3. Nuclear end of a very young *Salpa* A, just detached.
Fig. 3ª. Placenta and gemmiferous tube of this enlarged.
Fig. 4. Heart, placenta and gemmiferous tube of a young *Salpa* A, showing the rudimentary condition of the last structure.
Fig. 5. Ganglion, otolithic sac and languet.
Fig. 6. Young *Salpa* A, still attached by its placenta in the interior of *Salpa* B.

PLATE XVII. *Pyrosoma*.

Fig. 1. A single "zoöid," viewed from the right side.
Fig. 2. A single "zoöid," viewed from above.
Fig. 3. Part of the branchial network.
Fig. 4. The ovum with its pedicle *in situ*.
Fig. 5. Testis and ovisac *in situ*, both emptied of their contents.

Fig. 6. Ovum and pedicle.
Fig. 7. A young zoöid developed by gemmation.
Fig. 8. A young zoöid separated and enlarged. Viewed from the ventral side.
Fig. 9. Youngest form of gemma observed.
Fig. 10. "Ciliated fossa," with the ganglion and otolithes.

a. Anterior extremity.
b. Posterior extremity.
c. Endostyle.
d. Ganglion and otolithes.
e. Gill band.
f. Languet.
g. Heart.
h. Gemmiferous tube (single gemma in *Pyrosoma*).
i. Nucleus.
k. Muscular bands.
l. Solitary fœtus, or young *Salpa* A.
m. Placenta.
n. Sinus running specially to the placenta.
o. Dorsal sinus.
p. Testis.
q. Ovary, or rather ovum.
q'. Pedicle.
r. Buccal orifice.
s. Anal orifice.
t. Lobe of the stomach.
u. Tubular system.
v. Branchial bars.
w. "Ciliated sac."
x. "Ciliated band."
æ. Eleoblast.
α. External tunic.
β. Internal tunic.
γ. Partition of gemmiferous tube.
δ. Cell masses in *Pyrosoma*.

PLATE XVIII.

Fig. 1. *Appendicularia flagellum*. Much magnified.
Fig. 2. Still more magnified.
Fig. 2*a*. Extremity of the caudal appendage.
Fig. 3. Body of *Appendicularia* from behind.
Fig. 4. Individual in which the testis is much enlarged.
Fig. 5. *Doliolum denticulatum*, from the right side.
Fig. 6. *Doliolum denticulatum*, from below.
Fig. 7. A portion of the right wall to show the testis *in situ*.
Fig. 8. The "ciliated sac" and the origin of the "ciliated bands" in *Doliolum*.
Fig. 9. The intestine and heart, with the commencement of the branchiæ.

The letters have throughout the same signification.

a. Ganglion with the auditory vesicle.
b. Nerve.
c. Endostyle.
d. Respiratory or anterior aperture.
e, f. Ciliated bands.
g. Mouth.
h. Œsophagus.
i. Stomach.

k. Intestine.
l. Anus.
m. Axis of the caudal appendage.
n. A long membrane of appendage.
o. Bundles of striated muscular fibrils.
p. Testis.
p'. Efferent duct of testis.
q. Supposed ovary.
r. Heart.
s. Liver.
t. Muscular bands.
u. Ciliated sac.
B. The body of *Appendicularia*.
A. The caudal appendage.
x. Hypopharyngeal band.
y. Branchial bars.
z. The system of tubules embracing the intestine.

PLATE XIX.

The diagrams represent imaginary sections of the principal types of the Ascidian family. Without pretending to be strictly accurate, they are sufficiently so to give a just idea of the gradations in structure among the different genera, and of the essential unity of structure which runs through the group.

XXVI. *Researches into the Structure of the Spinal Chord**.

By J. Lockhart Clarke, *Esq.* *Communicated by* Samuel Solly, *Esq.*, *F.R.S.*

Received October 15,—Read December 5, 1850.

By reflecting on certain facts connected with the respiratory movements, and which seem to derive no explanation from our actual knowledge of the structure of the spinal chord, I was induced to undertake a series of observations with the view of determining, if possible, the relations which appear to subsist between the spinal nerves and the respiratory nervous centres. These observations, however, led me into a more extended inquiry than I at first contemplated, the results of which I now venture to lay before the Royal Society.

It is needless to point out the difficulties which attend, not only the minute investigation, but a clear and connected description, of a structure so intricate and delicate as that of the spinal chord. It may be proper however to state, with regard to the contents of this paper, that those facts only which were verified by cautious and repeated examination have been brought forward with confidence; while in cases where the results of my observations were less satisfactory, I have expressed myself with corresponding reserve. Yet, no labour has been spared in order to arrive at correct conclusions. My observations were made, by means of one of Mr. Ross's finest microscopes, on many thousand preparations of the spinal chord of Man, of the Calf, Sheep, Pig, Dog, Cat, Rabbit, Guinea Pig and Frog, and occupied a period of more than two years.

In making these preparations, the two following methods were adopted:—a perfectly fresh chord was hardened in spirits of wine, so that extremely thin sections, in various directions, could be made by means of a very sharp knife. A section so made was placed on a glass slide and treated with a mixture composed of one part of acetic acid and three of spirits of wine, which not only makes the nerves and fibrous portion more distinct and conspicuous, but renders also the grey substance much more transparent. The section was then covered with thin glass, and viewed,

* At the time these inquiries were begun, and nearly up to the period of their completion, I had not seen any of the works of Dr. Stilling, and knew no more of his investigations into the structure of the spinal chord than what I had gathered from the anatomical and physiological works published in this country. However nearly, therefore, a few of the facts brought forward in this paper may correspond to the results of his inquiries, I may state that they were made out without any knowledge of Dr. Stilling's views, excepting only those which regard the origin of the spinal nerves. On showing some of my preparations to Mr. Solly and Mr. Grainger, I was advised by those gentlemen to look over the works of Dr. Stilling; and having done so, I shall refer to them whenever they appear to oppose, or coincide with, my own observations.

first by reflected light with low magnifying powers, and then by transmitted light with higher ones.

According to the second method, the section is first macerated for an hour or two in the mixture of acetic acid and spirit. It is then removed into pure spirit, and allowed to remain there for about the same space of time. From the spirit it is transferred to oil of turpentine, which expels the spirit in the form of opake globules, and shortly (sometimes immediately) renders the section perfectly transparent. The preparation is then put up in Canada balsam and covered with thin glass. By this means the nerve-fibrils and vesicles become so beautifully distinct that they may be clearly seen with the highest powers of the micoscope. If the section be removed from the turpentine when it is only semi-transparent, we sometimes obtain a good view of the arrangement of the blood-vessels*.

All the observations described in this paper were made and verified also on sections in a perfectly fresh state and unaffected by any chemical re-agent; and whenever it was proposed to examine the natural structure of any particular part, as the nerves and vesicles, a specimen was selected from an animal immediately after death.

The drawings have been executed by myself with the greatest care, and may be relied on as faithful and exact delineations of what was seen under the microscope. The outlines of all were taken by means of a camera lucida, in order to ensure correctness.

Of the Grey Substance of the Spinal Chord.

At the lower end of the spinal chord the posterior grey substance consists only of a single mass, having a broad and rounded extremity, and formed, as we shall presently see, by the coalescence of the two posterior cornua along their inner borders, where, being increased in breadth, they meet each other in the middle line (see Plate XX. fig. 1). In consequence of this arrangement, the *substantia gelatinosa* extends uninterruptedly and horizontally across from one side to the other. From the posterior white columns it is separated, particularly on each side, by a kind of border composed of fine nerve-tubes, which wind round it and proceed from the posterior roots of the nerves.

The anterior grey substance, on the other hand, is here, as in other regions of the chord, separated by the anterior white columns into two cornua of considerable size. These curve somewhat inwards, taper to an irregularly rounded point, and nearly reach the circumference of the chord.

Here also the spinal canal is very distinct and large, being the $\frac{1}{100}$th part of an

* This mode of preparation succeeds best in cold weather; for in summer, the chord, however fresh when immersed in the spirit, remains more or less spongy, instead of becoming firm and dense in the course of five or six days. The spirit should be diluted with an equal quantity of water during the first day, after which it should be used pure. Certain modifications of this mode of preparation may be sometimes employed with advantage by a practised hand.

inch in diameter. It is situated much nearer the anterior than the posterior surface of the chord*.

The nerve-vesicles however are few in number, and scattered irregularly through the grey substance. They are not, as in some other regions, collected into large masses, but are most numerous in the anterior cornua, at the extremities of which three or four may be seen closely grouped together.

Nature and Arrangement of the Nerve-fibres in the Grey Substance of the Spinal Chord.

The nerve-fibres in the grey substance of the spinal chord are mostly of the white tubular kind, and of variable but small average diameter†. With regard to *direction*, they may be divided into two classes,—*transverse* and *longitudinal*.

Transverse Fibres.—These again may be divided into two orders,—*antero-posterior* and *latero-transverse*.

Antero-posterior Fibres.—On each side of the middle line, in the posterior grey substance, are several large bundles of fine nerve-tubes, which take a transverse direction from behind forwards (see Plate XX. fig. 1). These bundles may be traced through the posterior white columns, partly into the roots of nerves which arise in the same vertical (in Man, horizontal) plane, and partly into others above and below them. They are convex towards the middle line, but on approaching the anterior cornua, they break up into smaller bundles, which interlace with each other and form a coarse network, in the meshes of which the nerve-vesicles are contained. Many of these nerve-tubes, both singly and in bundles, extend outwards into the antero-lateral white columns‡. Many of them, also, appear to be continuous with the anterior roots of the nerves; but whether they be so or not, the anterior and posterior roots are mingled together in the network above mentioned§. Between the spinal canal and the anterior median

* As stated by Stilling and Wallach, it extends uninterruptedly through the whole length of the medulla spinalis. In many preparations that I have by me of the filiform extremity of the chord, where it measures less than a line in diameter, the spinal canal is even larger than in other regions.

† By Drs. Stilling and Wallach they are described as grey fibres, but seem to have been examined under strong pressure. Moreover, they ought to be examined in a perfectly fresh state, as spirits of wine considerably alters their appearance.

‡ After nearly reaching the circumference of the chord, they *appear* to form with each other a series of loops of various sizes; but of this arrangement I cannot speak with any confidence, for it is seen only in a section of a perfectly fresh chord, and under strong pressure, to which the appearance may be due. Stilling (Textur des Rückenmarks, p. 21) also states that the transverse (so-called) *grey* fibres form loops at the periphery of the chord, but since he made use of strong compression, his statement is open to the same objection.

§ This description corresponds, in some measure, with that given by Dr. Stilling (Ueber die Medulla Oblongata, p. 4), who states that the posterior nerves, after forming loops within the grey substance, become continuous with the anterior roots. I have never seen loops such as he describes, but the rest of his statement is probably correct; for although it is impossible to trace any *single* fibre from the posterior to the anterior roots, it is nevertheless certain that the latter are continuous in the grey substance with *bundles* which proceed from the former. According to Stilling, therefore, the anterior and posterior roots of the spinal nerves are united in the grey substance by intervening *grey* fibres, since all the transverse fibres are described as such; or to use

fissure, some of these fibres cross obliquely through the anterior white columns, and decussate with corresponding fibres from the opposite side (see Plate XX. figs. 1, 3, &c.).

Latero-transverse Fibres.—The second order of transverse fibres take a more or less horizontal direction between the opposite sides of the grey substance, forming the transverse commissure, and are arranged chiefly in loose bundles (see figs. 2 and 3). Those behind the spinal canal diverge on each side into the anterior and posterior grey masses, and extend into both the posterior and lateral white columns; a great number being evidently continuous with both the anterior and posterior roots of the nerves. One large bundle, in the centre, is particularly conspicuous (see figs. 2 and 3). Those in front of the spinal canal are much less numerous and curve principally forwards: they may be traced partly into the antero-lateral columns, and partly into the anterior roots of the nerves.

By Dr. Stilling some of the fibres of the posterior nerves are said to cross over, behind the spinal canal, into the anterior cornua of the opposite side. I have many times thought that such an arrangement really exists, but the inquiry is attended with great difficulty, and my own observations, though frequently repeated, are too uncertain in their results to admit of my confirming or denying the existence of these fibres. Hanover denies that there is any crossing of the fibres in the spinal chord. He says, "Une partie des fibres passe bien d'un côté de la moëlle épinière à l'autre, mais un entre-croisement général n'a pas lieu*."

Longitudinal Fibres.—The fibres of the *substantia gelatinosa* are, as usually described, chiefly longitudinal. When examined immediately after death, and uninjured by pressure or traction, they have all the characters and appearance of minute tubular fibres, presenting dark, single, and sharply-defined outlines, with frequent varicosities; but when stretched, or otherwise injured by manipulation, they lose their dark borders, and become exceedingly pale and delicate†. They vary but little in size, but are all extremely slender, their average diameter not exceeding the $\frac{1}{10,000}$th of an inch: some few are larger, but I have seen numbers as small as the $\frac{1}{15,000}$th or $\frac{1}{16,000}$th of an inch in diameter, as shown by the micrometer. They are usually distinguished as grey fibres, but differ in many respects from the grey fibres contained in the sympathetic system of nerves; for they are not only, on an average, finer, but, like the tubular fibres, are little or not at all affected by acetic acid, which fails to bring out any traces of nuclei‡. They have also been said to resemble the processes of the

his own words "Die Primitivfasern der Nervenwurzeln nichts Anderes sind als die unmittelbaren Fortsatze der queerlanfenden grauen Substanz des Rückenmarks" (Textur des Rückenmarks, p. 28). The transverse fibres, however, are really white tubules of smaller diameter than those which form the roots of the nerves.

* Recherches Microscopiques sur le Système Nerveux, p. 14.

† The method adopted for examining these fibres was as follows:—a thin longitudinal section through the *substantia gelatinosa* was made by a rapid stroke with a very sharp instrument, wet with albumen. The *substantia gelatinosa* was then carefully separated from the parts on either side of it, and examined without any pressure beyond that produced by the weight of the thin glass employed to cover it.

‡ Remak states that they are nucleated. Drs. Stilling and Wallach describe them as grey nerve-tubes

caudate vesicles, but in their natural state they certainly bear no resemblance to them whatever. Around the extremity of the spongy portion of the posterior cornua, these longitudinal fibres are collected into an arched and very dense band, of the same form, and about the same thickness as the *substantia gelatinosa*, with the lower border of which it is continuous. This band may be seen even with the naked eye, and then appears as a white opake stripe. The longitudinal fibres are also found, but in smaller numbers, through the rest of the grey substance.

Changes in the Form of the Grey Substance.

On examining the chord upwards towards the lumbar enlargement, certain modifications are found to take place in the form and disposition of the grey masses. The two halves of the posterior mass, with its two sets of antero-posterior bundles of tubular nerve-fibres, begin to separate from each other at the middle line (see figs. 2 and 3). Here the *substantia gelatinosa* is first interrupted, and the posterior border of the grey substance between its divided central extremities, or rather, the posterior fibres of the transverse commissure, begin to arch forwards towards the spinal canal, and to mark out the rudiments of the posterior cornua. At the same time, on each side, and a little behind, the spinal canal, a small but gradually increasing mass of caudate vesicles makes its appearance. These are the commencement of two longitudinal columns of vesicular substance, which extend through the whole length of the spinal chord, and which I name the *posterior vesicular columns*. They are traversed and surrounded by fibres from the posterior roots of the nerves, and also by arched fibres of the posterior transverse commissure, some of which, as already stated, are continuous with the anterior roots of the nerves. The changes just described continue to increase from below upwards, and reach their greatest extent in the middle of the lumbar enlargement, where the posterior cornua are broad and long, and widely separated from each other by the posterior white columns of the chord; while the space between the posterior border of the transverse commissure and the spinal canal, and which is almost entirely occupied by the former structure, is much reduced in breadth, for it measures only the $\frac{1}{180}$th of an inch, whereas at the lower extremity of the chord its diameter reaches the $\frac{1}{10}$th of an inch (see Plates XX. and XXI. figs. 1 and 6). At this central constricted portion, therefore, of the transverse commissure, its fibres, which before were but loosely arranged, are now compressed into a dense band, but still diverge on each side into the anterior and posterior cornua. In consequence of these changes, also, the two *posterior vesicular columns*, which have increased considerably in size, and were formerly situated behind, and at the sides of, the spinal canal, are now pushed up, as it were, and included in the

presenting a very small diameter and a bright golden colour; but add, that they are not easily examined with accuracy. Repeated and careful examination, however, has convinced me that the above description is correct; their golden colour, when seen by transmitted light, is due only to the granular matrix in which they lie; when viewed by direct light, they have an opake white appearance.

posterior cornua, some of their vesicles reaching as high as the *substantia gelatinosa*. As a further result of the same changes, the posterior white columns are much deeper and broader here than in the lower regions of the chord (compare Plates XX. and XXI. figs. 3, 4, 5 and 6).

While the posterior masses of the grey substance are undergoing the modifications just described, a series of somewhat similar alterations are found to take place in the form and arrangement of the anterior masses. In the middle of the lumbar enlargement, the anterior cornua have increased to a still greater extent than the posterior, and have assumed a shape the opposite of that which they possessed lower down in the chord. They now turn rather outwards instead of inwards, and have a large, irregularly club-shaped, instead of a pointed, extremity. The caudate vesicles have become exceedingly numerous, and are grouped together in several large masses, which are situated chiefly on the outer and middle parts of the cornua (fig. 6). They lie in the meshes of a net-work formed by bundles of fibres proceeding from the anterior and posterior roots of the nerves, and from the commissural bands behind the spinal canal*.

On proceeding upwards from the lumbar enlargement into the dorsal region of the chord, arrangements in the grey substance are observed to take place in the reverse order of those already described. By degrees the posterior cornua are reduced in length, and somewhat modified in shape; their inner sides extend towards each other and the middle line, while the posterior bands of the transverse commissure are drawn, as it were, gradually backwards, becoming at the same time less curved; so that the space between them and the spinal canal is now correspondingly increased. Into this space the two *posterior vesicular columns* advance between the fibres of the transverse commissure, together with the inner sides of the posterior cornua, which, in the middle of the dorsal region, coalesce and inclose them behind. (Compare Plates XXII. and XXIII. figs. 7, 8, 9 and 10.). Here, then, the posterior grey substance again consists only of a single mass, and again, also, the *substantia gelatinosa* extends uninterruptedly and nearly horizontally across from side to side. On the other hand, the anterior cornua are long, straight and narrow, and project almost directly forwards. The caudate vesicles, reduced considerably in number, are scattered irregularly through them, but are more numerous towards their extremities, where they are sometimes seen in the form of one or two small groups.

From the middle of the dorsal region to the cervical enlargement of the chord, the alterations in the form of the grey substance are once more reversed, being nearly similar to those which are found to take place on proceeding upwards from its lower extremity. The posterior mass renews the process of division into two parts, commencing with the *substantia gelatinosa* at the middle line, where the transverse com-

* It is interesting, in a physiological point of view, to find that the number of caudate vesicles is in direct proportion to the size of the nerves, which are known to be much larger at the lumbar and cervical enlargements than in other regions.

missural bands are gradually pressed, as it were, forwards, and thus mark out the posterior cornua; while in these, which again reach their greatest size in the middle of the cervical enlargement, the *posterior vesicular* columns, formerly situated behind, and at the sides of the spinal canal, at length resume their position and increased dimensions (see fig. 11). During the progress of these changes, the anterior cornua also undergo considerable modifications. They become gradually broader and longer, and contain a much larger number of vesicles, which in the cervical enlargement are grouped together, as in the lumbar region, into several large masses; indeed, the general arrangements of the grey substance at these two parts of the chord have a very striking resemblance*.

At the outer border of the grey substance, between the anterior and posterior cornua, is a small column of vesicular matter, which is softer and more transparent than the rest. This consists of caudate vesicles of inferior size and more regular shape than those usually found in the spinal chord. It is seen distinctly at the upper part of the lumbar enlargement (see Plates XXII. and XXIII. figs. 7 and 8 *m*), and increases somewhat in size in the dorsal region, where it projects slightly into the lateral column on each side. In the cervical enlargement it is less distinctly marked, but higher up it again becomes conspicuous, and is there seen to form the principal part of the nucleus of the spinal-accessory nerve (see fig. 12). On ascending the medulla oblongata, this column of vesicles gradually makes its way inwards till it reaches the space immediately behind the spinal canal, where it meets and blends with its fellow of the opposite†.

In the upper part of the cervical region the posterior cornua are long and narrow. The outer part of each, below the *substantia gelatinosa*, consists of a large and remarkably beautiful network of blood-vessels, which incloses in its meshes bundles of fibres of the lateral columns (see Plate XXIV. fig. 12). These bundles are of various sizes, and encroach gradually upon the cornua as they ascend to the medulla oblongata. Some of the fibres of the spinal-accessory nerve run transversely through this network; but a considerable branch, on entering the grey substance, bends *forwards*, and after passing through its vesicular nucleus, continues the same course *within the anterior cornu, where its fibres traverse and surround the caudate vesicles, in company with the roots of the anterior spinal nerves* (see Plate XXV. fig. 13). The space behind the spinal canal has slightly increased in breadth, and the *posterior vesicular columns* on each side of it have nearly the same position and relations as in the dorsal region. The anterior cornua are rather small and pointed, and contain each a circular or oval mass of vesicles.

The central portion of the grey substance surrounding the spinal canal is described

* The vesicles are more abundant, however, in the lumbar than in the cervical enlargement, and therefore correspond in number to the relative size of the nerves which belong respectively to these regions.

† The nucleus of the spinal-accessory nerve in the medulla oblongata has been already correctly described by Dr. Stilling, Die Medul. Oblong.

by Foville* as the grey commissure of the chord; and by Stilling† as a circular commissure composed of exceedingly delicate grey nerve-fibres. To me, however, its structure appears to consist of a circular layer of extremely fine fibrous tissue (for supporting the sides of the canal), left unobscured by the transverse commissural fibres which arch round it in front and behind. Indeed, it may be seen, at some parts of the chord, to be continuous with the areolar tissue which extends from the borders of the anterior fissure, through the white columns, in front of the canal. It varies, also, in shape at different parts of the chord, being fusiform from side to side in the higher regions, where the curves described by the anterior and posterior transverse commissures are least; and nearly circular in the lower regions, where those curves are greatest. The walls of the spinal canal are lined with a layer of columnar epithelium, which in a transverse section of the chord, viewed by direct light, appears as a white opake ring at the margin of the foramen‡.

Of the Nerve-vesicles of the Spinal Chord.

The vesicles found in the grey substance of the spinal chord are either circular, oval, pyriform, or otherwise irregular in shape; and all of them, except those peculiar to the *substantia gelatinosa*, have remarkably delicate processes issuing from their sides, like the neck from a flask or funnel (see Plate XXV. fig. 15). They are found chiefly in the anterior cornua and in the *posterior vesicular columns*§. They are also connected with each other by their processes, which divide and subdivide into smaller branches, so that the space between them appears to be occupied by a minute network of the most delicate fibrils‖. Many of these processes, particularly from vesicles situated near the border of the grey substance, run out into the white columns, through fissures which contain blood-vessels and pia mater. Whether they give off branches which follow the vascular network through the white columns, I have not been able to determine. That the caudate vesicles have some important relation to the functions of the nerves, there is every reason to believe, since we find that they not only *invariably exist in the vicinity of nerves*, but, as already shown, that *they increase also in number in direct proportion to the size of the nerves with which they are associated*. I am aware that several continental physiologists of eminence assert

* Traité Complet de l'Anatomie, &c. du Système nerveux Cerebro-spinal.

† Textur des Rückenmarks, p. 23.

‡ It was suggested to me by Mr. Bowman, of King's College, that as the spinal canal is continuous with the fourth ventricle, it is probably lined with epithelium, which, on careful examination, I found to be the case.

§ The situation and connections of these two longitudinal columns of vesicular substance render them extremely interesting. They would seem to have some intimate relation to the functions of the posterior roots of the nerves, many of which, as already stated, traverse and surround them, without, however, forming with their vesicles any apparent connection. At the upper part of the medulla oblongata they are reduced in size and ultimately disappear. The processes radiate from their vesicles on every side; some extending to the extremities of the posterior cornua, and others into the lateral and posterior white columns.

‖ A somewhat similar description is given by Todd and Bowman, Physiological Anatomy, p. 214.

that they have discovered in the lower vertebrata and in the invertebrata a direct union between the caudate vesicles and the tubular fibres; but I have never been able to make out satisfactorily any such connection in the spinal chord of mammalia, although I have constantly sought for it under very favourable circumstances*. The nerve-fibrils wind around, and apparently in contact with, the vesicles, but the connection does not seem to have at all the character of an attachment. Nor have I succeeded in tracing anything like continuity between the tubular fibres and the *processes* of the vesicles. It is very common to see one or two of these processes running outwards into a bundle of nerves attached either to the anterior or posterior cornua; but then we have seen that they pass out into the white columns from *all* sides, and therefore from parts which are not connected with nerves.

Blood-vessels of the Spinal Chord.—These enter through the anterior and posterior median fissures, through the smaller fissures in the white columns, and at the roots of the nerves. After giving off numerous branches to the white columns, they proceed inwards to the grey substance, along the whole periphery of which they form a remarkably beautiful network of loops†, intermixed with nerve-tubes and some of the processes of the vesicles. Within the grey substance they form an extremely minute network. Some of the vessels which enter at the anterior and posterior fissures and at the posterior white columns, anastomose with each other around the spinal canal; while others branch off, right and left, between the bundles of the transverse commissure, into the anterior and posterior cornua, where they follow the course of the transverse fibres, and running to the periphery of the grey substance, assist in forming there the loops already described. A series of loops is also found to exist near the extremity of the posterior cornua, along the border of the spongy substance.

Of the White Columns of the Spinal Chord.

The anterior white columns of the chord have no proper transverse commissure, but are united in the middle line, at the bottom of the fissure, by a fibrous band or raphè. In this situation, however, they are crossed transversely by horizontal and oblique tubular nerve-fibres and blood-vessels proceeding from the grey substance on either side, and which, in some regions of the chord, are so numerous that they nearly replace the longitudinal fibres‡. The transverse slits observed by Foville on each side of the raphè are fissures for the passage of blood-vessels.

* Hanover is, I believe, the only observer that professes to have seen it in all classes of vertebrata.

† According to Dr. Stilling this network consists of the (so-called) grey transverse fibres. He confesses, however, that it is not unlike a network formed of small blood-vessels.—Textur des Rückenmarks, p. 22.

‡ Stilling maintains that the separation of the anterior white columns is complete, for at the bottom of the fissure may be seen "the transverse grey fibres of the anterior commissure."—Ueber die Medul. Oblong. p. 6. The fibres, however, which give to this structure its grey or yellowish-grey appearance, are really blood-vessels and pia mater.

Nor are the posterior columns of the chord connected by a transverse commissure; for the posterior fissure, as stated, first, I believe, by Bellingeri*, reaches down to the border of the grey substance, or rather, to the posterior border of its transverse commissure, which has been mistaken and described by Foville as a commissure between the posterior white columns, formed by their coalescence at the bottom of the fissure†.

Of the Origin of the Spinal Nerves.

Posterior Roots.—The bundles of fibres which form the posterior roots of the spinal nerves are much larger, but less numerous, than those of the anterior; the fibres themselves, however, are mostly finer and more delicate‡. They are attached immediately to the *posterior* white columns *only*§, which they traverse obliquely inwards, interlacing and forming with each other an intricate plexus (see figs. 6 and 14). From this plexus straight and distinct bundles‖ enter the posterior cornua along their whole breadth, and cross the *substantia gelatinosa* both obliquely and at right angles; some being immediately continuous with fibres of the transverse commissure, while others, after plunging into the spongy portion of the cornua to unequal depths, break up and form a finer net-work which extends forwards into the anterior cornua: some of their fibres, after traversing the grey substance, pass out, as already described, into the posterior and lateral white columns.

Anterior Roots.—These are attached exclusively to the anterior parts of the antero-lateral columns. They do not always enter the chord directly, or at the points of

* De Medullâ Spinali, pages 7 and 8.

† He observes "Elle semble plutôt la simple coalescence des deux faces latérales, réunies au fond du sillon." *Op. cit.* p. 134.

‡ I find that this statement coincides with that previously made by Remak (Müller's Archiv, 1836) and by Henle, Allg. Anat. p. 669.

§ I believe that this statement is directly opposed to the opinions of almost every anatomist, except Sir Charles Bell, that has written on the subject of the spinal chord; and were it not undeniably proved to be true by the preparations which accompany this paper, I should feel some hesitation in coming forward to oppose so many eminent and recent authorities. According to Bellingeri (*op. cit.*) the posterior roots are attached to both the posterior and lateral, and the anterior roots to the anterior and lateral, white columns. Mr. Grainger and Mr. Swan join in referring the origin of both to the lateral columns only: the former observes (Spinal Chord, p. 30), "I have never been able to trace any fibres from the nerves into the fasciculi composing the anterior and posterior columns." A nearly similar statement is made by Dr. Todd. The two other most recent writers on the subject (Drs. Sharpey and Carpenter) have adopted the same views as Bellingeri. By Sir C. Bell (whose conclusion, however, was probably drawn from superficial examination) these roots are referred to the posterior columns only. "Each nerve," says he, "has two distinct series of roots coming out in packets or fasces, one from the posterior column, and one from the anterior column, of the spinal marrow."—Nervous System, p. 29, 4to.

‖ The primitive fibres of most of these bundles are very small, their average diameter being about the $\frac{1}{1000}$th of an inch; a few very large tubules are found amongst the rest, and here and there is seen a slender bundle composed of fibres of larger average size. The anterior roots of the nerves within the grey substance are composed of fibres of more irregular size, and contain a much greater number of large tubes than those of the posterior roots.

attachment, but sometimes run for a short distance transversely along its circumference (see Plate XXI. fig. 6). They then traverse the antero-lateral columns somewhat obliquely, and in straight and distinct bundles, which do not, like those of the posterior nerves, cross and interlace each other, but proceed directly to the anterior grey cornu*. Their connection with the latter is still doubted by many physiologists of eminence, but in the preparations which illustrate this paper, it may be seen as clearly and satisfactorily under the microscope, as their connection with the white columns is perceived by the naked eye†. On reaching the grey cornua, they break up into smaller bundles and separate fibrils, which diverge in various directions. Some of the fibres proceed to the right and left, crossed by others from neighbouring bundles (see Plates XXIII. and XXV. figs. 14 and 15). Of those proceeding within the external border of the cornu, a few pass out to the antero-lateral columns; while others, after winding round groups of caudate vesicles, curve inwards and join the fibres of the transverse commissure. Of those proceeding along the inner border of the cornu, a few pass into the white column at the side of the median fissure; while others, on reaching its base, curve round, and, in company with fibres from the posterior cornu, cross obliquely through the junction of the anterior columns in front of the spinal canal, where they decussate with corresponding fibres from the opposite side. The remaining bundles of the roots plunge into the central portion of the cornu, and winding among its vesicles, are lost in the intricate network (see fig. 14). A few of their fibres, after proceeding to some depth in the grey substance, bend round and take a more or less longitudinal direction.

* I have not yet ascertained whether any of the fibres of the spinal nerves ascend with the white columns without entering the grey substance, as maintained by Mr. Grainger, Mr. Solly, and Dr. J. Budge.

† Dr. Sharpey, one of the most recent authorities on the subject, observes, "The anterior roots have been said to reach the anterior grey cornu, by passing through the superficial stratum of white substance over it, *but actual demonstration of the fact is yet wanted.*"—Quain's Anat. 5th edit. p. 727. Bellingeri states merely that some of the anterior roots *perhaps* reach as far as the grey substance. His account of their origin is as follows: "Cernuntur filimenta radicum anteriorum nervorum spinalium partim exoriri à fasciculis anterioribus medullæ spinalis, et quidem filimenta aliqua nervea directè secedunt à superficie medullæ spinalis; alia verò filimenta nervea per canaliculum a piâ matre suppeditatum profundè descendunt in substantiam albidam medullæ, et *fortassè* nonnulla usque ad cinereum substantiam perveniunt."—*Op. cit.* p. 50. It is well known that Mr. Grainger was the first who professed to have traced the connection of the anterior roots of the nerves with the grey substance of the chord; and that Dr. Stilling has since described and represented it; but the statements of these eminent men failed to convince most of the physiologists of this country. I may take this opportunity, however, of observing, that we are much indebted to Dr. Stilling for having thrown considerable light on some of the most obscure points in the anatomy of the spinal chord and medulla oblongata; and for having originated and suggested new methods of further inquiry. His researches prove him to be not only a skilful, ingenious and laborious investigator, but, in general, a correct and acute observer. The plates attached to his second work, although a little embellished, are, with a few trifling exceptions, exceedingly correct.

Of the Nerve-tubes of the Spinal Chord.

The nerve-tube is endowed with considerable elasticity, and may be drawn out and attenuated to a great extent. It then loses its thick and sharply-defined borders, or double outline, and assumes the appearance of a much finer and more delicate fibre. When partially broken by traction, the two extremities are connected often by one of its borders only, and sometimes only by the axis cylinder, which has then the appearance of an almost imperceptible, shadowy line; in the unbroken portion of the fibre, however, the axis cylinder appears perfectly pellucid.

The following summary of the principal facts described in this paper may perhaps be found useful.

That the posterior grey substance, at the lower extremity, and in the dorsal region, of the spinal chord, consists only of a single mass; and that the *substantia gelatinosa* there extends uninterruptedly across from one side to the other.

That the nerve-fibres of the grey substance, including those of the *substantia gelatinosa*, are not grey fibres bearing nuclei, like those of the sympathetic, but fine tubules.

That two considerable columns of caudate vesicles (which I have named the *posterior vesicular columns*) in intimate connection with the posterior roots of the nerves, extend through the whole length of the chord; commencing small at its lower extremity, increasing in size in the lumbar and cervical enlargements, and terminating at the upper part of the medulla oblongata.

That the number of caudate vesicles, particularly in the anterior grey substance, is in direct proportion to the size of the nerves.

That the column of vesicles into which, in the cervical region, the spinal-accessory nerve may be traced, extends down the chord as far as the lumbar enlargement.

That a considerable branch of the spinal-accessory nerve, after entering the grey substance through the lateral column, may be easily traced to the caudate vesicles of the anterior cornu.

That the spinal accessory is the only nerve immediately attached to the lateral column.

That the posterior roots of the spinal nerves are immediately attached to the posterior white columns only; and the anterior roots to the anterior columns only; but,

That fibres from both these roots, after traversing certain portions of the grey substance, pass out again into the white columns.

That neither the anterior nor posterior white columns are connected by a transverse commissure.

That the central portion of the grey substance immediately surrounding the spinal canal is not a commissural structure, but a layer of fine fibrous tissue for supporting the walls of the canal, which is lined with a layer of columnar epithelium.

I have now stated all that I have to say at present on the subject of the spinal chord; but if the observations contained in this communication be considered by the Royal Society to throw important light on its structure, they will be followed by others that I have made on the medulla oblongata and cerebellum. I cannot conclude, however, without acknowledging the kind interest shown me, during these inquiries, by Mr. Solly, Mr. Grainger, and Dr. Marshall Hall. I am also much indebted to Mr. Frederick Brown (Member of the Microscopical Society) for several ingenious contrivances with which he kindly furnished me.

Explanation of the Plates.

PLATE XX.

The Plates represent transverse sections of the spinal chord of the Calf, all of which, except the two last, were prepared according to the first method described in the paper. The same letters indicate the same parts in all the figures.

Fig. 1. A section of the white and grey substances of the filiform extremity of the chord, scarcely more than one line in diameter; magnified 20 diameters. *aa*. Posterior white columns; *bb*. Antero-lateral columns of one side; *c*. Posterior lateral fissure, which separates the posterior from the antero-lateral white columns; *d*. Anterior median fissure; *d'*. Posterior median fissure; *e*. Foramen; *ff*. Anterior cornua; *gg*. *Substantia gelatinosa*. On each side of the middle line are seen antero-posterior bundles of fine tubular fibres.

Fig. 2. A section of the grey substance of the chord about an inch higher up; magnified 20 diameters. *h*. Arched band of tubular fibres; *i*. Light space surrounding the foramen and composed of a circular layer of fine fibrous tissue; the margin of the foramen is set with columnar epithelium; *c''*. Rudiments of the posterior cornua; on each side the *substantia gelatinosa* is broad, but is gradually narrowed at the middle line; the dark or spongy portion of the cornua is seen projecting into it like the matted hairs of a brush. This appearance is caused by oblique sections of bundles of nerves and blood-vessels which enter the grey substance in a more or less longitudinal direction, or with various degrees of obliquity; these, together with the bands of longitudinal fibres, are the chief cause of its opacity, and form the principal difference between its structure and that of the *substantia gelatinosa*.

Fig. 3. Represents the same appearances higher up; magnified 20 diameters. The bundles of transverse commissural fibres are distinctly seen, as well as the decussating fibres in front of the spinal canal; the two dark oval masses

behind these latter fibres are sections of thick bundles cut off by them from the anterior white columns; *i*. Anterior roots of the nerves.

Fig. 4. Another section of the grey substance nearer the lumbar enlargement; magnified 20 diameters. The commencement of the *posterior vesicular columns kk*, is seen at the sides of the spinal canal; other vesicles are also seen in the anterior cornua.

Fig. 5. A similar section through the lower part of the lumbar enlargement; magnified 20 diameters. The vesicles in the anterior cornua have considerably increased in number.

PLATE XXI.

Fig. 6. A section of the white and grey substance through the middle of the lumbar enlargement; magnified 20 diameters. The *posterior vesicular columns kk* are here situated within the posterior cornua, traversed and surrounded by fibres from the posterior roots of the nerves *ll*, part of which are seen also to extend as far as the large groups of vesicles in the anterior cornua.

PLATE XXII.

Fig. 7. Outline of a section of the grey substance, through the upper part of the lumbar enlargement; magnified only 10 diameters.

Fig. 9. A similar section through the lower part of the dorsal region; magnified 20 diameters.

PLATE XXIII.

Fig. 8. Another section three-quarters of an inch higher up; magnified 20 diameters.

Fig. 10. Another section of both the white and grey substance, through the *middle* of the dorsal region; magnified 20 diameters.

Fig. 14. A similar section through the lumbar enlargement, prepared according to the second method described in the paper. It represents the course of the fibres of the roots of the nerves, and of the transverse commissures, through the grey substance; the vesicles have been omitted to prevent confusion. The outline is magnified 20 diameters; the fibres were drawn under a power of 100. Notwithstanding the distinctness of the fibrous arrangement here shown, it is to be understood that this figure, like the rest, is not a scheme, but an exact representation of one of the preparations which accompany the paper, as seen under the microscope.

PLATE XXIV.

Fig. 11. A similar section of one side through the middle of the cervical enlargement, between the roots of the nerves; *vvv*, nerve-tubes and blood-vessels passing out from the grey substance; the bundles of nerves are seen breaking up amongst the clusters of nerve-vesicles; magnified 20 diameters.

Fig. 12. A section of the grey substance of one side, immediately below the first cervical nerves; magnified 20 diameters. *m*. Nucleus of the spinal-accessory nerve.

PLATE XXV.

Fig. 13. A section of the white and grey substance of one side through the lower part of the medulla oblongata; magnified 20 diameters. The spinal-accessory nerve, *n*, is seen entering the grey substance through the lateral column, and proceeding forwards to the vesicles of the anterior cornu.

Fig. 15. A group of caudate vesicles, traversed by the fibrils of the anterior roots of the nerves. From a section of one of the anterior cornua in the lumbar enlargement of the chord; prepared according to the second method, and magnified 220 diameters.

XXVII. *On the Meteorology of the Lake District of Cumberland and Westmoreland, including the Results of Experiments on the Fall of Rain at various heights, up to 3166 feet above the sea-level.—Fourth paper, for the year 1850.* By JOHN FLETCHER MILLER, *F.R.S., F.R.A.S., Assoc. Inst. C.E. &c.*

Received February 21,—Read March 13, 1851.

Introductory Remarks.

IN the month of December last I visited the Lake District, chiefly with the view of ascertaining approximately, the heights above the sea of some of the mountain gauges which I have hitherto been obliged to estimate. The heights of these stations were taken by means of an excellent aneroid barometer (previously compared with a standard), and a standard barometer read simultaneously, or nearly so, at the sea-level. By this method I found the height of Cockermouth above the sea to be 127 feet, Keswick 253 feet (Crosthwaite 258 feet), Bassenthwaite Lake and Lowdore (measured from Keswick) 214 and 224 feet respectively; Seathwaite (mean of two observations calculated from Lowdore) 368 feet; from Wastdale Head, 399 feet; and taken direct from the sea-level, 389 feet. I find the summit of Seatollar Common to be about 1590 feet, and the gauge 1388 feet above the sea, assuming the elevation of Seathwaite to be 368 feet; the gauge on Sprinkling Fell or the Stye, 948 feet, measured from Wastdale, and 936 feet by a simultaneous barometrical reading at the coast. Two distinct observations taken on the 16th and 17th of December, show the gauge near the top of Stye Head Pass to be 1443 and 1448 feet above the sea respectively, supposing the height of Wastdale Head village to be 247 feet. Mr. OTLEY states the summit of the Pass to be 1250 feet above the valley, or, taking Wastdale Head at 247 feet, 1497 feet above the sea; and an observation of Dr. DALTON'S from the same base, gives 1506 feet, both of which correspond very nearly with my own results, as the gauge is somewhat *below* the highest part of the road leading over the Pass. I have hitherto stated the height of this station to be 1250 feet, but it appears I have misinterpreted my authority (Mr. OTLEY), whose calculation represents the height above the *valley*, not above the sea. The gauges on Brant Rigg and on Lingmell appear to be 924 and 1778 feet respectively above the sea. At the latter station, the barometer fell to 27·00 in., while in the valley it stood at 28·67 in., and at Whitehaven at 28·86 in. Temperature in the valley 48°; at 1778 feet, 32°·8, wet bulb 32°·2, heavy rain falling. I was prevented from ascending to any of the higher stations by the unfavourable state of the weather; indeed the barometrical observation at Brant Rigg, and also a second reading on Stye Head were taken at night, after

having been confined to the valley the whole of the day by torrents of rain; I have consequently not had an opportunity of ascertaining the altitude of Sprinkling Tarn, but in 1812 Dr. Dalton states it to be 1860 feet; and another observation, either by Dalton or Otley, gives 1943 feet above the sea. I have therefore allowed the elevation of 1900 feet, previously given in my tables, to remain unaltered for the present. A complete and authentic table of the heights of our principal lakes and mountains is much wanted. Of the elevations given in the Guide Books to the Lake District, some are probably not far from the truth, but others are undoubtedly very erroneous; thus, Ennerdale Lake, by the Whitehaven Waterworks' Survey, proves to be 356 feet above the sea, instead of 246 feet, as stated in one of these treatises, showing an error or miscalculation of 110 feet. This is, no doubt, an extreme instance; but the altitudes of several of our lakes and mountains are yet unknown; and a careful measurement or remeasurement of the whole of them with accurate instruments and under favourable circumstances, is very desirable, as the results would be more or less interesting and valuable both to the meteorologist and the geologist, the botanist and the tourist.

TABLE I.—Synopsis of the Fall of Rain in the Lake District of Cumberland and Westmoreland, in the year 1850.

No.	I.	III.	IV.	V.			VI.	VII.	VIII.	IX.	X.	XI.	XIII.	XIV.	XV.	XVI.	XVII.	XVIII.	XX.	Prevailing winds.		
	Whitehaven.			Cockermouth, &c.											Westmoreland †.			Borrowdale.				
1850.	High Street, 90 feet above the sea.	St. James's Church Steeple, 79 feet above the street.	The Flosh, three miles south of Whitehaven.	The Town, 127 feet above the sea.	Tarn Bank*, 807 feet above the sea.	Broughton Moor, 410 feet above the sea.	Ennerdale Hills, 810 feet above the sea.	Keswick, 238 feet above the sea.	Vale of Gilderthwaite, Ennerdale, 306 feet above the sea.	Lowswater, 330 feet above the sea.	Foot of Crummock Lake, 260 feet above the sea.	Gatesgarth, 320 feet above the sea.	Eskdale Head.	Wastdale Head, 247 feet above the sea.	The How, Troutbeck, 300 ? feet above the sea.	Ambleside, 100 ? feet above the sea.	Langdale Head, 350 ? feet above the sea.	Seathwaite, 366 feet above the sea.	Stonethwaite, 330 feet above the sea.	At Crummock Lake, two observations daily.	At Seathwaite, one observation daily.	At the Coast, two observations daily.
January.....	in. 2·995	in. 2·797	in. 3·63	in. 4·14	in. 3·23	in. 3·56	in. 3·10	in. 2·944	in. 4·89	in. 4·130	in. 4·49	in. 4·13	in. 5·68	in. 7·26	in. 4·744	in. 5·20	in. 7·07	in. 7·34	in. 5·20	E. var.	N.W. var.	S.W.
February ...	4·836	3·104	6·18	6·53	4·84	3·96	6·40	8·582	12·00	8·775	10·87	15·66	9·52	15·25	15·504	14·45	21·66	22·58	17·01	N.W. & S.W.	N.W.	S.W.
March	·969	·553	1·65	1·33	1·03	·59	1·45	1·268	4·13	2·365	3·31	3·38	3·59	3·55	1·624	1·73	3·04	4·13	3·42	Easterly.	E. & W.	Westerly.
April	3·113	2·304	4·08	2·46	2·79	2·07	3·30	4·760	7·14	5·255	8·01	10·34	8·25	9·34	8·523	6·64	11·51	15·62	9·63	S.E.	S.E. var.	S.W.
May	1·329	·823	1·37	2·08	1·60	1·50	2·26	2·721	3·00	2·935	4·27	6·20	3·45	4·78	3·377	2·91	5·46	7·14	5·89	S.W. var.	Westerly.	S.W.
June	2·012	1·233	2·63	4·17	2·38	2·40	3·74	3·910	6·30	6·000	5·40	5·79	4·69	5·97	4·845	3·99	6·76	6·83	4·43	S.W.	S.W.	S.W.
July	5·358	4·131	6·66	6·18	5·65	4·53	5·35	6·254	8·65	8·265	8·26	11·56	6·46	9·75	7·189	6·54	9·50	11·20	9·45	N.W.	S.E. & S.W.	N.W. var.
August	4·501	3·073	5·74	4·92	4·53	3·63	5·00	7·205	10·43	7·840	10·73	15·75	8·47	11·28	8·362	9·27	13·40	16·22	12·50	N.W.	N.W.	N.W.
September...	2·643	2·158	2·52	2·07	2·54	2·03	1·51	3·234	4·03	3·110	4·01	5·21	4·02	3·93	2·810	3·82	4·12	5·85	4·46	E. var.	E. var.	S.W.
October.....	3·364	2·213	4·86	3·69	4·53	2·83	3·66	5·582	6·00	6·045	7·58	8·52	8·27	11·60	6·556	6·58	14·92	12·94	9·33	N.W.	N.W. & S.W.	N.W. var.
November...	6·258	4·152	8·13	7·44	4·84	4·75	7·24	8·966	12·34	9·545	12·72	15·45	12·16	17·82	9·798	13·26	20·55	22·60	16·94	S.W. & N.W.	S.W.	S.W.
December...	3·095	2·095	3·83	2·73	3·06	2·50	3·96	4·102	5·72	4·533	6·01	6·85	7·13	8·23	6·476	5·46	8·57	11·51	7·55	S.W.	S.W. & N.W.	S.W.
1850.	40·473	28·636	51·28	47·74	41·02	34·35	46·97	59·528	84·63	68·798	85·66	108·84	81·69	108·76	79 808	79·85	126·56	143·96	105·81	S.W. & N.W.	N.W. & S.W.	S.W.
1849.	38·999	28·210	45·13	38·39	38·71	31·82	40·00	48·80	76·41	55·28	70·21	97·09	71·22	107·22	75·420	65·32	107·83	125·47	94·27	S.W.	S.W.	S.W.
1848.	47·342	36·344	60·82	52·37	.....	.....	47·06	66·407	97·73	76·668	98·07	133·55	86·78	115·32	91·347	77·58	130·38	160·89	130·24	S.W.	S.W.	S.W.
1847.	42·921	30·713	47·80	42·55	.....	.....	44·45	58·286	80·13	66·296	82 32	106·25	74·93	96·34	78 004	.....	112·95	129·24	106·21	W.	S.W.	S.W.
1846.	49·134	35·422	55·16	52·41	.....	.....	.....	67·678	83·87	79·249	96·47	121·90	.....	106 93	77·719	.....	127·40	143·51		S.W.	S.W.	S.W.
1845.	49·207	33·489	53 00	46·93	.....	.....	.....	62·202	76·88	69·542	87·48	124·13	.....	108·55	76·305	.....	136 00	151·87	.....	S.W. var.	.....	S.W. var.
1844.	36·723	27·862	39·31	.....	.....	.....	.....	40·629	54·62	49·829	61·46	.....	.....	80·01	51·986							
Mean of 7 years	43·514	31·525	50·35	46·73	39·86	33·08	44·62	57·467	79·18	66·523	83·09	108·84	78·65	103·30	75·798	74·25	123·52	142·49	109·13	S.W.	S.W.	S.W.

* Tarn Bank bears S.W. from Cockermouth, distant four miles; the Broughton Moor Station is three miles N.W. of Cockermouth, and 2¾ miles S.E. of Maryport.

† At Kendal, the fall in 1850 is 49·57 inches (average of twenty-eight years, 54·43 inches); at Selside, six miles from Kendal, 76·39 inches (average of three years, 73·87 inches); and at Bowness, near Windermere Lake, 64·28 inches.

Table II.—Wet Days.

1850.	Whitehaven.	The Flosh.	Cockermouth.	Turn Bank, near Cockermouth.	Bassenthwaite Halls.	Keswick.	Lowes water Lake.	Crummock Lake.	Wastdale Head.	Troutbeck*.	Ambleside.	Langdale Head.	Seathwaite.	Stonethwaite.
January ...	12	15	9	12	11	13	12	12	16	9	13	13	12	12
February...	20	22	20	23	21	22	23	20	26	21	21	23	23	22
March.....	10	10	9	11	9	11	9	12	15	7	5	12	12	9
April	18	17	18	20	18	20	21	21	24	20	21	20	21	21
May	15	15	14	18	14	16	14	14	21	16	14	14	19	14
June	10	10	15	15	12	12	14	12	19	12	11	13	16	14
July.....	13	15	18	18	19	19	14	17	17	14	11	15	15	13
August ...	22	24	23	17	21	23	19	20	24	16	17	22	24	18
September..	11	10	10	9	10	12	12	12	13	9	8	11	12	10
October ...	19	20	19	17	18	20	22	22	25	22	22	22	24	24
November..	19	23	23	14	22	23	20	24	26	21	22	24	24	23
December..	20	20	21	19	19	18	18	18	22	18	17	22	21	19
1850.	189	201	199	193	194	209	198	204	248	185	182	211	223	199
1849.	189	185	191	.....	182	205	191	185	236	186	159	199	193	183
1848.	210	207	228	.....	196	229	217	207	243	201	.....	212	232	224
1847.	191	183	210	.....	199	204	190	199	226	188	.....	209	202	195
1846.	200	208	234	.....	.....	213	198	216	234	194	.....	213	219	
1845.	193	175	212	.....	.....	195	195	202	211	180	.....	.....	211	

Table III.—Showing the Quantity of Rain received by the Mountain Gauges in eleven months, between the 1st of February and the 31st of December 1850.

No.	XXI.	XXI.[a]	XXII.	XXIII.	XXIV.	XXV.	XIV.	XIII.	XXVII.	XXVI.	XIX.
							The Valley.		Borrowdale.		
1850.	Sca Fell Pike, 3166 feet above the sea.	Lingmell, 1778 feet above the sea.	Great Gabel, 2925 feet above the sea.	Sprinkling Tarn, 1900? feet above the sea.	Stye Head, 1448 feet above the sea.	Brant Rigg, 924 feet above the sea.	To the west, Wastdale, 247 feet above the sea.	To the south-east, Eskdale, height unknown.	The Stye, 948 feet above the sea.	Seatollar Common, 1388 feet above the sea.	The Valley, Seathwaite, 368 feet above the sea.
	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.
February†..	10·50	12·00	12·82	17·60	13·32	12·00	15·25	9·52	29·40	21·20	22·58
March.....	Frozen.	Frozen.	Frozen.	Frozen.	Frozen.	Frozen.	3·55	3·59	5·06	4·27	4·13
April‡.....	10·00	11·07	9·00	17·00	17·05	12·22	9·34	8·25	20·30	14·34	15·62
May	5·48	4·44	5·28	7·30	6·05	3·95	4·78	3·15	8·36	6·85	7·14
June	6·00	5·95	5·84	7·59	6·15	5·51	5·97	4·69	8·41	8·00	6·83
July.....	9·56	9·36	9·57	12·78	10·98	9·74	9·75	6·46	13·31	10·46	11·20
August ...	11·16	11·77	10·23	17·26	13·59	8·99	11·28	8·47	20·27	18·21	16·22
September..	3·53	4·60	3·41	5·86	5·76	4·80	3·93	4·02	9·91	5·78	5·85
October ...	Frozen.	9·70	10·96	13·18§	17·10	10·84	11·60	8·27	17·11	13·32	12·94
November..	Frozen.	Frozen.	Frozen.	Frozen.	Frozen.	Frozen.	17·82	12·16	28·06	24·15	22·60
December..	24·08‖	23·61	20·17	29·23	25·53	23·05	8·23	7·13	14·14	12·26	11·51
Inches.....	80·31	92·50	87·28	127·80	115·53	91·10	101·50	76·01	174·33	138·84	136·62

* At Kendal, 168; at Selside, six miles from Kendal, 193; and at Bowness, near Windermere Lake, 217 wet days.

† The month of January 1850 was included in the Tables for 1849, in consequence of the gauges being frozen up at the close of the latter year.

‡ April 30. This morning the summits of Gabel and Sca Fell were capped with snow ankle deep, and it froze keenly. The receivers were iced over at both stations, but not so strongly as to prevent the water being measured off.

§ In October, the normal proportion betwixt Stye Head and Sprinkling Tarn is inverted. On inquiry, I find the quantities as given in the Table are correct.

‖ The Sca Fell gauge was frozen on the last day of the year; the receiver was brought down to the valley and the ice melted. The funnel was filled up with snow, which accounts for the relatively small quantity of water received by this gauge during the last quarter. The fall of snow is very much greater on Sca Fell than on Gabel, although the difference in altitude is only 241 feet.

Table IV.—For the Summer Months.

No.	XXI.	XXI.[2]	XXII.	XXIII.	XXIV.	XXV.	XIV.	XIII.	XXVII.	XXVI.	XIX.
							The Valley.		Borrowdale.		
1850.	Sca Fell Pike, 3166 feet above the sea.	Lingmell, 1778 feet above the sea.	Great Gabel, 2925 feet above the sea.	Sprinkling Tarn, 1900 feet above the sea.	Stye Head, 1448 feet above the sea.	Brant Rigg, 924 feet above the sea.	To the West, Wastdale, 247 feet above the sea.	To the South-east, Eskdale, height unknown.	The Stye, 948 feet above the sea.	Seatollar Common, 1338 feet above the sea.	The Valley, Seathwaite, 368 feet above the sea.
	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.
May	5·48	4·44	5·28	7·30	6·05	3·95	4·78	3·45	8·36	6·85	7·14
June	6·00	5·95	5·84	7·59	6·15	5·51	5·97	4·69	8·41	8·00	6·83
July	9·56	9·36	9·57	12·78	10·98	9·74	9·75	6·46	13·31	10·46	11·20
August ...	11·16	11·77	10·23	17·26	13·59	8·99	11·28	8·47	20·27	18·21	16·22
September..	3·53	4·60	3·41	5·86	5·76	4·80	3·93	4·02	9·91	5·78	5·85
October ...	8·64	9·70	10·96	13·18	17·10	10·84	11·60	8·27	17·11	13·32	12·91
Inches.....	44·37	45·82	45·29	63·97	59·63	43·83	47·31	35·36	77·37	62·62	60·18

Table V.—For the Winter Months.

1850.	Sca Fell Pike.	Lingmell.	Great Gabel.	Sprinkling Tarn.	Stye Head.	Brant Rigg.	The Valley.		Borrowdale.		
							To the West, Wastdale.	To the South-east, Eskdale.	Seatollar Common.	On the Stye.	The Valley, Seathwaite.
	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.
February...	10·50	12·00	12·82	17·60	13·32	12·00	15·25	9·52	21·20	29·40	22·58
March ...	Frozen.	Frozen.	Frozen.	Frozen.	Frozen.	Frozen.	3·55	3·59	4·27	5·06	4·13
April	10·00	11·07	9·00	17·00	17·05	12·22	9·34	8·25	14·34	20·30	15·62
November..	Frozen.	Frozen.	Frozen.	Frozen	Frozen.	Frozen.	17·82	12·16	24·15	28·06	22·60
December..	15·44	23·61	20·17	29·23	25·53	23·05	8·23	7·13	12·26	14·14	11·51
Inches.....	35·94	46·68	41·99	63·83	55·90	47·27	54·19	40·65	76·22	96·96	76·44

Table VI.—Temperature at Seathwaite, Borrowdale, 368 feet above the sea-level.

1850.	Absolute		Mean of max.	Mean of min.	Approximate mean Temperature.	Mean at 9 A.M.	On Grass.				Prevailing winds.
	Max.	Min.					Absolute min.	Mean.	Radiation.		
									Max.	Mean.	
	°	°	°	°	°	°	°	°	°	°	
January	48	19	36·88	29·82	33·35	32·95	11	23·42	9·8	6·40	N.W. var.
February ...	49·5	29	45·48	38·90	42·14	41·14	19	31·87	16	7·03	N.W.
March	53	21	44·53	34·87	39·70	38·00	18	29·41	10·5	5·46	E. and W.
April	56	32	50·51	40·91	45·71	45·12	27	35·31	15	5·60	S.E. var.
May	68·5	30	54·98	43·80	49·39	48·29	19	35·58	14	8·22	Westerly.
June	73·3	42	61·99	52·53	57·26	56·66	37	46·65	13	5·88	S.W.
July	76	43	65·51	54·02	59·76	59·26	34	47·61	14	6·41	S.E. and S.W.
August	72	39·5	61·06	50·71	55·88	55·13	30·5	44·42	12·5	6·29	N.W.
September ...	61	39	57·46	47·50	52·48	51·18	31	42·60	15	4·91	E. var.
October	55	27	47·26	38·77	43·01	42·60	22	32·86	9·5	4·64	N.W. and S.W.
November ...	52	20·5	43·53	39·03	41·28	41·44	...†	.....	.....	.....	S.W.
December ...	51	22	43·47	36·00	39·73	40·48	...†	.....	.....	.....	S.W. and N.W.
1850.	59·6	30·3	51·05	42·23	46·64	46·02	.....	.....	.....	.....	N.W. and S.W.
1849.	61·8	31·1	51·78	41·81	46·79	46·18	21·5	35·67	12·4	6·23	S.W.
1848*.	62·4	30·5	52·15	42·06	47·10	46·76	20·5	35·18	12·9	6·91	S.W.
1847.	62·7	29·9	52·89	42·04	47·46	47·21					
1846.	63·0	33	53·77	44·05	48·91	48·13					

* On the 1st of May, 1848, the thermometers were removed from the garden wall (where they were affected by solar radiation) to the gable end of a building facing the north where the sun never touches them, but the change of position does not appear to have materially affected the *means* of the maximum readings.

† The results of the thermometer on grass for November and December, are omitted for the reason assigned in the remarks.

Note.—The mercurial thermometer got deranged in August, and a duplicate sent to supply its place was also found to be separated in the column on its arrival at Seathwaite, and consequently useless. Considerable delay took place before the instruments were adjusted, and the registrar did not receive them again till the middle of December. From about the middle of August till the 17th of December, the maximum was obtained by frequent examination of the spirit thermometer in the course of the day; the mean of the maximum for 1850 is in consequence about 1°·5 too low.

The mercurial thermometer, prior to its getting out of order, was considered to be nearly free from index error, and the night thermometer has been compared with it throughout the scale, and reduced to the mercurial as a standard.

Table VII.

Temperature at Whitehaven on the West Coast, 90 feet above the sea-level, and seventeen miles distant in a direct line, bearing W.N.W. from the hamlet of Seathwaite, Borrowdale.

1850.	Absolute		Mean of Maximum.	Mean of Minimum.	Approximate mean Temperature.	Mean. at 9 A.M.	Naked Thermometers on Grassplot *.							
							Absolute Minimum.		Mean.		Radiation.			
											Maximum.		Mean.	
	Maximum.	Minimum.					On Grass.	On Wool on Grass.	On Grass.	On Wool on Grass.	On Grass.	On Wool on Grass.	On Grass.	On Wool on Grass.
	°	°	°	°	°	°	°	°	°	°	°	°	°	°
January	48·5	21·5	37·62	30·85	34·241	33·87	12	8	25·94	22·74	11·8	15·5	4·91	8·11
February	52·5	33	46·00	40·00	43·000	42·55	29·7	26	37·00	34·91	8·3	12·0	3·00	5·09
March	52·5	25	45·89	37·30	41·590	40·59	16·2	8	32·35	28·65	10	16	4·95	8·65
April	63	35	54·47	42·83	48·650	49·01	26	19·5	37·73	35·04	10·5	16	5·10	7·79
May	70	32·5	57·35	43·75	50·550	52·00	26	18·5	37·86	34·03	13·5	17	5·89	9·72
June	78·5	44·5	65·55	53·46	59·505	60·53	41·5	37	48·33	45·62	12·5	15·5	5·13	7·84
July	84	47	68·14	55·53	61·835	62·93	42	33·5	51·36	47·88	9	14	4·17	7·65
August	73	49	64·92	52·51	58·715	59·45	33·5	30	46·84	44·10	12	15	5·67	8·41
September	66	39·5	61·83	48·50	55·165	55·55	28	24	42·71	40·01	13	18	5·79	8·49
October	61	32	53·20	42·38	47·846	47·43	26	20·5	37·86	35·16	9	14	4·52	7·22
November	57	24·5	48·90	42·50	45·700	45·38	16	13	37·25	35·74	8·5	12·5	5·25	6·76
December	52·5	25·5	45·61	39·21	42·412	41·97	15·5	12	33·35	31·33	11	17	5·86	7·88
1850.	63·1	33 5	54·13	44·07	49·104	49·28	26·0	20·8	39·04	36·26	10·7	15·2	5·02	7·80
1849.	62·3	33·7	53·24	44·15	48·696		23·5	18·8	38·04	35·05	14·0	18·4	6·11	9·09
1848.	62·9	32·6	53·77	43·79	48·785			20·2		35·73		15·9		8·06
1847.	62·3	33·7	53·85	43·50	48·679			20·5		35 95		15·1		7·45
1846.	64·8	36·1	55·95	45·75	50·858			23·1		38·30		14·6		7·45

* The results of the thermometers exposed to the sky at Whitehaven and at Seathwaite, are not strictly comparable. At Seathwaite, the thermometer on grass is a common spirit thermometer on a boxwood scale. Naked thermometers were used for a year or two, but the observations were so frequently interrupted by breakage, that it was deemed preferable to employ a less fragile instrument.

Remarks.

The fall of rain throughout the Lake District in 1850, is slightly above the average of the six preceding years. At Seathwaite, the depth is 1·77 inch over the average of this period. The largest daily falls in 1850, at the three principal stations, are grouped as under:—

	Wastdale.	Langdale.	Seathwaite.
	days.	days.	days.
Between ¼ an inch and 1 inch	38	39	39
Between 1 inch and 2 inches	29	37	34
Between 2 inches and 3 inches	5	6	13
Between 3 inches and 4 inches	2	2	6
Between 4 inches and 5 inches	...	1	...
Days in 1850 exceeding 0·5 inch in depth	74	85	92

Temperature.—At Seathwaite, the average mean temperature of the last five years is 47°·38; mean of maximum, 52°·32; mean of minimum, 42°·43. At Whitehaven, on the west coast, seventeen miles distant in a direct line, bearing W.N.W. from Seathwaite, the mean of the maximum for the same period is 54°·18; mean of minimum, 44°·25; average mean temperature, 49°·22.

The mean difference between the two places is, in the maximum, 1°·86; in the minimum, 1°·82, and in the mean 1°·84, the temperature at Whitehaven being higher than at Seathwaite by these quantities. The mean temperature at Whitehaven from eighteen years' observations is 49°; at Greenwich, the mean for seventy-eight years is 48°·3; and at Somerset House for sixty-nine years, 49°·5.

The radiation of heat from the earth's surface at night, as indicated by self-registering thermometers fully exposed to the sky on grass, appears on the whole to be greater in the mountain valleys than at the coast, and particularly in summer; but, occasionally in the winter months, the results are strangely and unaccountably anomalous. Thus, in November 1850, the mean amount or effect indicated was only 0°·90, and in December the mean reading of the thermometer on the grass was identical with that at 4 feet above the surface. At Whitehaven, the amount in those months was 5°·25 and 5°·86. Yet the same instrument (which has been in use at Seathwaite since 1846) in nearly all the other months of 1850, shows a greater extent of radiation than at Whitehaven. Results almost equally abnormal were presented in the winter of 1846 and 1847, and as such they were omitted from the Tables for that year*. I have examined the thermometer employed at Seathwaite for indicating the direct effect of terrestrial radiation; the column is perfect, and I am satisfied it has no material index error, and that it is correctly read off; moreover, it is exposed in the same place throughout the year. The cause of its occasional anomalous indications in the winter months must therefore be left unexplained for the present.

* Philosophical Transactions, Part I. 1849, p. 85.

The Mountain Gauges.—The phenomena exhibited by the mountain gauges in the year 1850, do not seem to call for any particular comment, as the results are very generally in accordance with the deductions embodied in former papers which have appeared in the Transactions of the Royal Society; and at the present time I am more anxious to eliminate new facts and to accumulate a mass of accurate observations and well-digested results made and obtained both in normal and abnormal seasons and under various modifying circumstances, than to theorize upon or draw from them inferences or conclusions which extended experience may modify, contradict or destroy. The following Table shows the excess or deficiency per cent. of the principal mountain gauges over or under the quantity of rain received by the adjacent valley of Wastdale, both in the summer and winter months, in each year since the instruments were erected in 1846.

The positive sign signifies that the quantity is greater, and the negative sign that it is less than the fall in the valley in the same period.

Summer Months.

Year.	Sca Fell Pike, 3166 feet.	Lingmell, 1778 feet.	Great Gabel, 2928 feet.	Sprinkling Tarn, 1900 feet.	Stye Head, 1443 feet.	Brant Rigg, 924 feet.
	per cent.	per cent.	per cent.	per cent.	per cent.	per cent.
1846*.	−13·5	...	− 7·5	+ 29·5	+12·0	−10·3
1847*.	−13·5	...	− 7·5	+ 29·5	+12·0	−10·3
1848.	− 1·0	...	− 6·0	+ 41·5	+20·5	−14·0
1849.	− 6·5	− 8·0	− 7·0	+ 17·3	+ 9·0	−18·5
1850.	− 6·2	− 3·2	− 4·3	+ 35·3	+26·1	− 7·3
Algebraical Sums.	−40·7	−11·2	−32·3	+153·1	+79·6	−60·4
Algebraical Means.	− 8·1	− 5·6	− 6·4	+ 30·6	+15·9	−12·0

Winter Months.

Year.	Sca Fell Pike.	Lingmell.	Great Gabel.	Sprinkling Tarn.	Stye Head.	Brant Rigg.
	per cent.		per cent.	per cent.	per cent.	per cent.
1846*.	− 42·5	...	− 38·5	+11·3	+ 4·5	−15·2
1847*.	− 42·5	...	− 38·5	+11·3	+ 4·5	−15·2
1848.	Leaked.	...	− 42·5	+ 1·5	+ 0·5	−14·6
1849.	− 43·5	−28·8	− 40·5	− 3·2	−21·2	−27·8
1850.	− 33·7	−13·9	− 22·5	+17·8	+ 3·2	−12·8
Algebraical Sums.	−162·2	−42·7	−182·5	+38·7	− 8·5	−85·6
Algebraical Means.	− 40·5	−21·3	− 36·5	+ 7·7	− 1·7	−17·1

The remarkable deficiency in the per-centage of rain both in the summer and winter months of 1849, is accounted for by the abnormal and relatively excessive fall of rain in the Vale of Wastdale in that year, as explained in my last report on the meteorology

* The per-centages in 1846 and 1847, show the mean of the two years, which were tabulated together.

of the Lake District. The *increase* in the per-centage in the winter months of 1850 is doubtless attributable to the fact of the deposition being almost entirely in the form of rain, the fall of snow on the mountains having been unusually small both in the early and latter months of the past year.

The most interesting and important circumstance connected with the experiments in 1850, is the discovery of a mountain station which promises to yield nearly one-third more rain than the celebrated hamlet of Seathwaite in Borrowdale, hitherto, and with good reason, considered to be the wettest spot in Great Britain. The new station is about a mile and a half distant from Seathwaite in a south-westerly direction, and 580 feet above it, or 948 feet above the sea-level, at the extreme southern termination of the valley; it is on the shoulder of Sprinkling Fell or the Stye, about 100 yards south of the road leading over the Stye Pass to Wastdale.

The actual quantity of water measured on Sprinkling Fell in *eleven* months of 1850, is 174·33 inches; but the receiver was found running over on four different occasions, by which I calculate 5 or 6 inches at least must have been lost to the instrument; hence, if we add 5·67 inches for overflow, and 9·49 inches for the computed depth in January (7·34 inches at Seathwaite), the result is 189·49 inches for the fall on the Stye in 1850, with 143·96 inches at Seathwaite*.

The wettest year since the commencement of the experiments is 1848, when 160·89 inches fell at Seathwaite; and, computing the fall at the new station for that year in the same proportion which the two localities bear to each other in 1850, we have 211·62 inches for the depth of rain on the Stye in 1848. An inspection of the following Table, which exhibits the fall at the coast during the last eighteen years, will show that the period (1844–50) over which the Lake District gauges have been in operation, has been far from a wet one.

Fall of Rain at Whitehaven (seventeen miles distant in a direct line from Seathwaite) during the last Eighteen Years, from 1833 to 1850 inclusive.

Month.	1833.	1834.	1835.	1836.	1837.	1838.	1839.	1840.	1841.	1842.	1843.	1844.	1845.	1846.	1847.	1848.	1849.	1850.
	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.
January ...	·712	9·169	4·643	4·133	2·824	1·785	4·576	4·586	3·674	3·429	4·875	4·119	4·541	4·604	1·873	3·745	5·683	2·995
February...	4·235	5·303	7·597	3·626	5·278	·943	2·768	4·376	1·536	2·657	·863	3·499	2·830	2·007	1·827	7·815	2·045	4·836
March	2·928	2·560	5·751	5·742	1·209	4·474	6·229	·396	3·800	4·912	1·927	3·746	3·735	4·460	1·370	4·588	·837	·969
April	2·346	1·404	1·367	2·939	1·789	2·355	1·463	·580	3·803	·551	6·046	2·658	2·587	2·848	2·560	·455	1·488	3·113
May.....	2·257	2·483	3·964	·010	1·148	3·238	·843	3·781	2·955	2·293	2·016	·262	1·480	2·317	3·428	1·798	3·037	1·329
June	6·783	4·682	1·601	6·642	3·647	7·125	4·003	5·893	4·512	1·861	4·497	3·878	4·090	2·311	2·912	3·867	1·224	2·012
July	4·384	5·065	5·451	7·146	7·245	4·893	5·681	8·138	4·569	3·782	6·104	4·183	2·900	9·061	·776	3·630	5·478	5·358
August ...	2·090	4·755	1·813	5·886	2·157	5·148	3·655	6·175	7·879	1·813	4·875	1·999	6·995	4·066	4·496	5·054	3·771	4·501
September.	2·339	4·281	6·507	6·399	3·305	2·289	6·714	5·754	5·670	2·871	·390	5·809	3·653	2·857	3·738	2·266	2·814	2·643
October ...	4·240	3·937	6·217	4·049	5·832	4·574	4·309	2·452	8·344	2·388	6·748	4·335	6·744	7·982	5·407	5·772	5·252	3·364
November.	7·094	2·749	4·996	6·150	4·620	4·481	4·048	4·420	3·955	3·702	5·569	1·926	4·022	4·671	7·937	3·507	4·974	6·258
December.	9·048	3·315	4·228	6·253	6·278	1·641	2·787	·893	5·276	4·434	2·296	·309	5·621	1·950	6·597	4·805	2·396	3·095
Total	48·456	49·703	54·135	58·975	45·332	43·946	48·976	47·444	55·973	34·693	46·206	36·723	49·207	49·134	42·921	47·342	38·999	40·473
Wet Days.	203	200	215	230	189	167	198	210	220	167	210	172	193	200	191	211	190	190

Average number of wet days, 197.

* A new and capacious gauge was placed on the Stye early in January of the present year (1851).

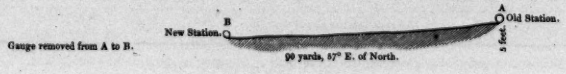
At Whitehaven, the average annual fall from 1844 to 1850 inclusive, is 43·543 inches; but in the eleven years preceding 1844, the average is 48·53 inches; and the average of the last eighteen years, from 1833 to 1850 inclusive, is 46·58 inches.

And if we analyse the period of seven years comprehended between 1844 and 1850, we find that only three of those years have exceeded the average; while of the remaining four, one year is characterized by drought, and the other three by unusual dryness. Even in the year 1848, when 161 inches fell at Seathwaite, the depth at Whitehaven was only 47·34 inches, or $\frac{3}{4}$ths of an inch above the average of eighteen years; whilst in 1835, the fall was 54·13 inches; in 1836, 58·97 inches; and in 1841, 55·97 inches.

It is not pretended that the gradation in quantity between Seathwaite and Whitehaven in any particular year will be the same in other single years, or that the differential mean of one term of years will correspond precisely with that of other terms of equal length, although I conceive that the proportion found to obtain for a group of ten consecutive years, will never be very widely departed from in future decennial periods; but we may, at least, fairly assume that a wet or a dry season at either station will bear a similar character at the other; and sufficient evidence has been adduced to show that the mean annual fall of rain in the Lake District has yet to be determined, by the incorporation of a future term of wet years with the comparatively dry period already on record. And this remark applies with still greater force to the maximum fall, as, judging from the records kept at the coast during eighteen years, no one of the last seven, during which the Lake District gauges have been in action, has any pretension to a character for excessive wetness.

Hence, the maximum annual depth in the mountain district of Cumberland may far exceed the computed fall of 211 inches at the Stye in 1848, enormous and almost incredible as is the quantity for a climate situated in the heart of the temperate zone. I may observe that the fall of rain at the coast in one year is rigidly comparable with any other year; the gauge having been in the same spot, or at least within a few feet of it, from the year 1832 up to the present time. Moreover, the same gauge and the same glass metre (graduated to the $\frac{1}{1000}$th part of an inch) have been used from the first; and the rain has been read off daily throughout the period.

In my paper printed in the Philosophical Transactions for 1848 (Part II.), allusion was made to the difference in the receipts of rain gauges within 100 yards of each other, when placed near the head of a valley. A still more remarkable instance is presented in the past year, but in this case the gauge is considerably elevated *above* the valley. On the 31st of August 1849, the gauge on Seatollar Common, 1338 feet above the sea, was removed 90 yards to the south-westward, nearly in a direct line, the difference in height between the old and new station being only 5 feet.



Below, are given the receipts of this gauge for the last five years, by which it will be seen that on the average of the three years ending with 1849, it has received 23·4 per cent. *less* than the valley; but in 1850, with the slight alteration in position just described, it has obtained 1·6 per cent. *more* rain than the valley, showing an annual increase of 25 per cent. consequent on the removal of the instrument 90 yards in linear distance, and a diminution of 5 feet in its height above the valley.

Year.	Seathwaite, 368 feet above the sea.	Seatollar Common, 1388 feet above the sea.	Deficiency per cent. at Seatollar Common.
	in.	in.	in.
1847.	129·24	104·55	−18·5
1848.	160·89	123·68	−23·1
1849.	125·47	108·97	
1849 to Aug. 31.	78·95	56·34	−28·7
1850.	143·96	146·18	+ 1·6

The records of the self-registering minimum thermometer on Sca Fell, in 1850, are as under:—

January and February, 31° below zero; March, 10° below zero; April, 10° below zero; May, 14°; June, 22°; July, observation lost; August, 9°; September, 7°; October, 7°; November and December, 15° below zero.

In the valley, the minima at 4 feet from the ground were,—in January, 19°; February, 29°; March, 21°; April, 32°; May, 30°; June, 42°; July, 43°; August, 39°·5; September, 39°; October, 27°; November, 20°·5; and December, 22°.

I have recently planted a minimum thermometer on the Gabel, and also one near Sprinkling Tarn, at the respective heights above the sea of 2928 and 1900 feet, and hope, in future, to obtain regular monthly readings at all the three stations.

The Observatory, Whitehaven,
February 6, 1851.

Erratum in Philosophical Transactions, Part I. *for* 1851.

Page 147, second line from top, *for* "severe *nights*" *read* "severe *droughts*."

XXVIII. *On the Annual Variation of the Magnetic Declination, at different periods of the Day.* By Lieut.-Col. Edward Sabine, *R.A., V.P. and Treas. of the Royal Society.*

Received April 30,—Read May 22, 1851; Revised October 1851.

THE interest which papers recently communicated to the Royal Society have excited in regard to the physical explanation of the Annual and Diurnal Variations of Terrestrial Magnetism, makes it extremely desirable that the facts which are to be explained should in the first instance be clearly and fully comprehended; and that for this purpose, the different classes of facts, which undergo much additional complication by being viewed together, should be distinguished apart, and that each class should be presented separately, combining at the same time, as far as circumstances may permit, facts of the same class obtained from different parts of the globe.

Under this impression I have deemed that an acceptable service might be rendered, by arranging in a small compass and presenting together the Annual Variations which the Magnetic Declination undergoes at every hour of the day at the four Colonial Observatories established by the British Government at Toronto, Hobarton, the Cape of Good Hope and St. Helena;—stations selected, it may be remembered, with the express view (amongst others) of affording, as far as any four stations of equally convenient access might be expected to do, the means of generalizing the facts of the Annual and Diurnal Variations in different quarters of the globe. I have attempted to accomplish this object by a graphical representation (Plate XXVI.), in which the Annual Variation at every hour is shown by vertical lines varying in length according to the amount of the range of the Annual Variation at each hour; each line having also small cross lines marking the mean positions of the several months in the annual range. The scale is the same for all the stations, being one inch to one minute of declination. The declination is that of the north end of the magnet at all the stations; the upper end of the line is always the eastern extremity, and the lower end the western extremity, of the annual range. The broken horizontal line which crosses all the verticals at each station, marks for each of the observation hours the mean declination in the year at that particular hour, obtained by adding together the daily observations of the declination at that hour, and dividing the sum by the number of days of observation in the year. This line is consequently not a line of uniform declination-value throughout, because the mean declination varies at different hours, by quantities which constitute the mean *Diurnal* Variation: but it is the line, or curve as it is sometimes called, of mean Diurnal Variation projected as a straight line, for the purpose of viewing the phenomena of the Annual Variation at

each hour, irrespective of the Diurnal Variation, or the changes which the *mean* declination undergoes at different hours. The hours are those of mean solar time at each station, the day commencing at noon, and being reckoned through the twenty-four hours; noon is therefore $=0^h$. The fractional minutes are occasioned by the observations having been made at the exact hours of *Göttingen* time, which differ more or less at each station from exact hours of local time.

The Annual Variations represented in Plate XXVI. were obtained at Toronto from three years of observation, viz. 1845, 1846 and 1847; at Hobarton from five years, viz. July 2, 1843 to July 1, 1848; at the Cape of Good Hope from five years, viz. July 2, 1841 to July 1, 1846; and at St. Helena from three years, viz. from July 2, 1844 to July 1, 1847. The secular change at the several stations during the years of observation has been eliminated; but what are frequently called the "irregular disturbances," are retained, as all observations are taken into the account. At Hobarton, the Cape of Good Hope and St. Helena, the Annual Variation is probably very little affected by the disturbances referred to; but it may be otherwise at Toronto, where they are larger, and especially influential during the hours of the night when the regular range of the Annual Variation is small. It is not probable, however, that the annual range during the hours of the *day* is materially affected by the disturbances, even at Toronto. The absolute values of the magnetic elements at the respective stations were nearly as follows in the mean of the years in which the observations were made:—

	Toronto.	Hobarton.	Cape of Good Hope.	St. Helena.
Declination	1° 33′ W.	9° 57′ E.	29° 07′ W.	23° 51′ W.
Inclination	75° 15′	−70° 34′	−53° 25′	−22° 07′
Total Force	13·90	13·56	7·48	6·01
Horizontal Force	3·54	4·51	4·46	5·57
Vertical Force	13·45	12·78	6·01	2·26

As the mode in which the subject of the Annual Variation is treated in this communication is I believe in some respects new, it has been suggested to me, since this paper was presented to the Society, that a fuller description than is given in the preceding paragraphs, of the process by which the graphical representation in Plate XXVI. is derived from the actual observations, may be desirable. I have therefore subjoined in Plate XXVII. a graphical representation for two of the stations, Toronto and Hobarton, of the direct and immediate results of the observations themselves, exhibiting according to their absolute declination-values the mean declination in each of the twelve months, at each of the twenty-four hours, of a *mean* year. The observations thus represented are, as before stated, at Toronto, three years of hourly observation, from January 1, 1845 to December 31, 1847 inclusive; and at Hobarton five years of hourly observation, from July 2, 1843 to July 1, 1848 inclusive. The *mean* or middle year at Toronto is therefore from January 1 to

December 31, 1846; and the mean or middle year at Hobarton is from July 2, 1845 to July 1, 1846; the mean of the results in the same month and at the same hour in each year of the series (three years in the one case, and five years in the other) being taken as the declination-values corresponding to the mean year; which may thus be regarded as a type of the actual changes taking place in the declination in the course of a single year, at the period when the observations were made; but with the advantage that the various declination-values in the mean or typical year are derived in the one case from three, and in the other case from five years of continuous and intercomparable observations, instead of from one.

The dark vertical lines in Plate XXVII. exhibit the actual annual *range* of the declination at each of the observation hours. The figures on the left of the lines (on the left in the Toronto figure, and accidentally placed on the right of the lines in the Hobarton figure) show the positions in the annual range at each hour of the mean declination in the different months as derived from observations at that hour only; the months being distinguished by numbers, according to their order of succession, from 1 to 12, commencing with January. The broken line MM shows the *mean* declination of the *whole year*, viz. the mean declination of all the months and all the hours. The dotted line DD represents the mean *diurnal* variation in the year, or the mean variation of the declination at the different hours of the day and night: it is drawn through and connects the points of mean declination in the year at each of the hours respectively. The scale of declination-value in this Plate is reduced for convenience to half the dimensions of that of Plate XXVI.; in Plate XXVII. an inch is the equivalent of two minutes of declination, and in Plate XXVI. of one minute. The declination in both plates is that of the north end of the magnet, or that end which in the middle latitudes points towards the geographical north.

On an intercomparison of the observations represented in their mean monthly values at each hour in Plate XXVII., it is at once obvious that they are affected by three distinct and easily distinguishable variations, two of which are periodical, and one is secular. 1°. If the mean declination of the year at the different hours of the day and night had an uniform value, the line DD would be a straight line, and there would be no mean Diurnal Variation, or variation whose period is a *day*:—but DD is very far from being a straight line. 2°. If the mean declination at any particular hour of the day were of uniform value in all months of the year, the dark vertical line corresponding to that hour would be reduced to a point instead of a line, and there would be no Annual Variation, or variation whose period is a *year*, at that hour:—but the dark vertical lines are very far from being points at any hour of the day or night. 3°. If, on comparing the mean declination observed in any particular month and at any particular hour in one year with the mean declination in the same month and hour in preceding or succeeding years, the values were found uniform, or presenting at most such differences only as might reasonably be ascribed to observation errors, or to what are usually called magnetic disturbances, we might infer that the line

MM had a constant value at the station to which the figure in the Plate referred, and would apply to any other year as well as to the one in which it represents the "sum of all the observations in the year divided by their number." This, however, is not the case: the line MM prolonged into other years would not be found to correspond with the mean declination in adjoining years, or in any other year than the one represented. Both at Toronto and at Hobarton the line MM would have a less westerly declination-value in years preceding, and a greater westerly declination-value in years succeeding those represented in the Plate. This Variation, since the observatories have been established, has at each station been progressive from year to year; its period, if it has a period, is unknown; it has therefore been called "secular," to distinguish it from those variations which have known periods. Its value for particular years (as for example, for the three years at Toronto, and the five years at Hobarton, which are under present notice) may be satisfactorily determined by comparing the mean declination of one year with that of succeeding years, or of particular portions of the year with that of the same portions of succeeding years. At the Colonial Observatories it has been obtained by intercomparison in different years of the mean declinations corresponding to *fortnightly* periods; and the small amount of the probable errors of these determinations shows their satisfactory character. The amount of the secular variation is comparatively small at Toronto, Hobarton and the Cape of Good Hope; but it is large at St. Helena, being an annual increase of nearly eight minutes of west declination. This large amount at St. Helena has proved, however, in one respect, an advantage, by the opportunity it has afforded of examining the character of the progression according to which the secular change takes place, and of ascertaining by the concurrent evidence of several years, that the progression is *equable and uniform in all parts of the year*.

The mean secular change during the years represented in Plate XXVII. being thus known, the mean declinations of each of the months at each of the observation-hours have received a correction, previous to their insertion in Plate XXVI., for the purpose of eliminating the influence of secular change on their actual positions in the annual range. This correction is in each case the proportional part of the secular change due to the interval of time occurring between the month in question and the middle period of the year. At Toronto, Hobarton and the Cape, the correction is small and little significant, even in the months most distant from the middle of the year; and the Annual *Variation* is consequently very little different at these stations from the annual *range*, either in magnitude or in the relative position of the several months. But it is otherwise at St. Helena, where the secular change is large, and its elimination therefore is indispensable, even to an approximate knowledge of the phenomena of the Annual Variation. Happily at St. Helena the hypothesis of an uniform and equable distribution of the secular change throughout the year, upon which the elimination is based, has been shown by the series of fortnightly mean declinations continued for several years, to be a correct representation of the facts of nature.

For the mean Diurnal Variation, it is manifestly indifferent whether the secular change be eliminated or not; the line DD retains in either case its position and form unchanged, for the sum of the monthly corrections for secular change in every case =0. If, therefore, the Annual *Variations* were substituted for the annual *ranges* in the dark vertical lines in Plate XXVII., we have reason to believe that a correct representation would be given of what would be the facts of the Annual and Diurnal Variations if no secular change whatsoever existed; or if a secular change were to take place (as has recently been the case in Britain) in the opposite direction to that which previously prevailed, and were eliminated by the same process.

But when the vertical lines of Annual Variation are thus placed on the Diurnal Variation, projected as in Plate XXVII. according to its mean declination-value, the complexity occasioned by the two phenomena being viewed together is considerable, and is greatly increased when, as in Plate XXVI., four different stations are exhibited on the same plate, for the special purpose of examining and comparing one class of the phenomena only, (those of the Annual Variation,) as presented at different stations. To facilitate this examination and comparison, the curve, if such it may be called, of Diurnal Variation, has been projected in Plate XXVI. as a straight horizontal line; and in that Plate consequently the Annual Variations at the several observation hours may be viewed independently of Diurnal Variation, as well as of Secular Change.

On directing our attention to this Plate, it is perceived at the first glance, that the range of variation at all the stations is considerably greater during the hours of the day than during those of the night; and that there is a great similarity, though not a perfect identity, at all the stations, in the relative amount of the range at different hours. The amount does not progressively enlarge to a maximum at any obviously natural epoch,—such as, for example, at or about noon, when the sun's altitude is greatest, or at the early hours of the afternoon when the temperature is greatest,—but it will be distinctly seen that at all the stations the increase of the range is most rapid in the first or second hour after sunrise, and that its extent at the hours from 7 to 9 A.M. (19^h, 20^h and 21^h) is not exceeded at any subsequent hour at Hobarton, the Cape, and St. Helena; whilst at Toronto the great enlargement of the annual range takes place even earlier, the hours of 6, 7 and 8 A.M. being exceeded by none, though they are equalled by a second increase at noon and the two following hours. This second enlargement is also perceptible at the same hours, though not to the same extent, at Hobarton and St. Helena.

On examining the distribution of the months at the different hours, or their relative positions to each other in the several vertical lines, we may perceive that certain months, which are found congregated at the one extremity of the range during the early hours of the morning, undergo a transfer towards the opposite extremity at a subsequent period of the day; thus June, July and August, which, in respect to their positions in the range, may generally be grouped together, occupy usually one extremity of the range,—and November, December and January, which,

from a similar reason, may also be grouped together, the other extremity,—in the early morning hours, and until from 8^h to 10^h A.M., about which time the two groups are respectively transferred, each towards the opposite quarter to that which it previously occupied. The period at which this transfer takes place is somewhat earlier at Toronto and St. Helena than at the Cape and Hobarton.

The comportment of the two equinoctial months, March and September, at the Cape of Good Hope and St. Helena, presents a contrast to that of the two solstitial groups which have been just described, and at the same time the two months are remarkably contrasted with each other. At the Cape, September is found exclusively on the *east* side of the mean line throughout the twenty-four hours, except at the observation hours of $0^h\ 34^m$ and $1^h\ 34^m$; and March as exclusively on the *west* side of the mean line throughout the twenty-four hours, excepting only at the observation hour of $2^h\ 34^m$. At St. Helena also September is found on the east side of the mean line at all hours of the twenty-four, except for one hour before and one hour after noon, when it is slightly on the west side; and March is found on the west side of the mean line with the exception of the hours from noon to 6 P.M. April participates with March in the peculiarity thus described both at the Cape and St. Helena. April is on the west side of the mean line at the Cape at all hours without exception, and at St. Helena at all hours with the exception of 11 A.M. and noon.

I abstain from dwelling on other particulars, fearing that, not possessing any clue towards an explanation of these remarkable phenomena, I may do harm rather than good by contributing to give an undue significance to what may not prove the most important features. I will only permit myself therefore to point out one or two practical considerations suggested by the facts now under notice.

1. Recent researches in meteorology have shown us, that when we confine our attention to the *mean* annual and *mean* diurnal values (of the temperature for example) in different parts of the globe, we derive but a small part of the instruction which the observation of nature is capable of affording. A similar remark applies with equal justice to the consideration of the variations of the magnetic elements. In the Annual Variation now before us, even a very cursory inspection is sufficient to show, that as the same months occupy positions on opposite sides of the mean line at different parts of the twenty-four hours, the *mean* Annual Variation, or that which is shown by the mean values in each month taken from *all* the observation hours, must be merely a residual quantity; and that consequently natural features must be more or less masked in deductions in which only mean values are brought into view. In fact, as has been shown in the published volumes of the observations at St. Helena and Hobarton, the *mean* annual variation at those stations is so small as to be scarcely sensible, more particularly at Hobarton, where its whole range amounts to not more than a small fraction of a minute of arc. But when we resolve these *mean* results into their respective constituents, viz. the Annual Variation *at each of the observation hours*, there is then at once disclosed to us an order of natural phenomena,

very far from inconsiderable in amount, systematic in general aspect, and apparently well deserving the attention of those who are occupied in the delightful and highly intellectual pursuit of tracing the agencies of nature.

2. We perceive in the variations of position of the several months in the annual range, the necessity of paying regard to the period of the year, as well as to the period of the day, at which observations have been made, which do not include long intervals, and from which nevertheless inferences are drawn in respect to secular change. Such observations, when not those of a fixed observatory, are usually made at some hour in the day-time, when it needs only a glance at the Plate to perceive that Annual as well as Diurnal variation-corrections are required, unless the month as well as the hour is the same in the earlier and later observations. A table of corrections for every hour of the day to the mean value in each month—corrections derived, as in the instances now before the Society, from a series of *strictly comparable* observations continued for several years—should be considered, not merely as a desirable, but as an almost indispensable provision, in countries where magnetic surveys are conducted with the degree of perfection of which they are now susceptible.

Woolwich, April 29, 1851; *revised October* 1851.

XXIX. *Investigations into the Structure and Development of the Scales and Bones of Fishes.*

By W. C. Williamson, *Esq., Professor of Natural History, Anatomy and Physiology, in Owens College, Manchester.*

Communicated by Edwin Lankester, *M.D., F.R.S.*

Received June 20,—Read June 20, 1850.

In 1849, I had the honour of laying before the Royal Society a memoir on *Lepidogenesis*, in which I chiefly directed attention to the structure and growth of the dermal teeth and scales of Ganoid and Placoid fish*. Since the completion of that memoir, a large proportion of such leisure hours as could be snatched from the active duties of professional life, have been devoted to a still wider range of inquiry connected with the same subject. During the interval, I have made a great number of sections and other microscopic preparations of scales belonging to M. Agassiz's Cycloid and Ctenoid orders, as well as of the Ostraciont family of his Ganoid order: the results of the investigation having convinced me, that the structure of these organisms has hitherto been very imperfectly known, I am not surprised that their genesis and development have been involved in considerable obscurity.

A list of the various writers who have preceded me in this inquiry, was given in my last memoir. From this, it will be necessary to select two of the most modern observers, and to notice what their respective views are, in order that we may comprehend the bearing of my more recent observations upon those of my predecessors in the study: these are, M. Mandl and M. Agassiz.

M. Mandl has published an elaborate memoir on Cycloid and Ctenoid scales, in vol. ii. of the 'Annales des Sciences Naturelles,' in which he has developed his peculiar views respecting them. He has arrived at the conclusion, that Cycloid scales consist of *two* portions. "Nous avons ainsi acquis la preuve que la plupart des écailles sont composées de deux couches superposées; l'inférieure offre la structure des cartilages fibrineux, la supérieure, celle des cartilages à corpuscules; cette dernière est pourvue en outre de lignes, dont nous démontrons l'origine, par la fusion de cellules primitives; ces deux couches sont parcourues par des lignes longitudinales, qui appartiennent aux deux couches†." Observing certain radiating lines proceeding from the centre towards the circumference, especially in the anterior portion of the scale, he concluded that they were nutrient canals, which conducted the fluids from the skin to the centre of the scale, which he designates "le foyer," and which he

* Philosophical Transactions, 1849, Part II. p. 435.

† Annales des Sciences Naturelles, vol. ii. p. 348.

regards as the centre of nutrition: "un lieu où le tissu se trouve dans son dévelopement." He then proceeds to explain the origin of his upper layer of "lignes cellulaires" as he terms them; meaning thereby, if I understand him aright, the concentric lines seen on the *surface* of the scale, and which he appears to imagine extend completely through its upper layer. "Nous avons par ces recherches acquis la certitude que ces lignes doivent leur origine à des cellules qui, primitivement, se forment dans la couche supérieure de l'écaille, placées sur une base; que peu-à-peu ces cellules se remplissent, s'allongent, et finissent par représenter des lignes plus ou moins larges, qui tout au plus, par un bord inégal, révèlent leur nature primitive*." M. MANDL then goes on to point out, very correctly, the structure of the inferior layer, which he describes as "composée de lamelles fibreuses, dont les fibres s'entrecoupent sous des angles réguliers, mais qui toutes suivent la même direction dans le même lamelle†." From these and other supposed facts, he draws the following general conclusion as to the way in which these scales have been formed—"Si nous voulons appliquer les résultats que nous avons obtenus dans l'étude de la structure intime des écailles, à l'explication de la manière dont elles se forment, nous verrons tout d'abord qu'il importe de bien distinguer la formation de la couche supérieure, et celle de la couche inférieure. La première, composée de cellules et de leurs bases avec le tissu qui contient les corpuscules, prend son dévelopement par des accroissements qui ont lieu dans la périphérie, autour des lignes cellulaires; au moyen, de pareils accroissements, ils forment, non-seulement plusieurs lignes cellulaires, mais les canaux longitudinaux eux-mêmes se trouvent allongés. Il est très probable que ces lignes cellulaires ne se forment pas, seulement, l'une après l'autre, mais que plusieurs lignes sont produites simultanément; nous en trouvons une preuve dans les écailles, qui dans leurs accroissements successifs, conservent les espaces marginaux, et dont les lignes cellulaires ou les cellules sont ainsi séparées en plusieurs groupes; nous citerons par exemple les écailles de *Cobitis fossilis*. Mais cet accroissement dans la périphérie n'expliquerait nullement, la grande épaisseur du milieu; nous en trouverons la cause dans la formation de la couche inférieure. Nous avons vu que celle-ci est composée de plusieurs lamelles. A chaque accroissement se forment toujours des nouvelles lamelles: les canaux longitudinaux, qui parcourent toute l'écaille, apportent les sucs nécessaires pour qu'une formation uniforme d'une nouvelle lamelle puisse s'opérer dans toute l'étendue de l'écaille. Il s'ensuit, que les anciennes lamelles étant plus petites, l'épaisseur doit s'augmenter, à mesure que l'on se rapproche du foyer‡."

I have introduced this long quotation, because, though abounding in errors, it is in some respects more correct than the hypothesis of other writers. The arrangement of the fibrous layers as well as of the fibres in each layer are accurately described. The longitudinal canals, we shall find, are no canals at all, and the *cells* which play so important a part in M. MANDL's views on *Lepidogenesis* are equally devoid of existence; we shall find that he has mistaken the nature of some solid calcareous

* *Ut supra*, p. 354. † *Ut supra*, p. 358. ‡ *Ut supra*, p. 363.

granules which are formed within the fibrous tissues of his inferior layers, and conceiving them to be cells, has built upon them the hypothesis just quoted.

The publication of this memoir, controverting as it did, some of the opinions of M. Agassiz, elicited from that distinguished Ichthyologist an elaborate reply which was published in vol. xiv. of the same journal. He then considered that each scale consisted of but one substance, and that the existence of two layers, except in certain cases, was an error*.

Subsequently, however, M. Agassiz discovered his own error on this point, and became convinced that each of these scales did really contain two different structures. In his great work on fossil fish, whilst correctly pointing out the non-existence of the longitudinal canals of M. Mandl, which he considers to be merely grooves in the upper layer, he observes, "On se convaince alors que chaque écaille de Cycloide est formée de deux couches distinctes et superposées, qui toutes deux sont lamelleuses, mais dont le plan de stratification est différent, la couche interne étant continuée entière, tandis, que la couche externe, qui seule montre des lignes concentriques et une grande partie des sillons, est souvent interrompue." Still, however, denying the existence of fibres in the lower layers, which he says are transparent, sometimes yellowish, never containing cells or corpuscles, and are softened by maceration, he proceeds to describe what he believes to be the structure of the upper layer. "Au dessus de cette couche se trouve une seconde couche qui porte en elle les conditions des ornemens de l'écaille. La substance de cette couche est plus dure, plus cassante et plus transparente que celle de la couche inférieure; elle n'a jamais cette teinte jaunâtre, et dans la plupart des cas, on n'y distingue aucune structure particulière. Dans les écailles minces c'est comme un vernis séché et racorni, dont on aurait couvert la superficie de l'écaille. Mais dans les écailles épaisses des Labres, on aperçoit distinctement sur les coupes verticales, des traces de stratification, mais d'une stratification discordante avec celle de la couche inférieure. Le plan des lames ne répond pas à celui de l'écaille tout entière, mais elles sont couchées l'une sur l'autre comme les tuiles d'un toit, et plus ou moins imbriquées†." With reference to the corpuscles of M. Mandl, he observes, "Les corpuscules, qui dit on, forment une couche particulière au milieu de l'écaille, se voient, il est vrai, assez souvent, sous la forme d'ovales ou de carrés à contours ombrés et indistinctement limités;" but he doubts their being true corpuscles, and rather refers the appearance to some solution of continuity between the upper and lower tissues‡: the latter of these tissues he conceives to be not fibrous, but of a horny texture, and an exuded secretion from the sac into which he considers the lower and anterior portions of the scale to be fitted. After pointing out the resemblance that exists between the scales of Cycloids and Ctenoids, M. Agassiz goes on to give his interpretation of the nature of the peculiar teeth in the margin of the scales of the latter order. In some, which have only *one* marginal row of teeth, as Corniger and Myripristis, he thinks that "ce sont tout simplement des

* *Ut supra*, vol. xiv. pp. 104, 105. † Poissons Fossiles, vol. i. p. 70. ‡ *Idem*, p. 72.

échancrures plus ou moins profondes du bord postérieur de la couche superficielle*." He observes others, however, which appear to be "corpuscules propres, implantés sur la couche supérieure de l'écaille; ce sont des formations tout-à-fait analogues à la couche supérieure et qui sont déposées sur cette dernière, comme les lignes et les bandes de la couche supérieure le sont quelquefois sur la couche inférieure†."

It will be seen from the details about to be brought forward, that both these distinguished observers have fallen into considerable errors, which affect alike their respective interpretations of the structure, and of the growth of Cycloid and Ctenoid scales. There is no doubt that M. Mandl is quite correct in distinguishing an inferior portion, composed of fibrous lamellæ, from the calcareous covering with which it is surmounted; and also in describing the fibres of each lamella as being parallel to each other, whilst they traverse those of contiguous lamellæ in a diagonal direction. But we shall find that no form of cartilage enters into the composition of these scales; their upper portions contain no cells or cartilage corpuscles; and so far from having been formed in the way described by M. Mandl, this superior calcareous layer consists of *two* very distinct substances, which differ alike in their structure and in their genesis. As an inevitable result, the whole of M. Mandl's hypothetical conclusions fall to the ground.

On the other hand, M. Agassiz has failed to detect either the fibrous nature of the inferior layers, or the existence of *two* distinct structures in the upper or calcified part of the scale; and whilst he has obviously seen the small calcareous granules which we shall find are distributed along the line of junction between the middle and lower layers, and which M. Mandl regarded as cells or cartilage corpuscles, he also has wholly mistaken their nature; the consequence has been the formation of an hypothesis which is as little tenable as that of M. Mandl.

The true structure of these thin scales can only be successfully studied by means of vertical as well as horizontal sections. The preparation of such sections from thin flexible scales, whose entire thickness is often not more than from $\frac{1}{300}$th to $\frac{1}{400}$th of an inch, is, as I have learnt from painful experience, only to be accomplished after long practice in the preparation of microscopic objects. Neither is it a task that can be well delegated to the lapidary. If the observations are to possess value and trustworthiness, the preparations must all be made by the observer himself; since an exact knowledge of the direction in which such sections traverse the scale is essential to the right comprehension and interpretation of the structures which they reveal. Without this knowledge all would be confusion and obscurity. Such sections may, however, be prepared even from the thinnest of scales. In all their essential characteristics, the Cycloid and Ctenoid scales are constructed upon the same plan. The differences which exist between them we shall find to be very trifling, and indeed some examples exist where it becomes difficult to determine to which of the two groups an individual scale belongs.

* Poissons Fossiles, p. 73. † *Idem*, p. 73.

The first scale to which I would direct attention is a very large one from the Bay of Dulse, on the western coast of Central America, and for specimens of which I am indebted to Sir Philip de Malpas Grey Egerton, Bart. I have not been able to learn the name of the fish to which it belongs, but from its large size, it furnishes such a beautiful illustration of the internal structure of this class of scales, that notwithstanding my ignorance of its name I avail myself of the information which it supplies: its natural size, and the aspect of its upper surface are represented in Plate XXVIII., fig. 1. It is of the Ctenoid type, but one of those examples which approaches very closely to the Cycloid form. Fig. 1 *a* is its posterior, and 1 *b* its anterior margin; each of which respectively present some peculiarities of structure. A few concentric lines are seen on various portions of its upper surface, which exhibits to the naked eye a frosted appearance, like that given to the vegetation by the hoar-frost on an autumnal morning. Posteriorly and laterally, bundles of divergent lines ornament this surface; they proceed somewhat irregularly, from near the centre towards the margin of the scale, being small grooves in its uppermost layer, which correspond with the "canals" of M. Mandl.

On making a vertical section of this fine scale along the line fig. 1, *a*, *b*, we obtain the structure represented in fig. 2, portions of which, still more highly magnified, are shown in figs. 3 and 4. We at once see that not only is the membranous lower layer (figs. 3 *a* and 4 *a*) distinct from the calcareous one, but that the latter portion consists of two distinct structures; a superficial (3 *c* and 4 *c*) and an intermediate one (3 *b* and 4 *b*). Fig. 3 represents a portion of the section (fig. 2) as seen at *a*, but viewed under a magnifier of still higher power. Inferiorly, it consists of numerous membranous laminæ (3 *a*) arranged in parallel horizontal lines. We learn from fig. 2 that these laminæ are most numerous in the centre of the scale, and diminish in number as we approach its margin, until we arrive at the extreme periphery, where but one exists. This is partly shown in fig. 4, which represents the portion of the section, fig. 2, as seen at *b*, but more highly magnified. We here see (4 *a*) the same membranous lamellæ as in fig. 3, only diminished in number and thickness, and which successively run out as we approach the margin of the scale. This margin is not quite reached at 4 *f*, but nearly so, since we find that the number of the visible membraniform laminæ is reduced to two.

Each of these membranous laminæ consists of numerous hair-like fibres, all those entering into the composition of one lamina being arranged in parallel lines; fig. 5 represents the horizontal aspect of portions of two such laminæ, showing the diagonal manner in which the fibres of each are arranged. The detached fibres seen at fig. 5 *a* exhibit a marked tendency to curl up in the way that those of the yellow fibrous tissue (ligamentum nuchæ) of mammals are well known to do.

Imbedded amongst these membranous laminæ, we see numerous isolated, lenticular calcareous bodies (figs. 3 *d* and 4 *d*), each of which exhibits a definite series of concentric lamellæ, with traces of a central longitudinal fissure. These small bodies have

not been developed *between* two of the membranous layers, their development causing the latter tissues to diverge, but are the result of a calcification of the membranous laminæ, which they incorporate into their structure. They commence as a small calcareous atom, and increase in size by the external addition of new concentric laminæ, the direction of the latter not being parallel with, or having any reference to that of the laminæ of fibrous membrane which they so amalgamate; thus they are not depositions *from*, but growths *in* the membrane; which growths, as they increase in size, retain their primitive tendency to assume a lenticular form.

On carefully examining the middle layer of the scale (figs. 3 *b* and 4 *b*), we perceive that it is chiefly composed of an aggregation of such small lenticular bodies, which appear to coalesce as they increase in size; the newer calcareous additions sometimes lose their lenticular contour from having to enclose several lenticular granules which have so coalesced. This is seen at fig. 4 *e*.

As we approach the margin of the scale, we perceive that this middle layer gradually diminishes in thickness until it becomes reduced to a few scattered calcareous granules, like those which remain isolated in the membranous portion of fig. 3; thus this calcareous layer is not only thicker, but is more consolidated towards the centre of the scale than near its margin. On becoming thus confluent and consolidated, these granules assume a new aspect. The layer, thus formed, splits up into horizontal laminæ, which correspond, as to their direction, with that of the membranous laminæ prior to their calcification; the latter having apparently influenced the direction of the former. They also exhibit numerous small vertical subdivisions disposed at irregular intervals.

On examining a horizontal section passing through these secondary calcareous laminæ, we meet with an appearance like that represented by fig. 6. The outlines of the small lenticular granules are still visible (fig. 6 *a*), presenting varying degrees of distinctness. But in addition to these, the section is also traversed by numerous small and slightly curved fissures; the principal ones in each lamina (fig. 6 *b*) have a decided tendency to run in one direction, which, as in the case of the fibres of the membranous portion, is different in two contiguous portions. Minor fissures also exist, either fringing the larger ones at an acute angle (6 *c*) or forming an additional series, arranged at right angles to those which constitute the principal group.

In some instances these fissures appear to take their rise from primary ones which pass through the centre of many of the lenticular bodies (fig. 3 *d*); but more frequently they commence in small open spaces left between these bodies, which, when the specimen is mounted in Canada balsam, are seen to contain air. These fissures obviously correspond with the small vertical ones which intersect the calcareous laminæ in the section fig. 4 *e*. Fig. 6 exhibits their horizontal arrangement as seen in three contiguous laminæ, when viewed by transmitted light.

The third or uppermost layer (figs. 3 *c* and 4 *c*) is very different from the pre-

ceding one, both in its structure and in the way in which it has been formed. It is the one, by various modifications of which are produced all the ridges and tubercles which ornament the surface of the scale: along the greater part of the vertical section fig. 2, it exhibits the appearance, which, when more highly magnified, is seen in fig. 3 *c*; it has an undulating outline, and presents indistinct traces of a lamellar structure, the more external lamellæ being parallel with the superior margin of the section. The surface of this portion of the scale, when viewed horizontally by means of reflected light, is seen to be studded with innumerable minute tubercles.

When we approach the anterior margin of the scale (fig. 1 *b*), we find that the radiating lines already spoken of, are produced by the absence of the superficial tissue along their course, whilst in the intervening ridges it is of considerable thickness; each ridge thus formed is transversely subdivided by very numerous minor ridges, the upper edge of each of which is sharp and crenulated. This structure will be readily understood by a reference to fig. 7, which represents a small portion cut out and viewed as an opake object; 7 *a* are the lines along which the uppermost layer is wanting. These lines are homologous with those which M. Mandl regarded as nutrient *canals*, an error which we have already seen to have been detected and in part corrected by M. Agassiz. But whilst the Swiss philosopher very properly pointed out their nature, he also erred in considering that they passed through the entire calcareous portion of the scale and reached the subjacent soft tissues. It is perfectly true that they do so at the margin of the scale, where the middle layer is not yet developed, as seen in fig. 7; but towards the central portion of this and all analogous scales, where the middle layer exists, these grooves do not pass through it, except in some scales where at the anterior margin *all the three* tissues are cut through and the border of the scale is converted into a series of digitations, as is the case in that of the Perch: this however is merely an incidental circumstance, and does not affect the true nature of these grooves as they exist on the upper surface of the scale. The mistake made by M. Agassiz was one easily fallen into, he not being aware of the duplex character of the calcareous portion of the structure.

Fig. 7 *b* represents the ridges of the upper layer with their transverse crenulated subdivisions. These latter are the true homologues of the concentric lines commonly seen on the surface of Cycloid and Ctenoid scales, but which in this instance are only developed on the surfaces of the radiating ridges; 7 *c* are the extremities of the ridges as seen in the vertico-transverse section, whilst at 7 *d* we have an analogous vertical section, only made parallel to the superficial ridges, and in which the subjacent membranous laminæ (7 *e*) are seen rising in succession towards the surface of the scale.

These subdivisions of the superficial layer are the most conspicuous at the extreme anterior and lateral margins. As we approach the centre of the scale they become less definite, and are ultimately lost amongst the irregularly disposed superficial tubercles, of which the ridges are only a modified form. But at the posterior or free margin we find this upper layer assuming a new aspect. The superficial tubercles

are not only more definite in their contour, but become isolated, and at the extreme margin assume the form of small, flat and regularly arranged hexagons. Their appearance is shown in fig. 8, which represents a fragment removed from this part of the scale, *a* being its posterior and *b* its anterior portions.

This uppermost layer covers the entire scale, even to its extreme periphery. In this respect, as well as in its internal structure, it differs from the subjacent calcareous portion. The latter becomes thinner, less consolidated, and finally ceases to exist at some little distance from the margins, as seen in fig. 4; on the other hand, the former exists on the upper surface of each of the membranous laminæ, as their margins successively rise towards the surface of the scale; and even at the periphery, where but one such lamina occurs, it is still covered over by the same superficial structure. This is a circumstance of importance, as tending to confirm the distinctness of the two calcareous structures. Though the way in which the superficial layer has increased in size is not very clear in this example, there are evidences, not only that it possesses a laminated structure, but that the lamellæ constitute so many additions that have been successively made to its upper surface. The examination of other scales, shortly to be noticed, will be seen to confirm this conclusion.

In the essential features of its internal structure, the scale now described may be received as a type of all the Cycloid and Ctenoid scales which I have examined. The increase in its size has evidently been accomplished by the addition of successive membranous lamellæ to the inferior surface of those previously formed; each new layer being rather larger than its predecessors. It is also obvious that the middle layer is produced by the formation and coalescence of the small lenticular bodies through the agency of which the calcification of the membranous laminæ is effected. This calcification permeates the entire extent of the upper and earlier-formed lamellæ, whilst with the exception of a few isolated granules, it has been confined to the margins of those which are inferior and of more recent growth; a distinction which will be readily comprehended on referring to the arrangement of these laminæ as seen in fig. 2. The scale of the Carp (*Cyprinus carpio*, Linn.) affords a beautiful additional illustration of the true nature of these structures, and more especially of the uppermost layer. A small portion of a vertical section of this scale, made in a direction parallel to the lateral line of the fish, is represented by fig. 9. The lower series of membranous laminæ are seen at 9 *a*; 9 *b* is the middle calcareous layer, and 9 *c* the superficial one. The lower membranous laminæ are arranged in precisely the same way as in the preceding scale, and the middle layer (9 *b*) only differs in the smaller size of the calcareous lenticular granules and in their being closely aggregated along the calcifying surface 9 *d*, instead of being also scattered through some of the lower membranous laminæ. After their coalescence and consolidation they break up, as before, forming a secondary stratification parallel with the original direction of the membranous layers; and when the inferior surface is viewed by transmitted light, it exhibits appearances which closely correspond with those seen in fig. 6.

On directing our attention to the uppermost layer, 9 *c*, we find a considerable difference presenting itself. When the scales of the Carp are *in situ*, the free portion of each, or that which is not covered over by its antecedent neighbours, is very rough, owing to the existence of numerous tooth-like points which project from its surface. It is this portion of the scale that has been traversed by the section fig. 9, and from it we learn that the existence of the superficial teeth is the result of an extraordinary development of the uppermost layer, which has not only produced these projecting appendages, but has also thickened the intervening portions. Nothing can be more distinct than the structures of the central and superficial tissues, as they exist in this scale. The superimposed lamellæ, of which the latter of these consists, are most obvious, even under a low magnifying power; and on tracing their direction, we see that it corresponds with the superficial outline of the section, evidently showing that its increment is effected at the upper surface. This appears to have been accomplished by the calcification of a very thin membrane with which the scale is covered, at the same time that the corresponding, though different, process was making additions to the *lower* surface of the middle layer along the undulating line, 9 *d*. The structure of these two portions is as distinct as their modes of growth. The middle tissue (9 *b*) exhibits the rough laminæ, filled with vertical fissures and traces of the lenticular granules of which it consists. The lamellæ of the uppermost one, on the other hand, appear to be perfectly structureless; consisting, apparently, of a smooth and homogeneous tissue, not to be distinguished, in the vertical section, from that seen in the ganoin of a *Lepidosteus* or some of the *Lepidoti*; thus the differences of aspect and of growth alike bring us to the conclusion that the scale of the Carp consists of three very distinct structures. At the *anterior* portion of the same vertical section, we find that the middle layer disappears as in the preceding scale, whilst the upper one rests immediately upon the membranous lamellæ. Here, though very thin when compared with its more largely developed posterior portion, the superior layer exhibits numerous minute tooth-like projections, which, whilst they exhibit the same laminated structure as the larger ones previously noticed, are more uniform in their size as well as more regular in their distribution. These projections are produced by the vertical and transverse section of the numerous concentric lines which give the cycloid aspect to this portion of the scale. It thus becomes manifest that these concentric ridges are not lines of growth, as thought by M. Mandl, but the result of a peculiar arrangement of the superficial tissue of the scale, a conclusion which accords with that arrived at by M. Agassiz.

The scales of the Pike, Salmon, and numerous other allied fish, are constructed on a similar plan to those already described, varying only in their minor details. A remarkably fine illustration of the type requiring a more special notice, is that of the Perch (*Perca fluviatilis*, Linn.), which also illustrates the peculiarities of the Ctenoid scale. In a vertical section of one of these, we observe that the number of the small lenticular granules which are isolated and dispersed through the membranous lamellæ,

is much greater than usual. On inverting one of the scales, and examining these lamellæ by means of transmitted light, we perceive that the calcareous granules are arranged on a more uniform plan than in the examples previously described; they are not only in layers, but the individuals of each layer have a tendency to follow one determinate direction, being arranged with their longer axes parallel to one another.

Fig. 10 represents the appearance presented by a horizontal section of this portion of the scale. When highly illuminated, the membranous tissues become so transparent as to be almost invisible, and the distribution of the calcareous granules then becomes very obvious. Those occupying the lowest plane when the scale is *in situ* (but which the inversion of the scale brings the nearest to the object-glass), appear as innumerable minute and almost invisible granules. As we successively bring the superior laminæ into focus, we observe that the granules composing each layer become progressively larger as we penetrate the scale. When isolated, they ordinarily exhibit an obtusely fusiform contour, but as they grow in size they frequently become confluent. The increase in their size is produced partly by the confluence of previously detached individuals, and still more generally by the addition of new layers of calcareous matter round their exteriors. Indeed the latter process has apparently been the direct cause of the former condition.

In fig. 10, three such layers are represented as they appear in the inverted scale: 10 *a* is one of the lower series; *b* is a layer of larger granules, some of which (*b'*) have become confluent; and at *c* we observe a faint indication of a still higher series, which when brought into view are found to have coalesced, forming in fact the inferior surface of the middle tissue of the scale. This representation also shows, what I have already alluded to, that whilst all the granules composing one layer exhibit a uniform tendency to arrange themselves in one direction, those of different layers cross each other more or less diagonally. Bearing in mind the arrangement of the primary fibres of which these laminæ of membrane consist, it seems probable that the direction of these fibres influences that of the calcareous granules.

As we proceed horizontally from the centre to the periphery of the scale, we find a similar change to occur as when we pass from the upper to the lower layers. After a certain time, the additions to the inferior surface of the calcareous layer almost cease to be made in the centre of the scale; at all events, they do not continue to be developed as rapidly as in an earlier stage of growth, and as they still are nearer the margin of the scale. Hence many of the laminæ which have become calcified towards their periphery, continue to be membranous in their central portions, or, at the most, only contain a few detached granules; these however are often of a large size, and by their successive junctions with contiguous granules lose their fusiform contour and become more or less square or cuboid; in this state they are manifestly the objects noticed by M. Agassiz, "sous la forme d'ovales ou de carrés à contours ombrés et indistinctement limités," and which he mistook for accidental lesions which separated his superior from his inferior tissue.

As we leave the centre and proceed towards the circumference of the scale, its inferior membranous laminæ successively enter the field of vision. These contain more numerous granules, but they are of smaller size. As we come nearer to the margin, the granules become still more numerous, but exhibit a corresponding diminution in their bulk, until, at the edge of the scale, the otherwise transparent membrane assumes a hazy aspect from the presence of the innumerable minute calcareous atoms which are diffused through its substance.

Whilst the scale of the Perch thus supplies a beautiful illustration of the way in which the calcareous middle layer is gradually produced, it also explains the peculiar structure which induced M. Agassiz to constitute the Ctenoid a distinct group from the Cycloid order, distinguishing it by the teeth which fringe the posterior margin of the scale.

Fig. 12 represents a small portion from the surface of the posterior part of the scale of the Perch, *a* being its marginal extremity. The marginal teeth (12 *b*, *c*) are long, pointed posteriorly and dilated anteriorly; they are not raised vertically, like those on the scale of the Carp, but are depressed to the level of the surface of the scale. Behind each of these teeth we see that there exist numerous bases of similar teeth (12 *e*), which in the earlier stages of growth successively fringed the posterior margin of the scale, but of which the apices appear to have been worn away prior to the development of a new series; the abraded extremity of each tooth abutting against the dilated and concave base of its successor in such a way as almost to give the impression of a true articulation. These teeth are arranged in an alternating order, as if, when each new marginal growth had taken place, new teeth had only been added to the front of every other perpendicular row; thus the individuals *b* and *c* belong to different series, the former being apparently of a more recent formation than the latter; the remains of those in the more central parts of each row exhibit the same arrangement. Fig. 12 *d* is the soft membrane within which these teeth are imbedded.

Fig. 11 represents a vertical section of one of these perpendicular rows of teeth, showing their relation to the other tissues of the scale. 11 *a* is one of the unbroken marginal teeth, whilst *b* are the remains of those of earlier growth; *c* is a thin membrane which invests the *upper* surface of all the teeth; *d* is the peripheral portion of the lowermost of the membranous laminæ of the scale, and on the surface of which, if not actually within its upper tissues, the calcareous tooth *a* has been developed; *e* represents a superior lamina, in which the very minute granules forming the middle layer are beginning to appear.

On tracing these marginal teeth back to the middle of the scale, we find that they are but modifications of its third or uppermost layer, thus corresponding with figs. 3 *c* and 4 *c*, being more especially the exact homologues of the tesselated hexagons seen in fig. 8, which, it will be remembered, represents a similar fragment taken from the corresponding portion of the large scale, fig. 1.

The question which now suggests itself is, what relation does the superior invest-

ing membrane (11 *c*) bear to the inferior fibrous portion (11 *d*)? We have already seen that all the scales just described require the existence of a superficial membrane to render the growth of their superior calcareous layer intelligible. This we found to be especially the case in the scale of the Carp, though the latter is but a highly developed example of the common type. In that of the Perch we have demonstrative evidence of its existence. Whether it is, in the first instance, merely the upper portion of a lamina (11 *d*), within the substance of which the teeth *a* and *b* are developed, or whether it is an *inflected prolongation* of the same lamina, modified in its structure, and which, after turning round the apex of the tooth, 11 *a*, is extended over the entire surface of the scale, I am unable to say. Analogical reasoning, based upon the structure of the ganoid scales, would lead us to the latter conclusion, in which case each new lamina of membrane, though apparently added to the inferior surface of the scale only, would really enclose the entire structure like a capsule. If, on the other hand, the former be the real interpretation, it is obvious that the *upper* surface of this superficial membranous structure must possess the power of perpetuating its existence by continuing to be developed superficially, as its *lower* fibres become gradually calcified and amalgamated with the substance of the uppermost calcareous layer. Be the process of its genesis what it may, we have here demonstrative evidence of the existence of such a superficial film of soft membrane as is essential to my hypotheses, accounting for the peculiar structure and growth of the uppermost layer. The production and growth of fibrous tissues has long been one of the most perplexing subjects in physiological science; and the peculiar forms of fibrous tissue existing in these scales, certainly do not diminish the difficulties with which the subject is already invested. Nothing here indicates a cellular origin of the fibres.

In the scale of the Grey Mullet (*Mugil Capito*, Cuv.), the posterior marginal teeth, though also pointed, are much shorter and broader, verging even to a rhomboidal form. Their persistent bases, as seen at a little distance from the margin of the scale, have received new calcareous additions to their upper surface, in the form of a crest of minute mammillæ, which are especially evident at the posterior border of the new growth. On examining the anterior margin of the same scale, we see that the concentric lines common to nearly all cycloid and ctenoid scales, are smooth at the periphery, whilst at a little distance from it, they also have received additions of the same mammillary character. These additions are the most obvious along the raised ridges which produce the tooth-like projections seen in fig. 3, only in the Mullet, as in the Perch, these ridges are much more acute, corresponding in this respect to the small transverse crenulated subdivisions of fig. 7. In these fishes, also, the continuity of these ridges is only interrupted by a small number of the grooves radiating to the anterior margin of the scale, so many of which exist in the Mexican example, 7 *a*.

An important homological question now suggests itself: what relation does this third tissue bear to those existing in the ganoid scales especially of Sauroid and

Lepidoid Fishes? I confess I see no way of distinguishing it from the true ganoin itself, using the term in the restricted sense in which it was employed throughout my last memoir already referred to. If the surfaces of these ganoid scales, instead of being smooth, as is usually the case, had been fretted and sculptured with as infinite a diversity of minute points and ridges as are seen on those of Cycloid and Ctenoid fishes, I suspect that the former would have lost very much of the shining aspect to which they owe their name. It appears to me that there is no real difference between the superficial calcareous layer of a perch or a salmon, and the ganoin of a *Lepidosteus*. This is, however, a point on which differences of opinion will doubtless exist; I confess that it is equally difficult to detect any difference between the same tissue as seen in the vertical section of a scale of a carp, and the laminæ in some of the bones of a pike and other osseous fish, the bones of which contain no lacunæ. In a subsequent portion of this memoir we shall find that the latter are merely the calcified lamellæ of a periosteal membrane, exhibiting no visible traces of minute structure. The lamellæ covering the scale of a carp are neither more nor less than this.

Additional light is thrown upon this portion of our subject by the study of some other scales, to which I will now direct attention. One of the most curious of these belongs to a species of *Balistes*, one of the File-fish, in the collection of Sir Philip Egerton.

In their external aspect these scales bear a close resemblance to those of many ganoid fish, exhibiting the rhomboidal form and peculiar tessellated arrangement seen in the members of that group, in which M. Agassiz has arranged it. On making a vertical section of one of these scales, we find that in its internal structure it approximates closely to the type prevailing amongst the Cycloids and Ctenoids, retaining however one or two curious points of the resemblance to the Ostracionts. A representation of a vertical section, made parallel to the lateral line of the fish, is given in fig. 13; *a* is its anterior extremity, which is overlapped as far as *b* by the free margin of the antecedent scale; *c* is its posterior border, the sloping inferior surface of which reposes in like manner upon the anterior portion of the scale behind it. As in the Cycloids, we have three very distinct horizontal tissues, lower, middle and upper.

The lower one (fig. 13 *d*) consists of parallel laminæ of membrane; but in addition to its horizontal fibres, we here see that there are numerous thick fibres which pass obliquely upwards from layer to layer, binding them together; we shall afterwards find that these oblique fibres belong to a peculiar type prevailing amongst the Ostracionts, where they produce some very remarkable and beautiful structures. In the present example I do not perceive that their existence materially affects the conformation of the middle layer, fig. 13 *e*, which is a calcareous one, formed in precisely the same way as the corresponding layer amongst ordinary Cycloid and Ctenoid scales, viz. by the development of small round and lenticular granules within the fibrous tissues of the membranous laminæ. In their aggregation these granules leave a

larger number of open interstices than occur in those scales already described. This may partly be owing to the existence of the oblique fibres just referred to. In other respects they present no material distinctive feature. The layer thins out as we approach the margin of the scale. The consolidated tissue first disappears, leaving only a layer of minute detached granules, which also soon cease to exist, and the marginal portions of the scale consist only of the uppermost and lowest tissues.

It is the peculiar aspect of the superficial layer which gives the chief interest to this curious scale. That portion of its external surface, which, when *in situ*, is not covered over by the antecedent scales, exhibits an aggregation of numerous large papillæ. The vertical section shows us that these are produced by an extensive development of the upper layer, which is so thick in this species, that it occupies one-third of the vertical diameter of the scale. At 13 f', where the preceding scale has rested upon it, this tissue is level and smooth; but at its anterior half its development has produced the large tubercles already referred to. All these various portions consist of numerous minute lamellæ, arranged in precisely the same way as the homologous ones in the scale of the Carp, fig. 9 *c*. The growth of the tissue has evidently been effected by the addition of new lamellæ to the upper surface of the pre-existing ones.

These lamellæ are perforated by a dense network of anastomosing canals which run in every direction; they are the largest at the inferior portion of the tissue, where it is contiguous to the middle layer, and where they exhibit a marked tendency to radiate from the centre to the circumference of the scale. These give off numerous anastomosing branches, which, as they ascend, diminish in size, and finally open by myriads of minute apertures at the external surface.

This peculiar structure is wholly new to me. It is very different from those dentine-like tissues, to which, in my former memoir, I applied the name of Kosmine. The anastomosing tubes have more of the character of the Haversian canals seen in the vertical sections of the scale of *Megalichthys*, only the lamellæ which they penetrate contain none of the lacunæ seen in that very beautiful example. They bear even a still closer resemblance to the small canals which penetrate the true *osseous* dorsal spines of some Siluroid fish, which pass through lamellæ crowded with lacunæ, and are obviously the homologues of the true Haversian canals of the higher vertebrata. If the lacunæ and their canaliculi were absent from the Siluroid spine, the canals in the two examples would correspond as closely as possible; we shall afterwards find that the existence of lacunæ is not at all essential to a true osseous structure, inasmuch as they are absent from the bones of a large majority of the osseous fish: they are an adjunct, but not a necessary one, especially amongst fishes; consequently this superficial structure in fig. 13 becomes but a slightly modified form of bone. It is equally obvious that it is also the homologue of the corresponding portions of the scales of the Carp, Perch and other Cycloid and Ctenoid fish; only we have here, in addition to the lamellæ which it possesses in common

with the examples just cited, these permeating and anastomosing canals. In their respective classifications, AGASSIZ, MÜLLER and OWEN have alike recognized the "ganoid" character of the scale of *Balistes*; hence this "ganoid" structure, homologous as it is with the superficial tissue of the scale of the Carp, becomes a strong argument in favour of my idea that the uppermost layer of the Cycloid and Ctenoid scales is identical in its character with the ganoin existing on those of *Lepidosteus*, *Lepidotus*, and their allies.

We obtain an additional illustration of the way in which these structures are modified and linked together from the scale of the Flying Gurnard (*Dactylopterus volitans*, LACEP.). Anteriorly these scales present a thick and expanded base, but posteriorly they taper away to a thin and narrow point. In the interior of the scale is a number of large irregular cavities with connecting channels of communication, and which give off numerous irregular anastomosing canals, running in every direction. On making a vertical section through the centre of the scale, parallel with the lateral line, we obtain the result represented in fig. 14.

With the exception of a slight modification of part of its inferior surface, this scale is obviously composed of one uniform tissue, which approximates more closely to the common form of bone as it exists amongst the osseous fishes than in any other example with which I have met. In its essential features it also bears a close resemblance to the uppermost layer of the scale of *Balistes*, fig. 13 *f*. It consists of numerous translucent laminæ variously inflected; the external ones lying parallel to the contiguous surfaces, whilst the internal ones follow the outline of the large central cavities which they invest. They are perforated by the canals, Plate XXIX., fig. 14, *a*, *b*, *c*, which run in every direction and communicate with the various surfaces; they are prolonged alike into the thin anterior basilar expansion, 14 *d*, which is implanted in the soft integument and into the posterior acuminate extremity of the scale, 14 *e*. Inferiorly, the horizontal lamellæ (14 *f*) are perforated by a number of branching tubes, which coalesce as they ascend, and finally open into the large central cavity, converting this portion of the scale into a tissue resembling kosmine.

If the uppermost layer of the scale of *Balistes* be really ganoin, as thought by M. AGASSIZ and Professor OWEN, and of which I have no doubt, then nearly the whole of the scale of *Dactylopterus* consists of the same tissue. I am unable to see any essential difference between the two examples. On the other hand, if we compare the structure of this scale with that seen in all the bones of the endoskeleton of the Pike, and especially with the central portions of the epitympanic (fig. 39) or the articular extremity of the opercular (fig. 40), we shall find that no essential difference exists between them, beyond a slight variation in their respective densities and degrees of hardness. In both we have the same inflected membraniform lamellæ, surrounding similar central cavities, and perforated by analogous Haversian canals; whilst a very considerable degree of resemblance exists as to their respective modes of growth, both being produced, as we shall afterwards find, by the analogous calcification of an

investing periosteum; only whilst in the cases of the bones we shall find that the growth is based *upon* a matrix of cartilage, we have not the slightest evidence that such a tissue ever entered into the composition of the scale. At the same time it is not easy to understand how such large cavities as exist in the interior of the latter structure can have been formed.

The same lamellæ, which at fig. 14 *a*, *b* and *c* are penetrated by Haversian canals, and appear to be at once osseous and ganoid, at 14*f* are penetrated by a different set of tubes converting them into kosmine, and allying them with tooth-structures. Connecting this fact with the analogous ones published in my preceding memoir, we can scarcely escape the conclusion that bone, ganoin, kosmine, dentine and the enamel of fishes' teeth, are but modified forms of one common tissue; and though for the purpose of facilitating the interchange of ideas it is necessary to distinguish these modifications by various names, we must bear in mind that no clear lines of demarcation can be drawn between them, or definitions given, which will not in numerous instances fail to be strictly applicable. Inosculating examples occur which may be classed with several of the tissues, and yet without agreeing exactly with any one. This statement applies to each of the characteristic properties exhibited by individual examples of these structures, and of course has reference to the relative degrees of glossiness presented by their external surfaces, as well as the hardness, thickness, colour, and transparency of their respective lamellæ. Hence it is perfectly possible to have "ganoin" without glossiness. We see something approaching to it in the fossil scales of *Dapedius*, and that such is really the case in examples recorded in the preceding pages, I have but little doubt.

Fig. 15 represents a vertical section of the scale of *Loricaria cataphracta*, Linn., a Siluroid fish from Brazil, for which also I am indebted to Sir Philip Egerton. The substance of the scale is of a truly osseous texture, closely resembling that seen in *Acipenser* and *Lepidosteus*, only wanting the curious parallel tubes seen in the latter of these examples. It consists of numerous lamellæ, arranged like those of the *Lepidosteus*. Towards the posterior margin of the scale (fig. 15 *a*) they are wholly parallel to the inferior surface; but at its anterior portion (fig. 15 *b*), where it has supported the free edge of its anterior neighbour, the laminæ turn upwards, and are prolonged over a portion of the upper surface. In this respect its conformation is almost identical with that of the *Lepidosteus*, which it also resembles in its numerous quadrate lacunæ, with their divergent canaliculi, spread out in layers between the lamellæ. An abundance of minute lepidine tubes penetrate the structure from below (fig. 15 *c*). In addition to these tubes, there are many large Haversian canals (fig. 15 *d*), resembling those which in my preceding memoir I described as existing in the scale of *Holoptychius sauroides*.

These canals run in various directions, but are especially spread out as an irregular network (15 *e*), immediately below the upper surface of the posterior half of the scale. The latter portion is thickly covered with large mobile recurved teeth,

arranged in rows which are more or less parallel with the mesial line of the fish*. Fig. 16 is a still more highly magnified representation of one of these teeth, with the basis upon which it is planted. Like the majority of the dermal teeth of fish, it is simple, containing an elongated pulp-cavity (fig. 16 *a*), from which numerous kosmine tubes are given off; at its lower extremity (16 *b*) it suddenly becomes contracted, the pulp-cavity being prolonged through the narrowed projecting base (16 *c*).

On the surface of the scale there are numerous small circular cavities (16 *d*), which communicate inferiorly with branches from the network of Haversian canals. Each cavity is surrounded by a narrow projecting rim (16 *e*), upon which the flanging shoulders of the tooth rest, whilst its constricted base is fitted into the inclosed hole, thus producing an arrangement which closely resembles a ball-and-socket joint, and which must allow of a considerable degree of motion in every direction. The tooth is apparently held in its place by a capsular expansion of the membrane (fig. 16 *f*) which covers the surface of the scale. From the direct communication that exists between the cavities (16 *d*) and the Haversian canals (16 *g*†), it is obvious that the latter furnish the pulp-cavity with its nutrient supplies. The one opens directly into the other. It only requires the tooth to be fixed instead of moveable, and depressed instead of acuminate, in order to render it the exact homologue of one of the areolæ in the kosmine of *Megalichthys*. It also reminds us of a very similar connection which exists between the ramified dentinal canals of the tooth of the Pike, and the corresponding ramifications of the Haversian canals in the subjacent bone.

By studying the structure of the above scale, along with that existing in Cycloid and Ctenoid scales, we are enabled to comprehend what was previously obscure, viz. that of the scale of *Macropoma Mantelli* from the Chalk. It is now obvious that the vertical section figured in my last memoir‡ reveals a structure which is identical with that of a cycloid fish; only in the process of fossilization, the lowest or membranous layer has disappeared, leaving the calcareous laminæ of the middle layer, as seen near the margin of one of these recent scales. But in the *Macropoma*, instead of being merely covered with a layer of what I have regarded as ganoin, as in the Carp, Perch, &c., this tissue is surmounted by an abundant development of kosmine, in the form of large pointed teeth§, which closely resemble those of the Siluroid fish just described. They differ little in the two cases, beyond the fact, that whilst in the Siluroid the teeth have contracted bases and are moveable, in the *Macropoma* these bases are expanded, and firmly fixed upon the upper surface of the scale. We thus see how

* Some similar teeth have been described and figured by M. Agassiz, as existing on the scale of *Hypostoma placostomus*, Poissons Fossiles, vol. i. tab. H. fig. 31 and 32. He speaks of their constricted bases and the circular holes into which they are fitted, but refers to the latter as existing in the enamel (émail). In the present scale we have no such substance; it is entirely composed of true osseous tissue; the details of the structure upon which the teeth are fixed are not shown in a very definite manner in the figures.

† Their continuity does not happen to be shown in the specimen from which the above figure was drawn.

‡ Philosophical Transactions, 1849, Part II. tab. 43, fig. 27.

§ *Ut supra*, fig. 28.

the types of several groups of scale structures are blended together in the latter fish. Whilst upon a truly *cycloid* scale are fixed numerous *placoid* teeth, in its perfectly ossified and kosmine-covered opercula, it approaches to *Dapedius granulosus* and other allied forms belonging to the *ganoid* order of M. Agassiz. Such is the extraordinary way in which the few primary elements entering into the composition of the ichthyal skeleton, are modified and combined in order to produce the greatest possible diversity in the resulting structures! How impossible must it be for any truly physiological system of classification to be framed, if it is based only on the relative degree in which any one of the elements composing the dermal skeleton are developed!

I have now to describe some still more singular and beautiful textures existing in the scales or dermal plates of fish belonging to the genus *Ostracion*. It is well known that their exoskeleton is composed of numerous small angular pieces, which do not overlap each other, as is the case with most other dermal scales, but are fitted side by side like the tiles of a tessellated pavement, on the plates constituting the shell of an *Echinus*. I am not aware that their structure has been subjected to any investigation beyond that of M. Agassiz, briefly noticed in the Poissons Fossiles, where, he observes, "la couche inférieure est une substance *cornée*, déposée par lames, et affectant des formes très-diverses dans la même écaille; elle est recouverte d'une couche épaisse de dentine très-bien caractérisée par ses tubes calcifères ramifiés, qui ressemblent en tout aux tubes calcifères des dents." The idea which this distinguished savant had adopted as to the *horny* nature of the fibrous portions of Cycloid scales, has again biassed his judgement and led him into a similar error in interpreting the appearances presented by the dermal plates of *Ostracion*. He gives a sketch* of the vertical section of a scale of *Ostracion triqueter*, the leading outlines of which are very correct, but the more minute structural details are not delineated.

Fig. 17 represents the upper surface of one of these scales (double the natural size), for which I am indebted to J. E. Gray, Esq. of the British Museum. As in *Balistes*, this surface is usually covered with large tubercles. The inferior surface on the other hand is smooth, and exhibits numerous concentric lines, like those represented by fig. 18; only towards the centre of the scale the angular concentric lines become less distinct than in this section, owing to the intervention of successive laminæ of fibrous membrane. Numerous minute apertures, which exist in these laminæ, are the orifices of canals which pass obliquely upwards towards the centre of the scale.

Fig. 19 represents a vertical section of one of these structures made in the direction of the dotted line, fig. 18 *a a'*. On looking at the latter drawing, it will be perceived that, whilst in its central portion the section runs parallel with one of the concentric lines a little to the right of the centre of the scale, at its two extremities it more or less directly traverses those of the two contiguous series.

From the above section, we learn that this curious scale is partly calcareous and partly membranous; the two substances being arranged in alternating horizontal

* Poissons Fossiles, vol. i. tab. H.

layers and vertical subdivisions. The dark concentric lines (*c* in fig. 18) are *membranous* portions, whilst the light intervening spaces (18 *d*), as well as the radii (18 *e*) proceeding from the centre to each angle of the scale, are calcareous. In this preparation a little of the lower surface has been ground away, in order to bring the arrangement of the more internal parts into view. In fig. 19, the upper surface (*a*), the vertical pillars (*b*), the horizontal lamellæ (*c*), as well as the oblique ones (*d*), in which the two last series (*b* and *c*) meet, are all calcareous. On the other hand, the central portion (*e*), the square lateral areolæ (*f*), and the inferior dark longitudinal lines (*g*) are permanently membranous. It will thus be seen that the scale consists of a combination of membranous and calcareous tissues, variously arranged. On examining these different portions still more minutely, beautiful structural details reveal themselves; some of these will be better understood after making a reference to fig. 20, which represents a still more highly magnified view of a portion of fig. 19. The uppermost tissue of the scale, or that to which the superficial tubercles seen in fig. 17 belong, consists of very thin superimposed lamellæ (fig. 20 *a*), traversed by numerous minute branching tubes, which open at the surface of the scale. The outermost of these lamellæ are arranged parallel to the superficial undulating outline of the section, which circumstance, combined with the direction followed by the permeating tubes, leaves no doubt that the growth of these layers, like that of ordinary ganoin, has been the result of successive additions made to the upper surface of the scale.

The structure thus produced is obviously one form of kosmine; but beneath it there is another form which assumes a very different arrangement. Numerous large branching canals (fig. 20 *b*), fringed all round with minute tubuli, enter at the margin of the scale, and proceed towards its interior, traversing the concentric plates seen in fig. 18, nearly at right angles. Their general direction will be best understood on a reference to fig. 21, which represents one angle from the horizontal section of a scale, made in the plane of this series of canals. In the latter figure, they commence by open orifices (21 *a*) at each margin of the scale, and proceed at right angles to that margin, and in nearly parallel lines, towards its interior; sometimes they are simple throughout, whilst at others they unite with contiguous canals by means of anastomosing branches. Owing to their parallel direction, few of these reach the centre of the scale, but meeting similar canals coming from the opposite side, unite with them almost at right angles; whilst a single trunk (fig. 21 *b*), running along the top of the radiating calcareous septum (fig. 18 *e*), combines all these various branches into a common system of canals, which furnish the upper tissues of the scale with a portion of their nutrient fluids. Sometimes these canals send small branches upwards (fig. 20 *c*), which open at the exterior of the scale, whilst others, proceeding in the opposite direction, open at its inferior surface. At fig. 21 *c*, the minute kosmine tubes given off by these canals have been omitted from the drawing, in order to show the undulatory lamellæ or lines of growth, which run parallel with the external surface, as we also saw to be the case in the vertical section, fig. 20 *a*.

On returning to the latter preparation, we at once see that the square membranous compartments (19 *f* and 20 *d*) are really the concentric lines (18 *c*) cut across, whilst the intervening septa (20 *e*) correspond with the concentric calcareous plates (18 *d*). In the former, some large bundles of fibres are seen to proceed horizontally inwards; and we shall subsequently find that the dark lines traversing the vertical calcareous septa in the same direction, are produced by prolongations of these fibres which pass through the septa without becoming calcified. I have already shown that the large central area (19 *e*) is the result of the unilateral direction taken by the section (fig. 18 *a a'*), which here traverses one of the membranous compartments (19 *f*) *longitudinally* instead of *transversely*, as is the case with those right and left of it. The numerous small orifices at fig. 19 *k* are those of the kosmine canals, 18 *h* and 20 *b*, which are here traversed at right angles, owing to the same cause.

Below these concentric compartments we find a solid calcareous layer, 19 *i* and 20 *f*, forming a conical, six-sided dome, the margins of which approximate towards the inferior surface of the scale as they approach its periphery. It is perforated by kosmine canals, 19 *i'* and 20 *g*, resembling those existing in the upper tissues, but here they are neither so numerous nor so extensively developed, being more confined to the marginal portions.

Beneath the dome-shaped septum there are several calcareous laminæ, 19 *c* and 20 *h*, extending from side to side, the upper ones being necessarily smaller than those which are inferior. At their margins they are continuous with the substance of the conical dome. Like the vertical septa, these laminæ are also penetrated by bundles of uncalcified fibres, 20 *i*.

It is impossible to study the distribution of these calcareous tissues without being struck with the way in which they are adapted to the purpose of strengthening the scale, and by maintaining its slightly arched form, enabling it to resist a large amount of external pressure. The dynamic principle displayed in the arrangement of the horizontal laminæ (19 *c*), strengthening the dome (19 *d*), is precisely the one acted upon in the construction of the light iron roofs of modern railway stations; whilst the small compartments with which the dome is surmounted, vividly remind us of the "cells" which constitute so important a dynamical element in the construction of those recent triumphs of engineering art,—the tubular bridges. Surely the engineer might study some of nature's contrivances for resisting pressure with great advantage, even though embodied in so insignificant a structure as a fish's scale.

Before proceeding to a more minute examination of the fibrous membranes, I would direct attention to a modification of these calcareous structures, as seen in the abdominal scales of *Ostracion cornutus*, for examples of which I am again indebted to Sir Philip Egerton.

The dermal plates of this species are constructed on the same general plan as those just described; but they exhibit a little variety in the arrangement of their details. There only exist two or three of the vertical calcareous septa, seen to be so nume-

rous in fig. 19, and a corresponding difference exists in the number of the subjacent horizontal laminæ. The superficial portion also presents some points of difference. Fig. 22 represents a vertical section of the upper textures, as they appear at the extreme margin of the scale. The concentric lamellæ (22 *a*) are here seen to be largely developed and perforated by even a still greater profusion of minute tubes, the orifices of which open at the external surface. These tubes give off numerous branches, and an immense number of minute recurved, falcate twigs, which are peculiar and characteristic.

The extraordinary development of this superficial system of tubes is compensated for by the absence of the secondary twigs fringing the large canals (22 *b*), which enter from the margins of the scale. The principal trunks recur, and are distributed in the same way as before; but the small branches which gave them their character of kosmine tubes are wanting; they are thus reduced to the condition of mere Haversian canals. The terminal twigs of the superior series of tubes begin to appear almost immediately above, instead of at a distance from them, and thus traverse a part of the space which, in the previous example, was occupied by the tubuli branching from these horizontal canals. The subjacent calcareous portions resemble those already described.

The singular structure of the membranous tissues of the decalcified scale of *O. cornutus* affords a good illustration of that of the preceding species, and of the group in general. Fig. 23 represents the centre of a vertical section of a scale subsequent to its immersion in weak hydrochloric acid. From this specimen we perceive that the distinctions seen in the scale prior to decalcification, partly exist in the membrane also. The upper tissue (fig. 23 *a*) still exhibits its wavy lamellæ, and traces of the vertical tubuli. The horizontal series of canals (23 *b*) are also conspicuous, communicating with the large vertical prolongations (23 *c*) which proceed downwards towards the lower surface of the scale. The two lines (23 *dd*) sloping away right and left in the downward direction correspond with the calcified dome-shaped layer (*i*) of fig. 19. The lower portions of the decalcified scale consist of numerous thick laminæ, which are horizontal in the centre (23 *e*), but turn upwards as they successively reach the line 23 *d*. Thus we see that the dome-shaped layer divides the membranous as well as the calcareous laminæ into two series; those which are superior to it being vertical to, and those which are inferior being parallel with its surface. The horizontal and vertical portions are, however, continuations of each other.

Each lamina, as seen at 23 *e*, consists of two sets of fibres; when a horizontal section is made near the centre of the scale, those belonging to one of these series are observed to form its chief substance; they are very minute, and arranged horizontally in parallel lines, being only deflected here and there so as to allow bundles of the second series to pass between them in a more or less vertical direction from one lamina to another. The latter series are especially obvious in the central and upper

portion of the section (fig. 23), from which point they proceed in divergent lines towards its inferior border. As the *horizontal* fibres of each lamina approach the line 23 *d*, they gradually lose their parallelism, and unite to form bundles of various thicknesses; owing to the manner in which the fibres forming each bundle redistribute themselves and contribute to the formation of other divergent bundles, there is produced a curious reticular structure with oval or fusiform meshes. On examining the upturned extremities of the laminæ, we discover the same tissues, only in a modified form which exhibits great beauty, whether viewed in reference to its adaptation to the primary purpose of strengthening the organism, or to the exquisite appearance of the interlaced fibres. Fig. 24, Plate XXX., represents a vertical section made almost parallel with the outer margin, but with sufficient obliquity to cause it to traverse three of the vertical laminæ at a very acute angle. Fig. 24 *a* is the superficial tissue (kosmine), and *b* the subjacent horizontal canals, divided nearly at right angles to their course; *c* are the lower or horizontal portions of several laminæ; *d*, *d* represent the three vertical laminæ, each consisting of an elaborate network of interlacing fibres; the latter being continuations of those which form the parallel horizontal series in the horizontal portions of the laminæ, and which have been already shown to assume a reticular arrangement as they approached the line fig. 23 *d*. Here this alteration in their distribution has attained its climax. The structure is not produced like those woven fabrics, where a warp and a weft interlace one another in opposite directions; the fibres, originally spread out on one plane like the threads of a warp, are looped up into bundles at one point, only to be separated again and redistributed at another, when the individual fibres enter into the composition of new and divergent bundles, the interlacings of which leave numerous small meshes between them*. These tissues appear to be more dense and strong towards the outer, than at the inner surface of each lamina.

The meshes thus left are occupied by bundles of the second series of fibres, which pass through them at right angles. Corresponding with those which radiate from the centre of the section (fig. 23), and traverse the horizontal laminæ, they are still more fully developed in the upturned vertical portions of each layer. Though the fibres are aggregated into large and strong bundles, they do not interlace so as to form a network; all the fibres which enter into the composition of one fasciculus, continue to do so; and even the separate bundles retain their parallelism with each other. Fig. 25 represents a portion of one fasciculus, from which it will be seen, that whilst it is flattened out at intervals (25 *a*), the intervening parts are cylindrical (25 *b*). The bundles of fibres, thus constructed, proceed horizontally from the centre to the circumference of the scale, following the direction of the canals in fig. 21. Fig. 26 represents their appearance in a *horizontal* section of a portion corresponding to that divided *vertically* in fig. 24 *d*. 26 *a*, *a* are the outer and more con-

* The fibrous membrane lining the canal of the chorda dorsalis of the common Dog-fish (*Spinax acanthias*) exhibits a very similar structure.

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solidated portions of the respective vertical laminæ, and correspond with 24 *d*. 26 *b*, *b* are their inner and more lax portions, corresponding with 24 *e*. These vertically disposed tissues are traversed by the numerous horizontal bundles of fibres, 26 *c*, which proceed, as already described, towards the periphery of the scale. Whilst the flattened portions of the latter pass through the meshes of the dense outer tissues, 24 *d* and 26 *a*, their more contracted and cylindrical parts traverse the inner and looser textures of the corresponding laminæ, 24 *e* and 26 *b*.

At their inner extremities, nearly all these latter bundles of fibres *appear* to terminate along lines which correspond with the radiating calcareous septa, fig. 18 *e*, but in reality they are only deflected in the direction of these lines towards the centre of the scale. A reference to the corresponding direction of the kosmine canals in fig. 18 will render this point intelligible.

After decalcification, we are no longer able to identify either the vertical or horizontal laminæ which were originally calcareous; their membranous portions have evidently no peculiarity of structure distinguishing them from those which were not calcified. We thus find that all the various complicated tissues existing in the lower portions of this curious group of scales, consist primarily of but two sets of fibres, running in different directions. The same arrangements exist in the decalcified scales of all the Ostracionts which I have examined. The next question to be decided is the relation which the calcareous elements bear to these different sets of fibres. Whatever may be their modifications, it is only those fibres which, in the central and inferior laminæ, are horizontal and parallel, that ever do become calcified. On exposing some of the vertical septa (18 *d* and 20 *b*) to the action of boiling Liquor Potassæ, I got rid of the merely membranous portions, and preserved the calcareous elements intact. Fig. 27 represents the lateral aspect of a fragment of a septum thus acted upon, which is obviously nothing more than a calcification of the interlaced fibres seen in fig. 24 *d*. It is a calcareous network with numerous lenticular meshes, 27 *a*, through which the flattened portions of the bundles of horizontal fibres have passed without calcification. The calcareous margin of each mesh is incised by a number of minute vertical fissures, 27 *b*. I have not been able to ascertain with certainty what function these fissures have fulfilled; but I suspect that separate fibres raised from the surface of the common bundle have fitted into these small incisions, which would thus contribute in a very important manner to the fixation of the bundle in its place and increase the strength of the organism.

The horizontal laminæ (20 *i*) are perforated, and penetrated by the uncalcified vertical fibres, in precisely the same way. In the vertical section of the extreme margin of one of these scales, fig. 22, we see that these horizontal bundles penetrate the lower portions of the uppermost calcareous layer. Thus they are seen passing outwards at 22 *d*, whilst at *e* their transversely divided extremities are rendered visible, owing to the direction of the line of section, as explained at page 662. They do not ascend as high as the horizontal canals, 22 *b*. The laminæ which enter into

the composition of these, and of the superincumbent tissues, consist wholly of prolongations of the reticulated fibres, fig. 24 *d*, which here cease to be aggregated into bundles, lose their reticular distribution, become much contracted in size, and spread out, forming thin membranous laminæ which are turned inwards, so as successively to invest the upper surface of the scale, and where, by their calcification, they form the large tubercles with which each is covered. Thus we perceive that amidst all their singular modifications, the scales of Ostracionts maintain a close conformity to the type prevailing amongst those of the ganoid order of M. Agassiz.

Another question to be considered bearing upon these structures, has reference to their mode of growth. This however is one which continues to be involved in some obscurity. That the new laminæ have successively invested the entire scale does not admit of a doubt. The origin of the new layers of fibres is a difficulty which affects the history of these scales in common, not only with those of other groups of fishes, but with all periosteal growths whatever. The additions to the second class of fibres forming the radiating bundles, appear to be made to their extremities, partly by the prolongation of the pre-existing fibres, and partly by the gradual incorporation of new ones. The latter process would of course be rendered necessary by the constantly enlarging area which they would have to occupy. On examining the line of junction between the margins of two contiguous scales, the large horizontally disposed bundles of fibres (26 *c*) may be seen projecting from the edge of each plate into the intervening space. This is obviously the place where in some way or another the additions to their length are made. In the inferior margin of several specimens I have observed an appearance like what I have represented at fig. 24 *f*, where an investing membrane appears to be raised from the lower surface of the scale, supported as upon arched pillars, by bundles of fibres, proceeding from the previously formed layer. This formation of the radiating fibres, prior to that of the horizontal layers through which they pass, is in accordance with what I have just described with reference to the homologous fibres at the margin of the scale. It is possible, that after they have been formed, the space enclosed within this series of expanded arches may have become filled up with the horizontal portions of each lamina: but this must still be regarded as a doubtful question. It is however obvious, that at the lateral margins, the corresponding fibres, which now become horizontal, project free into the open space, their investment with the upturned and reticulated portions of the central horizontal laminæ being apparently the result of a subsequent process.

The formation of the uppermost part of the scale is evidently accomplished by the successive additions of thin lamellæ to the surface, such laminæ being continuous with those which ascend from below: this continuity is well shown by fig. 22. As the vertical series of reticulated fibres approach the surface, their areolæ gradually become smaller and more circular and soon disappear altogether; at the same time the fibres spread out to form very thin lamellæ, which soon become calcified. The large canals, 20 *b* and 22 *b*, as well as the smaller tubes (22 *a*) entering from the sur-

face, are formed, not by any prolongations of cells or cell-nuclei, for none such exist; but are the result of small apposite apertures left out in each successive layer as it is added to the preceding ones.

I have examined scales from several other examples of Ostraciont fish, and find that they are all constructed after one common type, varying only in the extent to which the process of calcification has affected the membranous tissues. In one small species the scale is almost wholly membranous. A thin superficial layer of kosmine, into which the tubes enter from the external surface, rests upon an equally thin homogeneous layer of calcareous substance, which has evidently been formed by the aggregation of minute granules, not unlike those of a Cycloid scale. Below these, near the margin, one single, very thin vertical lamina unites with an equally thin horizontal one, extending from side to side; whilst a third portion proceeds downwards from the junction of the former two, along the line 19 *d*—*i*', to the lower edge of the scale.

The imperfect development of the calcareous elements of the latter scale contrasts strongly with that of the kosmine in some other portions of the fish. Three very large dermal spines project from its dermal covering, like those which give the name to *Ostracion cornutus*; one of these proceeds forwards from the upper anterior angle, immediately above and between the eyes, whilst the other two occupy the respective posterior inferior angles on each side of the tail. Throughout the greater portion of their extent they are cylindrical, and have their surfaces ornamented with numerous longitudinal ridges and intervening sulci. At their bases, where they are depressed vertically, the ridges terminate in rows of small tubercles, like those which stud the surface of the scale.

The internal structure of this spine is highly interesting, partly from the fact that it is merely one of the ordinary scales in which the superficial element is disproportionally developed and drawn out longitudinally, and partly because it explains the true nature of some well-known fossil ichthyolites. It contains a hollow pulp(?)-cavity, which exhibits no appearance of having been occupied by a cellular pulp, but is partly filled by a membrane composed of reticulated fibres, like those found in the ordinary scales. Around this is placed the calcareous portion of the spine, which wholly consists of kosmine, and includes the homologues both of the horizontal canals, 20 *b*, and of the more superficial tubuli, 20 *a*. The former of these are seen in the thicker portions of the spine, running parallel with the pulp(?)-cavity, and give off small kosmine tubes; the latter enter from the outer surface.

On making a transverse section, we obtain an exact fac-simile of the corresponding section of the fossil *Cœlorhynchus*, figured in my last memoir*. The superficial longitudinal ridges are seen to be the outer edges of long plates radiating from the centre of the spine to its circumference, separated by thin intervening layers of reticulated kosmine canals, which anastomose at their inner extremities with those which run

* Philosophical Transactions, 1849, tab. 43. figs. 35, 36, 37.

parallel with the pulp(?)-cavity. With one or two unimportant exceptions, the description of the structure of the fossil *Cœlorhynchus* applies, even in its most minute details, to that of the recent spine. In the latter example, I have not seen the long semilunar canals represented by figs. 36 *g* and 37 *d**. There is also some difference in the construction of the bases of the two examples. *Cœlorhynchus* has always been regarded by ichthyologists as representing the true premaxillary bones of the fish to which the fossil originally belonged. This idea received some confirmation from the circumstance, that whilst at its apex it possessed a single central cavity, at its base this canal was not only divided into two by an intervening calcareous septum, but even the septum itself was duplex, consisting of two plates which were readily separated. Hence the opinion was adopted that the organism consisted of two separate bones, closely united, especially at the apices. I had always some difficulty in reconciling my observations with the above interpretation of the homology of this structure, especially since I could not detect a very clearly defined line of demarcation between the two supposed bones near their upper extremities. At length, however, I thought I had detected it, and figured it accordingly. Since then I have obtained more perfect sections of the fossil, through the kindness of Sir Philip Egerton, and am now convinced that no such line does exist. The structure is one entire organism, only having a bifid base and a corresponding bifurcation of the central canal. I have hitherto seen no dermal spine exhibiting a kosmine structure, made up of two lateral elements, as is sometimes the case with the true *osseous* dorsal rays. *Cœlorhynchus* is, I am convinced, no exception to this general law, and from the light now thrown upon it by the study of the recent examples just described, I have no hesitation in arriving at the conclusion that it is a fossil example of a dermal spine of a large Ostraciont fish. The future discovery of the scales is a doubtful matter. It is possible, that like those of the specimen from which my spines were taken, they may have been principally membranous; still it is highly desirable that they should be sought for at the localities where the *Cœlorhynchus* is found. The new aspect which this fossil now assumes may render it desirable to change its generic name, since the present one conveys a false idea as to its true anatomical homology.

Having completed the examination of as many modifications of scale-structures as fell within my reach, it became a question of considerable interest to ascertain what physiological relation could be traced between their structure and mode of development and that of the bones constituting the endoskeletons of this group of vertebrate animals. The overthrow of many of the old notions respecting the genesis of mammalian bones by Professor Sharpey, has rendered it desirable that all forms of ossific growth should be subjected to a new and rigorous examination, in order to ascertain how far these collateral witnesses give support or otherwise to the highly philosophic views that he has enunciated.

I have employed the same method of investigation in this portion of the inquiry

* *Ut supra.*

that I had previously applied to scale-structures. This especially consists in making sections, of each bone to be examined, in every direction that appeared likely to reveal any new feature of its structure, and examining the same bone, as far as was possible, in fishes of different ages. The result has satisfied me that Professor SHARPEY's views not only receive the clearest possible confirmation from the study of the developing bones of osseous and cartilaginous fish, but that a large amount of additional light is thrown upon those views by such a study of the less advanced members of the vertebrate series.

Dr. SHARPEY has pointed out the fact that two distinct processes are put in operation during the growth of a mammalian bone. The first is the development of a cartilaginous matrix and the deposition of calcareous matter in its intercellular substance; the second is a deposition of similar matter in the fibrous membranes of the investing periosteum and perichondrium, prolongations of which, dipping into the interior of the organism, exhibit the same tendency to become calcified.

Both these processes exist amongst fishes, leading by their various modifications to very diversified results. In a great number of examples they are both seen in operation in the same bone. In some of these cases, the earlier cartilaginous bone-growth has been re-absorbed, and its place occupied by new bone developed in true fibrous membrane, in a manner that resembles, though it is not absolutely identical with, the process which Dr. SHARPEY has shown to take place in mammalian bones. In other instances, the two structures, so distinct both in their origin and in their aspect, are developed side by side, and continue to retain their respective positions even in the most matured animals. This is especially the case in some fishes, which, like the Pike, permanently retain a large amount of cartilage in connection with their true osseous elements.

But in addition to these, there also exists a third group, apparently including the whole of the Sharks and Rays, in which only the first of these processes has gone on. The calcareous elements of their skeletons are nearly all constructed on the type of the temporary and transitional bone-growths developed in the intercellular substance of the cartilage along the line of ossification; only in these cartilaginous fishes, that which is transitional among the higher vertebrates, is here permanent. These endo-skeletons are nearly all what may be termed *chondriform*. The two processes of growth being essentially different, it is desirable to apply to them distinct and intelligible names, and thus avoid the necessity of employing descriptive phrases every time that each one is referred to: the terms *chondriform* and *membraniform* appear sufficient for this purpose; the former being employed to designate those calcareous growths which are formed within the substance of the true cartilage, and the other being applied to the analogous growths which are formed by the calcification of lamiuæ of fibrous membrane. I would first direct attention to some examples of chondriform bone, as constituting the simplest type of true osseous structure.

Several writers have recorded the fact, that in the Plagiostomous fishes the carti-

laginous skeleton is covered with "an osseous crust, which is composed of a great number of small plates or rod-like portions of bone united in the manner of mosaic work*." Mr. Millar has also described the structure somewhat more in detail, as forming the osseous covering of the cartilaginous skulls of the Sharks and Rays†; but neither of these authors has noticed the minute structure of these plates, or recognized their true physiological relationship to other forms of ichthyal and mammalian bone. Professor Owen, in his admirable lectures, advances somewhat farther; he says, "in some species (of fishes) ossification commences at the periphery of the animal mould or basis, and is always limited to a thin outer crust of the bone, the rest remaining cartilaginous or gelatinous. In some of the higher cartilaginous fishes, for example, an osseous crust is formed upon the periphery of certain cartilages in the form of prisms, which contain oval calcigerous cells, but without conspicuous radiated tubes‡." Still even the accomplished author of the above passage does not appear to have observed the difference between these "calcigerous cells" and the canaliculated lacunæ of more highly organized bone; they are however perfectly distinct.

Fig. 28 exhibits a vertical section of the semicartilaginous column of the *Raia clavata*, or common Thornback. The section has been made *transversely*, crossing one of the vertebræ midway between its two concave extremities, the neural spine being removed; the neural canal (28 *a*) is surrounded by a layer of small calcareous plates, 28 *b*, and a corresponding series (28 *c*) covers the entire exterior of the structure, including not only the centrum and all the vertebral apophyses, but existing also along the lines of demarcation (28 *d*) separating the ossa intercalaria, which, in this group of fishes, often enter into the composition of the neural ring.

Fig. 29 represents a horizontal section of a portion of this osseous crust, taken from the lateral surface of the neural spine, and shows the way in which the small calcareous plates are fitted together. Each plate thus examined exhibits six or eight short, broad processes, 29 *a*, which meet corresponding ones projecting from the contiguous plates, 29 *b*. Sometimes these processes are not all arranged on one horizontal plane, but are on different levels, as at 29 *c*, where the upper pair is wanting, their deficiency being supplied by others deeper down. The uniform direction of these processes, modifying as they do that of the internal cells shortly to be noticed, give a stellate appearance to the surface of each plate; and their partial apposition through the projecting processes causes the existence of numerous small circular and oval interspaces, 29 *c* and *d*, which are occupied by a modified form of cartilage.

Fig. 30 represents one of these ossified plates still more highly magnified. The portions 30 *a*, *a'* are the projecting processes which have met corresponding ones from contiguous plates, whilst the intervening indentations (30 *b*) have contributed their

* Müller's Elements of Physiology: translated by W. Baly, M.D., 2nd edit., 1840, p. 393.

† Footprints of the Creator, p. 41, fig. 7.

‡ Lectures on the Comparative Anatomy and Physiology of the Vertebrate Animals, Part I. p. 33.

share to the formation of the small oval interspaces seen in fig. 29. The surface of the section is seen to be crowded over with small round or oval cavities in the solid calcareous substance. These cavities are frequently linked together by narrow necks or irregular tubular canals, and in the projecting portions exhibit a strong tendency to assume a linear arrangement in groups which radiate from the centre, those of each group being more or less parallel to one another. In the calcareous interareolar substance, numerous concentric lines of growth are faintly visible, especially traversing the projecting processes at right angles to their direction.

Fig. 31 is a vertical section of one of these osseous plates, with a portion of the subjacent cartilage. At 31 *a* we perceive that the cartilage-cells present the ordinary ichthyal form, being gathered together in small detached groups. As we approach the inner surface of the osseous plate these groups begin to break up, and the cells re-arrange themselves in divergent lines (31 *b*), radiating from the centre of the calcareous structure. At the same time that their distribution is thus changed, each cell becomes more turgid, and exhibits more definite margins than before. The edges of the calcareous plate when thus intersected are seen to be thin, whilst its centre projects considerably into the cartilage; it presents the same array of internal cells that were seen in the horizontal section, fig. 30, but here their direction is changed, being nearly the same as that of the contiguous cartilage-cells. A careful examination of the line 31 *c*, where the two substances are in contact, makes it very evident that the calcareous matter is deposited, in the form of minute granules, in the intercellular spaces, which granules resemble, in their more essential features, those forming the calcareous laminæ of Cycloid scales, only being round instead of lenticular. They obviously increase in size by the addition of concentric layers, and by the coalescence of previously isolated granules. The small cavities correspond with what were originally cartilage-cells, but which, not having been filled up with the calcareous matter, have remained as small circular cavities in the solid structure. The point 31 *d* has evidently been the centre of ossification, and the new additions have been made both to the inner surface and to the peripheral margins of the plate; whilst the cavities in the centre of the structure have been arranged more or less vertically, the linear rows nearer the margin have been so deflected as to be almost parallel with the outer surface of the ossifying cartilage. This correspondence between the direction of the cavities and those of the contiguous cartilage-cells, which is important, as establishing the connection that exists between them, is also seen on examining one of the small circular interosseous points of cartilage visible on the surface, 29 *d*. Fig. 32 represents one of these cavities, into the composition of which portions of three osseous plates have entered (32 *a*). The cartilage-cells are here arranged in three arched series, and the cartilage itself assumes a semi-fibrous aspect, the fibres running in the same direction as the cells, which again corresponds with that of the small cavities in the contiguous processes of the calcareous plates. When one of these plates is decalcified, the animal basis retains the contour of the

original structure. It has wholly lost its cartilaginous aspect, but its inner surface merges in the true cartilage and is inseparable from it.

There can be no doubt that each of these osseous plates is formed on a plan which is identical with that already referred to in mammalian bones, as explained by Professor Sharpey. The same processes are exhibited in both instances and follow the same order: the altered arrangement of the cartilage-cells, the calcification of the intercellular substance, and the production of small cavities in the bone corresponding with the position of, and owing their existence to, the cartilage-cells, are phenomena common to the growing bone of the mammal and the plates covering the cartilages of the Ray. But whilst in the former case this type is only transitional, leading to the formation of a still higher and more complex series of structures, by which its place is permanently occupied, in the latter instance it is the permanent state. Its physiological condition remains the same in the matured individual as in the fœtus, being only affected by the increase of bulk; and it furnishes another example of that permanent arrest of an early process in the development of the higher animals, of which so many instances are now on record.

On turning from the surface to the interior of the vertebral column of the Thornback Ray, we find that a modified form of the same physiological condition exists. It is well known that in the interior of the soft vertebral column of many of the cartilaginous fish, there exists a chain of calcareous elements, representing a series of osseous vertebral centres, but which have not advanced to a complete state of development. In the Ray, each of these exhibits the two concave extremities, connected together by divergent plates*, whilst a continuous canal is left open through their centres, within which the chorda dorsalis is lodged. In fig. 28, the vertical section has divided this osseous centrum, midway between its two concave extremities, exposing the canal of the chorda (28 *e*), the calcareous ring by which it is surrounded (28 *f*), and the fine radiating laminæ which connect the two extremities together. Of the latter, one proceeds upwards (28 *g*) to form the floor of the neural canal, two pass upwards and outwards (28 *h*) towards the parapophyses, and the two lower ones, which are bifid externally, are directed downwards and outwards (28 *i*).

On examining these several elements of each centrum under a still higher magnifying power, we see that they have precisely the same internal structure as the external osseous plates, modified only by minor differences in the forms and distribution of the small cavities, dependent upon the relative portions of the centrum which they occupy. In that which immediately surrounds the canal of the chorda (28 *f*), they exist as flattened cells, arranged in concentric rows; but in the projecting plates, 28 *g*, *h*, *i*, they are spherical and are disposed in radiating lines directed from the centre towards the periphery.

On examining the condition of the cartilage near the line of junction with the bone, we find very similar appearances to those seen in fig. 31, only the direction of the rows

* See Mr. Millar's Footprints of the Creator, fig. 8 *b*, p. 43.

of cells is altered, in accordance with that which they have assumed in the contiguous portions of the bone. In the deep sulci intervening between the osseous plates, the cartilage has assumed the fibrous aspect already noticed, the fibres being arranged in more or less regularly divergent radii, of which the canal of the chorda has been the converging point. The rows of turgid and isolated cells follow the direction of these semi-fibrous lines in the cartilage. The calcareous additions are made to the whole surface where the growing centrum is in contact with the cartilage, but owing to the radiating distribution of the cartilage-cells, such of them as are incorporated with the *sides* of the plates, 28 *g*, *h*, *i*, are arranged in rows nearly parallel with the line of increment, whilst the cells at their *peripheral margins* are arranged at right angles to that line. I have been particular respecting the relative positions of the cartilage-cells and the small cavities existing within the calcareous centrum, because we thus learn the important relationship which the one series of structures bears to the other.

When a transverse vertical section of the same vertebra is made near one of its extremities, so as to cut off a ring of bone from the margin of one of the terminal cones, we find a new modification in the arrangement of the cells. As in the central portion immediately surrounding the chorda, they are arranged in concentric circles, in which the cells exhibit so strong a tendency to coalesce, as to produce in many instances elongated concentric tubes, alternating with intervening ribs of calcareous matter. By carefully noting the gradual transition from these tubular appearances to those portions of the structure where the cavities continue to be isolated and spherical, it is easy to see that the differences are merely the result of a modified arrangement of similar cells. On examining a thin superficial section, taken from the surface of one of the growing plates, 28 *h* and *i*, we perceive that the first deposition of the earthy matter takes place immediately around the inflated cartilage-cell, each one being surrounded by a granular calcareous fringe. These granules soon coalesce; and the irregular rings thus formed continue to receive additions to their exterior until their more salient and contiguous points coalesce. Small angular interspaces exist for a while, but in process of time these also become filled up; and thus the cells are reduced to the condition of mere cavities in a solid calcareous structure, in which latter, by a careful management of the light, numerous small concentric rings may be traced, surrounding the permanent cavities and marking the lines of growth.

These centra contain none of the cancelli or Haversian canals seen in the ordinary forms of bone; the conditions which lead to the formation of both the one and the other have obviously no existence here. Bearing in mind the physiological relationship of these tissues in the Ray to those of the higher mammals, this deficiency of the higher osseous elements will be readily understood. Another important fact also to be remembered is, that the small circular cells existing in these bones and dermal plates, are not the homologues of the canaliculated lacunæ, known as the corpuscles

of Purkinje. The former exhibit none of the beautiful radiating lacunæ which are so characteristic of the latter objects. It is true, we have frequently a small tubular canal connecting two such cavities together, but these are very different from the beautiful stellate objects seen in the bones of an eel or the scales of a *Lepidosteus*. The fact is, that though analogous they are not homologous. They differ in their structure, and still more in their origin; and the neglect of these differences has contributed, in no small degree, to produce the amount of obscurity and confusion that has hitherto invested the history both of the corpuscles of Purkinje and the genesis of bone, an obscurity which Dr. Sharpey was the first to clear away.

Turning from the flat Plagiostomes to the Sharks, we find that their osseous structures are essentially the same as amongst the Rays, though presenting numerous minor modifications, both in their modes of development and in the forms which their osseous centra ultimately assume.

The common Picked Dogfish (*Spinax acanthias*, Cuv.) presents one of the simplest types, and is one which can be studied with facility owing to the readiness with which examples can be obtained in different stages of their growth. The small plates with which the surfaces of the cartilages are covered, are not above one-fifth or one-sixth the size of those seen in the Ray, and are consequently more numerous in the same area; in other respects they closely resemble them. Mr. Millar has already pointed out the fact that the ossified centrum of the vertebra of the Dogfish is shaped like a hour-glass*. It is in fact an osseous cylinder, constricted in its centre and surrounding the canal of the chorda. If a vertebra be taken from a very young Dog-fish, and a section of it be prepared in the direction of that of the Ray (fig. 28), viz. vertically, midway between the two concave extremities, we shall perceive that the canal of the chorda dorsalis is surrounded by a very narrow ring of bone. Between the inner surface of this ring and the fibrous membrane lining the canal, there exists a considerable interval which is occupied by cartilage, in which the cells are uniformly diffused, excepting in the immediate vicinity of the bone, where they exhibit a slight disposition to assume a concentric arrangement. External to the osseous ring is the great bulk of the cartilage forming the vertebra. In that portion of it which surrounds the bone, the cells are arranged with great regularity in lines running from the ring towards the periphery. In the more external portions this radiated disposition is less obvious. Nearly the whole of the peripheral portions are invested by a thin osseous film, consisting of the small plates already spoken of. Similar ones line the greater portion of the neural canal, but are absent from its upper part, both externally and internally. These two series of external and internal plates are connected by means of the canals which allow of the exit of the spinal nerves; they also being lined with similar plates. The floor of the neural canal is occupied by one plate of much larger size than the rest, whilst external to it, but within the canal, is a small mass of cartilage, so that this bone is invested with cartilage on both its

* Footprints of the Creator, p. 43, fig. 8 *a*.

surfaces. At the inferior border of the centrum, or what would constitute the roof of the hæmal canal of a typical vertebra, there is a similar enlarged plate of bone, also invested by cartilage on every side.

On preparing a similar section from a vertebra of an older fish, we find that the same general arrangements of the parts exist, only some of them are altered in their respective dimensions. The osseous ring has encroached upon the cartilages, both at its inner and outer surfaces. In the additions made to the former portion, the cavities in the bone are compressed and arranged concentrically. In the external additions they are spherical, and arranged in radii, corresponding with those of the uncalcified cartilage. The two large plates occupying the upper and lower surfaces of the centrum have also increased in thickness, by additions made to each of their individual surfaces. Their enlargement has been accompanied by a corresponding expansion of the portions of cartilage which are external to them, thus providing for their further development.

On making a vertical section of one of these vertebræ in the longitudinal direction, or at right angles to the last, we find that the cartilage-cells external to the osseous ring still display the same radiating disposition, the lines diverging as they approach the periphery. The arrangement of the small cavities in the cylindrical bone is also analogous. Those which form the surface of each concave articulating extremity are very large, and are arranged in rows which are nearly parallel with that surface, having been developed around the cartilage-cells with which the latter portion is lined. Those which, on the other hand, constitute the exterior of the constricted osseous cylinder, are considerably smaller, and towards its terminal margins become subcompressed. We now also find that the osseous plate, constituting the floor of the neural canal, extends across the entire vertebra, and that its anterior and posterior extremities blend with the contiguous margins of the osseous cylinder. It is in fact a rudimentary vertical plate, serving the same purpose as that of the Ray, fig. 28 *g*; only instead of being developed, as in that example, direct from the osseous cylinder surrounding the chorda, it is formed at a distance from it, being attached to it only by its anterior and posterior extremities, the central portions of each being separated from one another by a semicircular mass of cartilage. As the growth of the fish advances still further, this cartilage continues to be encroached upon both above and below. I have never yet met with an example in which it was completely obliterated and its place wholly occupied by bone, but in very large and old fish it is nearly so, the plate connecting the two concave extremities being there very thick, the result of successive additions made to both its surfaces. The preceding remarks also apply to the corresponding plate on the opposite inferior surface of the centrum, only it never becomes enlarged to anything like the same extent as the superior one.

It will be seen that whilst in these bones of the Dog-fish the process of ossification is essentially the same as in the Ray, it differs in one important particular. In the latter fish all the new growths are added to *one* surface only; in the former, the

additions are made to *two*. We have seen this to be the case, not only with the osseous cylinder immediately surrounding the chorda, but also with the superior and inferior laminæ; and there also appear to be indications of two *lateral* laminæ, formed in the same way, immediately above the parapophyses. We shall shortly find that this is the first step in some important physiological changes.

In the vertebræ of the large *Carcharias vulgaris*, Cuv., so common in osteological collections, the two terminal cones are united by four transverse bony plates; the two lateral ones are very large, whilst the intermediate ones are much smaller. These plates, as Müller, Professor Owen and others have already pointed out, are separated by deep, subcompressed, conical cavities, corresponding with the positions of the neurapophyses and parapophyses. Besides these large plates, there projects into the intervening cartilages four smaller ones; consequently a transverse section of a dried vertebra made midway between the two terminal cones will present the appearance of fig. 33, Plate XXXI.

In this section, the four cavities (33 *a*) have been occupied by cartilage, whilst the entire shaded surface is composed of chondriform bone. It consists of an innumerable series of minute areolæ or cavities, which, when more highly magnified, present the appearance seen in fig. 34. Professor Owen describes the vertebræ of the Tope (*Galeus communis*), and most sharks possessing the nictitating eyelid, as having their external surface, as well as the terminal cones of the centrum, covered by a smooth osseous crust, except at the openings of the four cavities. In the present example, the bone is not merely a crust, but exists as four solid cones, the apices of which unite at the middle of the centrum, from whence also are given off the small thin intervening plates, 33 *c*. The whole of this structure has been formed in the same way as the corresponding portions of the vertebræ of the Ray and Dogfish. The areolæ (34 *a*) are large, irregularly oval, and more or less compressed. The intervening calcareous portions (34 *b*) are small in proportion to the size of the areolar cavities, especially when compared with the relative development of the same textures in the Ray, where the earthy element is so much more abundant. These calcareous septa in the *Carcharias* are rough and somewhat irregular in their distribution, with the exception of a few more definite rows which take their rise from the lateral margins of each cone, 33 *d*, and proceed towards their peripheries. These rows are probably the result of lateral additions made to the cones during their process of increment.

A vertical section of the same vertebra, made at right angles to the last, along the dotted line, fig. 33 *e e*, reveals a somewhat different arrangement of the areolæ. The contour of such a section, unmagnified, is seen in fig. 36. The central portion of each half (36 *a*) is occupied by areolæ, like those, of which a magnified representation is given in fig. 34; but at each concave articular margin (36 *b*) there is an external layer, having a somewhat different structure. Fig. 35 represents a small portion removed from 36 *a*, and highly magnified. At the extreme margin (35 *a*) the areolæ

are arranged in oblique vertical lines, which successively run out at the border, as they approach the upper and lower surfaces in the two halves of the section. In this respect, as well as in their somewhat elongated contour, they exactly correspond with the same areolæ in the vertebræ of the common Dog-fish. Within these there exists a second series, 35 *b*, which occupy the greater part of the lightly-shaded margins in fig. 36. The latter areolæ are small, almost square, and arranged in curious undulating chains, which proceed from the central mass of spherical cavities towards the concave articular surfaces, the various rows being separated by rods of calcareous matter.

Not having been able to obtain the bones of *Carcharias* in a very young state, I cannot speak with absolute certainty as to the directions in which ossification has extended itself; I suspect, however, that it has combined the leading features of both the Ray and the Dog-fish; whilst, like the former, the greater portion of the new bone has been added to the exterior of the osseous cylinder, primarily formed round the chorda: there have also been some additions (35 *a*) made to its internal surface, especially at its two concave extremities, as in the case of the Dog-fish. The principal point about which I am in doubt, has reference to the way in which the four divergent osseous cones have been formed; whether they have been directly developed from the osseous cylinder, as in the Ray, or whether their ossification has commenced at the surface, and been developed internally, until the whole of the subjacent cartilage has been incorporated, and they have thus become united throughout with the inner cylinder. It is most probable that the former of these modes has been adopted; but the point can only be determined by an examination of the bones of some very young examples of the fish.

On turning from the vertebræ to the jaws of the same fish, we obtain results which substantially coincide with those obtained from the Ray and the Dog-fish. The cartilage of the jaw has long been known to be invested by a layer of vertically elongated calcareous prisms. Their structure closely corresponds with that of those covering the vertebral and cephalic cartilages of the Rays, only instead of being merely flattened plates, a rapid vertical development has caused them to be more elongated; but their internal areolæ are arranged in much the same way as those of the Thornback, and their formation has been preceded by a corresponding arrangement of the cartilage-cells, which their calcareous walls have invested. The jaws of *Cestracion* exhibit an identical structure.

The several bones which enter into the composition of the rostrum of the Saw-fish, are also largely composed of an aggregation of similar small calcareous prisms. An imperfect sketch of a portion of this structure is represented in my last memoir*. I had noticed the difference between its aspect and that of ordinary bone, but not having then been able to interpret its meaning, I spoke doubtfully as to its osseous character. There is however no doubt that the entire rostrum which supports the lateral teeth consists of two kinds of bone, covered over with a fibrous skin in which

* Philosophical Transactions, Part II. 1849, fig. 34 *a*.

are implanted the numerous minute dermal teeth. The calcareous chondriform prisms or rods exhibit some peculiarities. Their *internal* extremities exactly resemble the whole structure of those existing in the jaws of *Cestracion* and *Carcharias*; but the outer half of each osseous plate is very different. In order to obtain so considerable a bulk of solid calcareous matter as is required to give the requisite strength to the rostrum of the Saw-fish, nature has slightly modified her plan, but without abandoning the type prevailing amongst the other Plagiostomes; she has accomplished the object by adding largely to the length of each of these calcareous prisms; and where several of them proceed inwards from different sides of an angular cartilage and meet near its centre, they are twisted about in a peculiar manner, so as to fit in between one another, and thus increase the solidity of the structure.

It is evident that each of these prisms in the Saw-fish consists, when divided vertically, of two distinct portions, an internal and an external one. At its rounded internal extremity, the areolæ are usually arranged in a fan-like semicircle of radiating lines; the same series is prolonged outwards like a hollow cone, which invests the inner half of the structure, and in which the cavities are arranged at right angles to its longer axis. The centre of this portion is occupied by a cone of a different structure, containing cells differently arranged. The external half of the bone corresponds with the central cones. It is also full of cavities, but they are much less spherical, and in the transverse section form an irregular network. They appear to have been left by the imperfect coalescence of elongated botryoidal rods, of which this part of the structure consists. Each half of these prisms has obviously been formed in a different way from the other. Instead of the primary point of ossification having been at the outer extremity or base, as is certainly the case with those of the Ray, and apparently also of *Carcharias* and *Cestracion*, in the present example I believe it to have been in the middle of the prism at the apex of the central cone; and that whilst new additions have been made to the sides and inner extremity of each prism by calcification of the cartilage in the way already described, cognate additions have also been made to its base; but these latter growths, *which become continuous throughout the entire bone, and not subdivided into vertical prisms*, have not been produced by the calcification of ordinary cartilage. They consist of a congeries of elongated botryoidal rods separated by long irregular cavities, the appearance of which in the transverse section has been already referred to. Though these cavities owe their existence to the imperfect calcification of the soft tissues, they are different from the areolæ formed by the investment of the globular cartilage-cells, at the opposite extremity of the prism. This continuous external portion has evidently been developed in a modified form of fibro-cartilage in which the cells have been less obvious. In the case of the Dog-fish, where we found that both external and internal additions were made to many of the ossifying points, I have already noticed that the cartilage in which the external growths were effected, assumed a much more fibrous aspect than that which

was more internal; its cells also are very much less conspicuous, though it is continuous with the true cartilage, and merely a modified form of it. The external portions of the rostral cartilages of *Pristis* have undergone some similar but more marked modification, producing the peculiar differences that exist at the opposite extremities of each of the calcareous prisms. *The external continuous layer is obviously a rudimentary form of membraniform bone.*

The existence of these peculiar plates and prisms of chondriform bone, which appear to distinguish the recent sharks and rays from other fishes, is of some importance in its practical application to geology. In my memoir already so often referred to, I pointed out the circumstance, that in connection with what had always been regarded as the skin of the fossil *Hybodus reticulatus* from the lias of Lyme Regis, there existed a deep-seated layer of calcareous granules, which appeared to have been formed within the membranous tissues of the skin*. The more extended examination of recent forms which I have since made, has given me reason to conclude that these granules have belonged to the surface of the cartilaginous skull. The disappearance of the membranous portion of the skin, and of the soft cartilage, during the process of fossilization, would cause the dermal teeth implanted in the former, and the thin layer of calcareous granules investing the surface of the latter, to come into contact, and be permanently preserved in that state of close contiguity; so that what, along with other observers, I had always regarded as a layer of true skin, appears to be a combination of the calcareous elements of both the skin and the skull. On examining portions of the jaw in which the teeth are planted, and which is certainly a part of the endoskeleton, I find that it consists of a congeries of granules which are identical with those just referred to. In the memoir, I pointed out the existence, in the interior of each granule, of numerous small brown points, as well as of concentric lines of growth; the former of these I thought might possibly be merely the result of mineralization; I have now however no doubt that they have once been true areolæ formed by cartilage-cells, and that both these characteristics are homologous with the similar features presented by fig. 30. Some of the areolæ appear to have been filled up with a yellowish substance, and are no longer to be distinguished from the rest of the structure, which now presents the same hue; but the others have become occupied by brown carbonaceous matter, and are still obvious. We thus learn that the same structural peculiarities which are exhibited by the skeletons of the recent sharks, have also existed in their fossil allies of the Liassic æra.

But we can recede even still further into the past and obtain the same results. In the coal-measures of Leeds and Manchester, there are occasionally found small lustrous fragments, composed of a congeries of minute resinous-looking granules. When thin sections of these objects are examined under the microscope, we find that they have an internal structure closely resembling that of *Hybodus reticulatus*. Similar concentric lines of growth exist in each granule, following the outline of the numerous projecting points with which its surface is studded; in the interior is

* Philosophical Transactions, *ut supra*, p. 466, fig. 33.

seen a beautiful series of minute rings and concentric circles, which I believe to owe their appearance to the original areolæ; other traces of the former existence of these areolæ are preserved in the numerous brown semicrystalline points which are crowded together in the interior of some specimens. From the repeated conjunction of some of these fragments with the curious teeth of *Diplodus gibbosus*, I have been led to the conclusion that they have both belonged to one fish. On mentioning my idea to Sir Philip Egerton, I was glad to find that it accorded with the conclusion arrived at by our highest British authority in the field of fossil ichthyology.

We are thus furnished with two very decided examples of bone in a fossil state, belonging to the chondriform type, which, as far as my investigations have been carried, appears to characterize the recent Plagiostomes. Many others will I have no doubt be eventually found to exist; fragments, however small, when submitted to a careful microscopic examination, may thus be distinguished from the bones of the true osseous fishes.

After having ascertained some of the leading features attending the structure and growth of chondriform bone when unaccompanied with that of any other type, I turned my attention to the same topics in connection with the bones of the osseous fishes. In this portion of the investigation, the inquirer is often tempted to draw general inferences from partial and limited data; a proceeding, which I have learnt from experience to be especially dangerous in ichthyological studies; I have so frequently been astonished to find that the results obtained were widely different from what I had anticipated, owing to the endless diversity of the modifications of the osseous structures of fishes. Still there are certain great groups, the individual members of which are constructed upon a common plan, and whose osteo-genesis is very similar. By submitting well-selected examples from each of these groups to a careful investigation, we may obtain a large amount of detailed information, and be enabled to deduce generalizations which will be found applicable to large numbers of the osseous fishes, without necessarily tracing the progress of every bone in each individual species.

On examining a large number of Cycloid and Ctenoid fishes, we find that whilst the bones of some are so wholly osseous that scarcely a trace of cartilage remains in connection with them, others again preserve a large amount of cartilage in connection with their skeletons to the latest period of their existence. The Pike may be taken as an example of the latter, and the Cod, Haddock, Perch, &c., as belonging to the former of these classes.

Selecting the Pike (*Esox lucius*, Linn.) as a special subject of investigation, I have examined one by one, nearly every bone entering into the composition of its skeleton; making numerous sections of each, not merely in one fish, but in as many examples at different stages of their growth as I could conveniently obtain. From this series of observations I have been enabled to satisfy myself, in nearly every instance, as to the relations which the various osseous and cartilaginous portions have borne to each other.

There exists in nearly all the bones of the Pike more or less of cartilage, which, whilst it is partly internal, invariably makes its appearance at some portion of the external surface of each bone, and especially at the articulating margins. Around this cartilage there is developed a film of chondriform bone, which in some cases attains a considerable thickness, and which is again invested with layers of membraniform bone, into the direct composition of which no true cartilage structure ever enters.

The relative positions of these two osseous tissues, as they exist in the Pike, will be best comprehended by commencing the inquiry with some of the bones which are the least complex in their form. The carpals and stylo-hyals will serve this purpose*.

Fig. 37 represents a longitudinal section of a carpal bone from a Pike weighing about four ounces. It chiefly consists of a cylindrical rod of cartilage, very much dilated at its two extremities. At each of the latter portions (37 *a*, *a*) the cartilage-cells exhibit the common ichthyal aspect, being arranged in groups, each of which appears to be the result of successive divisions of a primary cell; or rather perhaps of successive developments of cells within those previously existing, as amongst the vegetable Algæ, reminding us strongly of the similar groups seen in a *Palmella* or a *Hæmatococcus*. At each tumid extremity the cartilage is only invested by a perichondrium; towards the centre it becomes much constricted, and its cells assume a new arrangement. The groups break upand the individuals become re-arranged in interrupted rows, those of each extremity curving somewhat inwards; near the centre of the shaft (37 *b*) these cells are globular, turgid, and almost in immediate contact with one another. The central point (37 *c*) is occupied by a congeries of minute spherical calcareous granules, which have become so aggregated at the surface of the cartilage as to produce a solid calcareous tissue. These granules are obviously chondriform bone-growths in an early stage of development, and correspond very closely, both in structure and in arrangement, with what exist amongst the cartilaginous Plagiostomes. Besides this central production of chondriform bone, a thin cylindrical layer of the same substance (37 *d*, *d'*) invests the exterior of the cartilage as far as the points, fig. 37 *e* and *e'*. It consists of an aggregation of granules, which, though smaller, in other respects resemble those of the centre of the organism. They are seen to be developed in the intercellular portions of the cartilage, leaving large round open areolæ, each of which is occupied by a cartilage-cell; additions to this chondriform bone are made by the production of new calcareous granules at its inner surface, within the tissue of the cartilage.

Investing this combination of cartilage and chondriform bone, we have a second osseous cylinder (37 *d*), of a very different aspect and origin. It is composed of numerous parallel superimposed lamellæ, many of which may be traced over the entire length of the bone, the more external ones being successively longer than those

* I may observe that throughout this memoir I have followed the definite and philosophical nomenclature of Professor Owen, as employed in his beautiful work on the Homologies of the Vertebrate Skeleton.

which they invest, and consequently extending further along the surface of the cartilaginous matrix. At fig. 37 *g* is a small tubular space, caused by a slight inflection of the lamellæ, and at 37 *g'* a large cavity has been produced in a similar way; in the latter instance, a narrow passage proceeds obliquely upwards and outwards, through the successive laminæ, to the surface. This portion of the bone increases in size by the addition of new layers to its external surface, such layers being the result of a constantly progressing calcification of the fibrous periosteum with which it is covered; whilst at each extremity of the membraniform osseous cylinder the extending calcification takes place in the perichondrium, which appears to be merely an extension of the periosteum around the two cartilaginous extremities of the bone. No lacunæ exist amongst the osseous layers.

The original matrix of this bone has obviously been a small, cylindrical cartilaginous rod, in which chondriform bone has been primarily developed as a superficial ring midway between the two extremities; a nearly connate layer of membraniform bone being formed by the calcification of a portion of the perichondrium. The two extremities have continued to be cartilaginous. The cells of the latter tissue have multiplied in the ordinary manner at these extremities, where the cartilage has increased both in length and thickness. As this primary process has advanced, the membraniform and chondriform bones have continued to increase, as already described. Whilst the two osseous tissues have been developed by independent processes of growth, they appear to maintain some peculiar relationship to one another: thus at the four points (fig. 37 *e*, *e'*) they terminate together; the external or intramembranous growth being but a very slight degree in advance of that formed within the cartilage.

The above description will apply with the utmost exactness to the stylo-hyal bone of the same fish; belonging to the same cylindrical class, it presents identical appearances. In the early state of the large cerato-hyal, precisely analogous details of structure and growth may be traced out; and even in the matured fish, where the osseous elements are highly developed, the expanded cartilages of the two articular extremities are connected by means of a small narrow rod of the same tissue which passes through the centre of the bone.

Before turning from the cylindrical to the flat bones, it is desirable to note the structure of one or two which present an intermediate character. The epitympanic will best serve our purpose; since whilst in its depressed central portion it is closely allied to the flat bones, it is furnished with three long cylindrical processes or condyles. In its early state, a narrow layer of cartilage passes completely through the centre of this peculiar bone. In the adult fish this cartilage only appears externally at the extremities of the three condyles, which articulate respectively with the prefrontal, mastoid and opercular bones. But conical prolongations of these terminal cartilages extend along the interiors of the condyles, in the direction of the centre of the bone; and in a very young fish these cartilages meet near the base of the external

process, which connects the epitympanic with the upper extremity of the pre-opercular bone. As the young fish increases in size, the central portion of this cartilage becomes absorbed, and is replaced by large cancellated cavities which are subsequently filled with cells of adipose tissue.

Fig. 38 represents a vertical section of the extremity of the opercular condyle as it exists in a Pike of about 3 lbs. weight. All the essential phenomena which are seen in the carpal bone are repeated here, only in a still more obvious form. At 38 *a* the cartilage-cells are distributed in small detached groups. At 38 *b* their arrangement is altered; they are disposed in interrupted vertical rows, passing from side to side, at right angles to the ossifying surface; a disposition which maintains throughout that portion of the cartilage which is invested by bone. At 38 *c* there is a layer of chondriform bone, produced in the same way as that seen in the corresponding portions of the carpal; only here it is much more extensively developed than in that example. It exhibits an areolar structure, which resembles, in the closest possible manner, some of the forms existing amongst the exclusively chondriform bones of the Sharks and Rays. This tissue is surmounted by successive lamellæ of membraniform bone, 38 *d*, exhibiting various arrangements of Haversian canals, 38 *e*. Sometimes these canals exist as large cavities left between two contiguous lamellæ, being the result of their mutual divergent inflections, and resembling those seen in the scale of the Sturgeon and the *Holoptychius*. At others they are the result of apertures left in the successive lamellæ, which have not been developed at the points along which the canal was intended to pass. This especially applies to the long oblique canals which proceed from the interior towards the surface of the bone. The membraniform lamellæ (38 *d*) are obviously growths successively added to the exterior; they encroach more and more upon the perichondrium surrounding the extremity of the cartilage as each new addition is made, being usually a little in advance of the subjacent chondriform bone. The latter circumstance we have already observed in the stylo-hyal and carpal bones.

Fig. 39 represents a similar section of the proximal half of the opposite condyle (or that which articulates with the postfrontal) from a young Pike weighing about six ounces. At 39 *a* we have the usual internal cartilage. At 39 *b* there is a very thin layer of chondriform bone, surrounded by a much thicker cylinder of membraniform bone (39 *c*) composed as usual of numerous parallel lamellæ. The chondriform layer is very thin in this instance, as compared with that of fig. 38. I have observed that it appears to be developed with greater rapidity at the extremities of these processes than at their bases, though it exists at both points. Fig. 39 *d* is part of the base of the external process extending from the centre of the epitympanic towards the preopercular. It consists of variously inflected layers of membraniform bone, perforated by innumerable irregular Haversian canals, each of which has been formed in the way already described, by the omission of the portions of the successively added lamellæ, opposite the pre-existing orifices of the canals. The internal cartilage has not been

extended into this process, the bone being wholly membraniform in its nature and origin. Internally the Haversian canals terminate in a large irregular cavity, 39 *e*, which occupies the centre of the bone. This has once been filled with cartilage, which, with its chondriform bone, has been removed by a process of absorption. The process is still going forward at the point 39 *f*, where the cartilage has become detached from the inner surface of the investing membraniform bone, leaving an intervening space which gradually diminishes in extent up to the point 39 *g*, where the cartilage and bone still retain their contiguity. The same process occasionally goes on in other parts of the structure. Thus at 39 *h* a small portion of the cartilage has become absorbed, and the interior of the cavity thus formed has been lined with a layer of membrane, in which calcification has produced a thin film of bone, thus forming a sort of cancellated cavity communicating with the Haversian canal, 39 *k*.

If we examine corresponding sections of this bone from individuals of different ages, we shall find that the inner extremity of the cartilage recedes further from the centre of the bone as the fish advances in age; the irregular cavity left by its absorption being then more or less filled with adipose tissue.

In the paroccipital, exoccipital, alisphenoid and parietal bones, we find that a still more considerable marginal development of the cartilages exists. These bones being in contact with others at nearly every part of their periphery instead of touching at a few salient condyles, their cartilages are not confined to the interiors of cylindrical processes, but form thickened margins round the bones. An extension of this marginal cartilage traverses the interior of the bone, separating the upper from the lower osseous portions; but notwithstanding this modification of the type, the essential process of growth in all these instances has been the same as in the previous examples; chondriform bone being produced at the two surfaces of the cartilage, and membraniform bone still more externally. Thus in young states the latter constitutes two independent unconnected layers, separated, even in the most central portions, by a very thin layer of the cartilage, which is found to increase rapidly in thickness as we approach the periphery. In the bones of matured individuals these two layers are more or less united by the production of cancellated cavities and canals, which pass from the one to the other through the intervening cartilage. In the first instance these have merely been open spaces, produced by the partial absorption of the cartilage; but they have soon become lined with a membrane, which, as in fig. 39 *i*, has been converted into bone by a calcification of its lamellæ, chondriform bone being produced at the same time in the contiguous cartilage. I have especially observed these secondary bone-growths to be developed in the centre of the parietals and alisphenoids.

In the supraoccipital, the modes of increment are the same as in the bones just described, only throughout the greater part of this bone there has existed a very marked difference in the development of the superior and inferior osseous layers. Whilst the former is of considerable thickness, the latter is very thin. This fact prepared me for the existence of a still more marked difference in the case of the

frontal. The greater part of this bone consists of an expansion of the upper layer only. The lower layer is developed, but to a very small degree, and that chiefly in the central portions of the bone. In some parts of the periphery, the thin membraniform structure appears to rest, not upon its own proper cartilaginous matrix, but upon that of one of the contiguous subjacent bones.

It will not be necessary to pursue the details of this portion of the Pike's skeleton any further. The examples already given suffice to show that all the bones entering into the composition of its cranium, have been produced according to one common plan, which, with various minor modifications, is identical with that on which the less complex carpal bone was constructed. There are, however, some other examples requiring a more special notice.

In the very young Pike, a small cylindrical rod of cartilage extends through the dentary, from the symphysis of the lower jaw, to the angle where it receives the anterior extremity of the mandible, and which is even prolonged a considerable way across the inner surface of the latter bone. This small cartilaginous rod has been the primary matrix of the dentary. In older fish its anterior extremity appears to become absorbed, the cavity being occupied by secondary cancelli, containing adipose tissue. The rest of the bone has been of primary intra-membranous growth.

The premaxillary exhibits similar features. It contains a central oval rod of cartilage, around which are arranged the investing layers of membraniform bone. In the latter portions there are some large cavities, which appear to have been formed, not by absorption of the cartilage, but by inflections of the membranous lamellæ prior to their calcification. In the dental surface of this bone, as seen in a transverse vertical section, we observe a curious arrangement of the Haversian canals. Immediately beneath the base of each tooth, there exists a group of small anastomosing canals, scarcely distinguishable from those seen in the contiguous portion of the tooth itself, and into which they open. Bearing in mind the intra-membranous origin of this portion of the bone, the above fact has a significant bearing upon the origin of the teeth. The central laminæ of the premaxillary contain a remarkable series of very minute parallel canals or tubes, which pass obliquely through them.

The presphenoid and vomer exhibit a peculiarity of structure, which at first sight appears to distinguish them from the other bones which I have examined; but this difference is more apparent than real. A thick cartilage intervenes between the vomer and the nasal bone, and a prolongation of the same cartilage runs backwards along the upper surface of the presphenoid. When vertical sections are made of the latter of these bones, we find that it appears to be developed on the inferior surface of the cartilage. The new lamellæ, which are membraniform, are added partly to the inferior surface of those pre-existing, but chiefly to their superior one, where a thin space, apparently lined with membrane, separates the bone from the contiguous cartilage. The inferior surface of the latter has not only a broad longitudinal depression receiving the upper surface of the bone, but also a long narrow central groove,

into which is fitted the thin osseous vertical plate, which contributes to the formation of the interorbital septum. Though this bone is everywhere in close connection with the cartilage and bears a definite relationship to it, the two are really separated by an appreciable intervening cavity, in which the new membraniform bone is chiefly produced. No chondriform bone is visible in the cartilage. The anterior extremity of the presphenoid in an advanced stage of its growth exhibits some similar appearances. I do not believe that the above has been the primary fœtal condition of these bones. It is probable that both the presphenoid and the vomer are but exogenous prolongations from the sphenoid, and that the primary centre of their osseous development has been in the interior of the latter bone, which has been developed in conformity with the plan followed in other parts of the skeleton. The presphenoid and vomer would thus be the result of an unsymmetrical development of the lower half of the osseous structure produced around the primary cartilaginous matrix. We shall obtain some additional light on this point from an examination of the same bone in some other species of fish.

In the palatine bone a thin film of cartilage extends through its interior, and appears externally between the two condyles which respectively articulate with the maxillary and prefrontal bones, where it is of considerable thickness. Throughout a great extent of the thin posterior squamous portion of the bone the cartilage does not exist, the superior and inferior series of membraniform lamellæ being in direct contact.

None of the topics suggested by ichthyotomy have been productive of more elaborate disquisitions and debates than the opercular bones. Whilst some authors have regarded them as constituting a part of the endoskeleton, others have referred them to the exoskeleton, and considered them as being enlarged and modified scales. The latter opinion, which is the one entertained by M. Agassiz, has been recently combated by Prof. Owen, who, from an examination of the operculars of the Carp and Goldfish, concluded that their development "is effected in precisely the same way as that of the parietal and frontal bones. The cells which regulate the intus-susception and deposition of the earthy particles, make their appearance in the primitive blastema, in successive concentric layers, according to the same law which presides over the concentric arrangements of the radiated cells around the medullary canals in the bones of the higher vertebrata*." The above description, based as it is upon the long-prevailing idea that the lacunæ of the Haversian canals amongst the vertebrata owed their existence *directly* to the cells of the cartilage, does not, of course, agree in its details with my own observations; but it is perfectly accurate in its general bearing upon the moot question as to the nature of the opercular bones. The scales of the Pike are formed in precisely the same way as all the cycloid scales previously described. The structure of the opercular is wholly different, but is closely accordant with that of the bones of the endoskeleton.

* Lectures on Comparative Anatomy, vol. ii. p. 139.

Fig. 40 represents a vertical section of the articular portion of this bone, passing through the articulation towards the posterior inferior angle. The concave articular cavity is partly lined with a fibrous structure (40 *a*) and partly with true cartilage (40 *b*), the latter of these tissues being chiefly confined to the central and internal portions of the cavity. A thin film of chondriform bone (40 *c*, *c'*) exists in the cartilage along its line of junction with the osseous structure.

Within the interior of this thick extremity of the bone is a large irregular cavity (40 *d*), which is of considerable width near the articulation, but contracts to a narrow space as it approaches the more flattened portions of the bone. It is traversed by a few thin and irregular laminæ (40 *e*), and is bounded internally by others (40 *f'*), all of which are composed of membraniform bone. Anteriorly, the latter terminate at an obtuse angle at the surface of the chondriform bone 40 *c'*; corresponding in this respect with the relative positions of the two tissues in the condyle of the epitympanic (fig. 38), and differing only in the case of the opercular bone in the less oblique manner in which they come into contact. Similar lamellæ exist in the upper part of the section (40 *g*) forming the outermost portions of the bone; only instead of terminating abruptly at the articular cavity, they bend downwards and inwards (40 *g'*), so that each additional growth not merely enlarges the area of the articulation, but adds to the thickness of its walls. No chondriform bone exists at this point, and the inflected lamellæ are in contact with a fibrous tissue (40 *a*), instead of with cartilage. In the thin squamous portions of the bone, all the three series of lamellæ, viz. the outer, middle and innermost, 40 *e*, *f* and *g*, are prolonged in parallel layers; those occupying the two free surfaces are extended as far as the several margins of the bone, whilst those which are internal appear to stop short of them, thus accounting for the cycloidal markings seen on the surface of the bone, which are really lines of growth. In the thicker portion of the structure, these membraniform lamellæ are penetrated by curiously-formed Haversian canals (40 *h*). These do not however extend far into its thinner portions. The latter appear to receive their supply of nutritive fluids through numerous minute branching tubuli, which enter the bone at right angles from both its surfaces. They bear a close resemblance to those which are so abundant in the scales of *Lepidosteus* and other allied fish, and to which I gave the name of Lepidine.

There can be little doubt but that the primary matrix upon which this interesting bone has been developed, has been the small portion of cartilage still existing at 40 *b*, but which, owing to the peculiar requirements of the articulation into which it enters, has never attained to any considerable size. Whether or not a prolongation from it once filled the large cavity 40 *d* is uncertain; for though not improbable, I have seen nothing enabling me to conclude that it has done so. The existence of the thin film of chondriform bone at *c*, *c'*, proves clearly that this is not merely a superadded articular cartilage, but a veritable part of the growing structure. The relations of the several parts shows that the genesis of this bone conforms, as to its type, with what we have

seen in all the other portions of the endoskeleton, the arrangement of the various structural elements being modified in accordance with the peculiar form of the bone into which they enter. On the other hand, they separate it wholly from scale structures, in connection with which cartilage and chondriform bone are alike unknown. It is scarcely necessary to add, that no visible cells enter into the composition of the membraniform lamellæ of the opercular of the Pike, any more than into the other analogous osseous structures which have passed under review.

The only remaining element of the skeleton of the Pike requiring a special notice is the vertebra, the growth of which is highly interesting: whilst the principles which have regulated the construction of the other bones are still adhered to, they have been subjected to such modifications as were rendered necessary by the peculiar form of the bone, and the functions which it had to fulfil.

Fig. 41 represents a transverse section of one of these vertebræ, made at right angles to the spinal axis and midway between its two terminal cones. In the centre is a small cavity, 41 *a*, through which the remains of the chorda dorsalis have been prolonged, and which is surrounded by a very thin ring of membraniform bone; from this ring there radiate eight conical segments having different structures; four of these are osseous, alternating with the other four which are cartilaginous. One of the osseous segments (41 *b*) proceeds upwards, to form the floor of the neural canal. Two others, somewhat larger than the last, 41 *c*, *c*, pass upwards and outwards, whilst the remaining one, occupying above one-fourth of the area of the section, passes both downwards and outwards, constituting the most substantial part of the centrum. When the vertebra is entire, these osseous segments appear as longitudinal plates, passing from the one terminal cone to the other.

Each of these segments is composed of large and irregular cancelli of membraniform bone, the walls of which have a laminated structure; but towards the periphery the laminæ lose their irregular distribution, and form a series of small radiating marginal plates, 41 *e*, between which are numerous open spaces, allowing a free communication to exist between the exterior and interior of the bone. These plates do not exist along the middle of each segment, which is occupied by a deep excavation, especially large in the inferior segment, 41 d^1. These excavations correspond with longitudinal grooves which run along the surface of the radiating plates.

The lamellæ entering into the composition of the osseous laminæ, or walls of the cancelli, usually follow the same direction as the exteriors of these laminæ. But in the marginal plates (41 *e*) they appear in the form of investing cones, arranged parallel to the external outline of each plate, and being evidently the result of successive growths added to its exterior. The space intervening between these osseous segments are occupied by four corresponding ones of cartilage, 41 *f*, *g*, but which are of more uniform size. Of these, the peripheral portions of the two uppermost correspond with the bases of the neurapophyses, 41 *h*, whilst the lower ones bear the same relation to the parapophyses, 41 *i*.

In the central convergent portions of each of these cartilaginous segments, there is a limited formation of chondriform bone (41 *k*) arranged around the canal of the chorda, and which must contribute materially to the strength of the bone by cementing together the thin internal borders of the wedge-shaped osseous segments. Internally, this chondriform bone is consolidated, presenting only the usual small cellular areolæ; but external to this more solid structure is a fringe, consisting of innumerable minute spherical granules of various sizes, and which obviously represent the early conditions of the same tissue. As in the case of the lenticular granules seen in the fibrous membranes of cycloid scales, these granules increase in size, partly by the addition of concentric calcareous layers applied to their exteriors, and partly by the amalgamation of detached granules, which are bound together by the addition of common concentric coverings. Thus the chondriform bone is constantly encroaching upon the inner extremities of the four cartilages, the cells of which are remarkably elongated and fusiform, and radiate outwards in irregular lines, their long axes being disposed in the same direction. No chondriform bone invests the sides of the four osseous segments. Near the periphery of each cartilaginous segment the cells are more spherical, but still detached from each other and not arranged in the ordinary ichthyal groups seen at a greater distance from ossifying surfaces. On examining the outer margin of each cartilage, at its junction with the apophysis with which it is surmounted, we perceive that here also the former is being encroached upon by the latter. Each apophysis (41 *h* and *i*) consists of large cancelli, formed by laminæ of membraniform bone, and appears to be produced in the way already described, when speaking of some of the cranial bones. An irregular cavity is formed by the shrinking or partial absorption of the cartilage, which is afterwards lined by a fibrous membrane. This membrane becomes subsequently calcified, a copious development of chondriform bone taking place at the same time within the contiguous cartilage; hence a considerable amount of the latter structure (41 *l*) fringes the growing bases of these apophyses. There is great beauty in the way in which the various elements of the structure thus preserve their needful adaptation to each other. The enlargement of the divergent osseous segments by peripheral additions of membraniform bone, produces a corresponding increase in the intervening areas; the more external cells of the cartilages by which the latter are occupied multiply consentaneously in the ordinary way. Though the apophyses are not in this instance anchylosed to the osseous centrum, but detached, it is still necessary, in order to obtain a degree of firmness, that the bases of the former should be immediately in contact with the radiating plates of the latter, and not merely perched upon the top of the terminal prolongation of the cartilage which always projects into their interior. As the superficial area of each cartilage is enlarged, a corresponding expansion of the growing base of each apophysis is produced, and the requisite adaptation of the one to the other is thus maintained.

The four portions of chondriform bone (41 *k*) grouped around the canal of the

chorda, are apparently analogous to the central nucleus, seen in the centres of the carpal and stylo-hyal bones (37 *c*), having a similar structure and origin. In each case its *continued* development is independent of the immediate proximity of any progressing growth of membraniform bone. The chondriform bone fringing the growing cancelli of the apophyses, on the other hand, bears more relation to that seen in the enlarging condyles of the epitympanic (38 *c*) and in the centres of the parietals and alisphenoids. When the cartilage at the base of each apophysis is absorbed, to make way for new cancelli, the contained granules of chondriform bone must necessarily disappear along with it.

We thus see, that, notwithstanding the great difference existing between the complex form of a Pike's vertebra and its more simply constructed carpal bone, the two are developed according to a common plan. Whilst the distinction between chondriform and membraniform bone is maintained in both cases, the two tissues invariably bear the same relation to each other.

Before leaving this portion of the subject, the close resemblance between the general direction of the divergent segments of *membraniform* bone in the vertebra of the Pike, fig. 41, and those of *chondriform* bone in the corresponding portion of the Thornback Ray, fig. 28, deserves a passing notice. The chief difference arises from the circumstance, that, whilst in the latter the two inferior plates (28 *i*) are separated by an intervening prolongation of cartilage, in the former (41 *d*) they have been united from the first. The substitution of the one kind of bone for the other in the two cases is an interesting fact.

After having thus acquainted myself with the structure and growth of the bones of the Pike, I pursued the same plan in reference to the bones of a number of other osseous fish, into the permanent composition of whose skeletons a much smaller amount of cartilage enters.

In the skeleton of the Perch (*Perca fluviatilis*, Linn.), I found that nearly all the bones, excepting the vertebræ, exhibited similar conditions to those existing in the Pike, modified only by the greater extent to which the cancelli have encroached upon the respective cartilages. The epitympanic bone taken from each of these fish presents a good illustration of the difference between them; on viewing their external aspect, the two bones appear to be very similar; but on making vertical sections of that of the Perch, we find that instead of the terminal cartilage being prolonged through the interior of each condyle, and almost reaching the centre of the bone, as in the case of the Pike, it only exhibits at its proximal part a very obtuse conical termination, penetrating but a little way into the concave extremity of the osseous portion of the condyle; its ossifying surface however still presents the peculiar film of chondriform bone. The remainder of the interior of the condyle consists of numerous membraniform cancelli, formed by successive encroachments upon the cartilage; whilst the external cylinder, also membraniform, has been produced in precisely the same way as the corresponding portion of the epitympanic of the Pike, fig. 38 *d*. The

petrorsals, alisphenoids and other bones of the cranium also correspond in their mode of growth with the same bones in that example; only, in accordance with what has just been remarked, the cartilage in the centre of each bone has been more or less absorbed, and only exists permanently as an irregular external ring, separating the peripheries of the two osseous layers; being thick at its margin and becoming thinner as we approach the centre of the bone. The cancelli of the latter portion appear to have been formed as before.

The dentary bone also exhibits a cylindrical rod of cartilage projecting from its posterior angle, and extending across the inner surface of the mandible. The vomer and presphenoid are developed upon the under surface of the interorbital and nasal cartilage; only the projecting osseous lamina, contributing to the formation of the interorbital septum (which is almost rendered complete by a thin upward expansion of the cartilage), is still larger than in the Pike. In fact it may be briefly stated, that, with one exception, the processes of growth are the same in the Pike and in the Perch, only modified in the latter fish by the greater extent to which the osseous tissue has displaced the cartilaginous.

The exception to which I here refer is the vertebra. This bone is wholly osseous. No cartilaginous segments enter into its composition, and the various apophyses are firmly anchylosed to the bone. I have not been able to obtain any evidence indicating that cartilage has ever entered into its structure. It appears probable, that in the first instance an osseous ring has been developed in the membrane surrounding the chorda dorsalis, and that the sole way in which its subsequent growth has been effected, has been by peripheral additions of the same kind of bone, variously arranged.

The same peculiarities exist in the vertebræ of the Common Plaice (*Platessa vulgaris*, Flem.). I find no trace of cartilage in connection with them, even in the youngest examples which I have been able to obtain. Numerous irregular longitudinally disposed plates radiate from the centre towards the periphery, and are connected at right angles by other smaller laminæ which bind them together. There is an attempt at the formation of four distinct segments like the osseous ones in fig. 41, and it is just possible that in a *very early* fœtal condition, cartilage may have existed in the intermediate spaces. The way in which the bones of the Pike appear permanently to typify the earlier stages of the osteo-genesis of many other fish, would lead us to suspect that such may have been the case. If so, the cartilage has wholly disappeared very early; since we soon find that these intervals are also traversed, like the rest of the vertebra, by a few transverse laminæ, which thus convert them into large cancelli occupied by fat-cells. In the Plaice, the parapophyses are certainly nothing more than prolongations of the radiating plates, and not formed from independent centres of ossification: I have some doubts whether the neurapophyses have not been produced in the same exogenous way.

In the vertebræ of the young Haddock (*Morrhua æglefinus*, Cuv.), we find the same peculiarities as in those of the Perch and Plaice, but the outlines of the deep lateral

fissures in the vertical section are still more strongly defined than in those examples. They are however filled up in the same way as in the Plaice, by transverse laminæ of membraniform bone. If the osseous neurapophyses have ever been independent of the centrum, they have become united at an exceedingly early age, since many of the lamellæ which enter into their composition may be readily traced down into the interior of the centrum.

On making a vertical section through the vertebra of the Cod (*Morrhua vulgaris*, Cuv.), parallel with the spinal axis, we see very clearly that the growth of the two terminating articular cones, and that of the laminæ by which they are connected together, have proceeded consentaneously. The osseous substance of these cones forms two thick and well-defined margins in a section so prepared. At the first glance, each of these terminal tissues appears to have been formed by the addition of new laminæ to their external or articular surfaces. This appearance however is fallacious, and arises from the existence of a series of minute radiating nutrient tubes, which permeate this portion of the bone at right angles to the real lines of growth; in the vertico-longitudinal section the latter are really transverse, and parallel with its peripheral surfaces. Nothing is easier than to show that the lamellæ, of which the longitudinal plates consist, are also prolonged into these marginal cones, and that the latter have been developed entirely by the successive additions of concentric lamellæ to their peripheries. The whole vertebra thus resembles a succession of thin closely-fitted cylinders, placed one within another, each one being successively larger than that which it invests; only instead of these cylinders being entire throughout their whole length, they are so only at their two extremities; the intervening portions being alike flexed and excavated, so as to produce the less regular laminæ forming the cancelli occupying the interior of the vertebra. The edges of the cylinders thus superimposed constitute the concentric rings seen in the concave articular extremities of each of these bones.

I am unable to see how this structure and mode of growth can be reconciled with the idea of the terminal cones being separately developed from two independent osseous centres, as thought by Professor Owen*. Corresponding sections of the vertebræ of the Pike, Plaice, Haddock, Perch and other fish, all lead me to corresponding conclusions as to the way in which these portions of the bones have been formed, though none of my sections show it with the same degree of clearness as those of the Cod. The small nutrient tubes in the latter example have been formed in the same way as the Haversian canals in the other bones of the Pike, by the omission of apposite points from the substance of the successively added lamellæ, which, as already observed, these tubes penetrate at right angles to their plane.

I am well aware that these views respecting the development of many vertebræ and their apophyses from one common centre of ossification, will not escape opposition, controverting as they do the ideas of some of our most justly distinguished

* On the Archetype and Homologies of the Vertebrate Skeleton, p. 82.

ichthyotomists. Still I am convinced that an unbiassed examination of sections carefully prepared from fishes of different ages, will fully corroborate the general accuracy of the preceding descriptions.

The other bones of the Cod, Haddock and Plaice agree in all material points with those of the Perch. The radius, ulna and femur of the Haddock are all produced by the development of membraniform bone on the two surfaces of a thin cartilage; but their thin osseous lamellæ have projected far beyond the cartilaginous matrix, as in the case of the opercular bones, thus producing the thin squamous expansions which constitute so large a portion of their substance.

Whatever may be the variations either in the form of the bones or the position occupied by the cartilaginous matrix, some portion of the latter always makes its appearance at the surface of the former up to a comparatively advanced stage of growth. As far as my present observations have enabled me to judge, this appears to be an invariable rule. In some bones, such as the coracoids and premaxillaries, scarcely any trace of the original cartilage can be found. Still even in these, though they consist almost wholly of membraniform bone, a small aperture may usually be detected at some point, where either a small portion of cartilage is retained, or which conducts to cancelli filled with fat-cells, by which it has been permanently supplanted.

The structure of the presphenoid in the very young Haddock illustrates my remarks respecting the same bone in the Pike. A slender cylinder of cartilage runs along its interior and is surrounded by concentric lamellæ of membraniform bone. This investing osseous cylinder can be distinctly traced backward far beyond the posterior apex of the cartilage, even when the space left by the absorption of the latter tissue has become occupied by well-marked, secondary cancelli. As we proceed anteriorly, we find that the continuity of the cylinder becomes interrupted by a small fissure dividing its upper walls, through which an expansion of the internal cartilage escapes, forming the interorbital septum: this fissure gradually becomes wider as the contained cartilage increases in size, forming the expansion of the same tissue which rests upon the vomer; the anterior portion of the presphenoid being at length reduced to an unsymmetrical prolongation of the lower laminæ of the primary cylinder; a trace of its original cylindrical character being preserved in the shallow groove marking its upper surface, on which the superincumbent cartilage still rests. In this case I have little doubt that the primary cartilaginous matrix occupied the centre of the sphenoid bone, of which the presphenoid appears to be only a vegetative, exogenous prolongation.

None of the bones hitherto described contain anything homologous with the canaliculated lacunæ seen in human bone. In those of the Eel (*Anguilla acutirostris*, Yarrell), such lacunæ, of the peculiar quadrate type so prevalent amongst fishes, are abundant; consequently I turned to their examination with a considerable degree of interest, being anxious to see whether the presence of these structures in any degree altered the genesis of the bones. As far as I have been able to comprehend

this process in the bones of the Eel, they appear to conform to the leading phenomena seen in those of the Perch and its allies. The condyles of the epitympanic, for example, exhibit a very similar distribution of the cartilage and the bone, both chondriform and membraniform, as in those of the Perch. The lacunæ in the membraniform lamellæ appear to be merely additional appendages to the structure, and in no way influenced by the relations between the bone and the cartilage. In the vertebræ of the same fish we see no trace of cartilage, and yet the lacunæ are abundant; their plane and that of their spreading canaliculi following the direction of the contiguous flexed lamellæ, between which they are grouped. It is manifest that they are mere spaces which were left open when these lamellæ were successively added to the various surfaces of the growing bone, and are perfectly distinct from the analogous, but not homologous, areolæ, seen where the chondriform bone has invested the superficial cartilage-cells in the ossifying surfaces of the same fish. I may remark, that in this example also the laminæ of the parapophyses and neurapophyses are evidently continuous with those passing into the interior of the centrum.

The only other fish which I propose to notice in the present memoir is the Salmon (*Salmo salar*, Linn.), the bones of which present a singularly anomalous structure. In their external contours they do not appear to differ materially from those of the other osseous fishes, with the exception that the ichthyotomist finds them very difficult to dissect and disarticulate, owing to the circumstance that there exists in connection with them a large amount of cartilaginous tissue of a tough leathery consistency.

On making a vertical section of a vertebra in the same direction as that of the Pike (fig. 41), we perceive that there is a similar arrangement of the osseous and cartilaginous segments. The osseous tissue is full of areolæ of various forms, resembling those of the chondriform bones of Plagiostomes. They vary in shape, being sometimes spherical and at others elongated and fusiform. Instead however of growing merely at their extremities, the osseous elements increase wherever they come into contact with the cartilage, the increase proceeding the most rapidly at the peripheral portions of the osseous segments or plates. Where the cartilage is in contact with the sides of these plates, it is converted into a very marked form of *fibro*-cartilage, the fibres being exceedingly distinct, running in radiating lines from the centre of the organism, and of course being parallel with the growing lateral surfaces of the plates or segments. This altered character of the cartilage is equally obvious where it invests the peripheries of the radiating plates, and in all cases the cartilage-cells are retained amongst its parallel fibres, though considerably altered in their form and appearance. At the internal angle of each cartilaginous segment and at the bases of the neurapophyses, there exists a small amount of chondriform bone of a character which more closely resembles that in the corresponding portions of the vertebra of the Pike. In the Salmon, its areolæ are larger in proportion to the amount of the solid calcareous substance with which they are surrounded.

All the other bones exhibit a similar process of development to those of the osseous

fishes previously described, only in every instance the bone retains its areolar character, and the investing periosteum appears to have more the character of a fibro-cartilage than of a simple fibrous membrane. The ossifying surfaces exhibit a film of chondriform bone like that seen at the bases of the neurapophyses of the vertebra; whilst laminæ of bone, resembling that seen in the centrum, successively shoot into the cartilage, the intervening portions of which are absorbed as they advance leaving long parallel cancelli, abounding in fat-cells. It is obvious, that a slight difference exists between the chondriform bone contained in the cartilage at the extremities of these elongating laminæ and that of the laminæ themselves, as well as of the osseous cylinder in which they are contained: the latter tissues approximate more to the nature of true membraniform bone, and yet they cannot be regarded as such. As far as I am able to comprehend their curious structure, all the bones of the Salmon appear to be of a chondriform character; but whilst in some cases, where the bone is designed to be a temporary growth, the calcareous matter is deposited in the unaltered intercellular substance of cartilage, in others, where the bone is designed to be permanent, the cartilage is converted into a fibro-cartilage previous to calcification. At the same time that the bones are increasing at their extremities in the way already described, a prolongation of the same altered cartilage or fibro-cartilage, containing fusiform cells, and investing their exteriors as a periosteum, is increasing their thickness by successive external additions, in which, as in the other parts of the skeleton, the presence of cells in the fibro-cartilaginous matrix leads to the existence of corresponding areolæ in the bone.

During the examination of the other osseous fishes referred to in the preceding pages, the question frequently suggested itself to me as to whence the fibrous membrane which lined the cavities encroaching upon the cartilage is derived; is it an altered condition of the surface of the cartilage itself, or after the latter was partially absorbed did the cavity become lined with an entirely new structure, which calcification was subsequently to convert into membraniform bone? Whatever was the origin of the tissue, it appeared to be of the same character as the perichondrium and the periosteum. In endeavouring to trace a line of division between the cartilage and the perichondrium I usually failed; whilst at the bases of the neurapophyses of the Pike, where the external periosteum and the subjacent internal fibro-cartilage came into contact, there appeared to be a commingling of the two tissues, rather than a mere juxtaposition. Many of these circumstances combine to produce a conviction, that the cavities about to be transformed into cancelli were *not* lined by an entirely new membrane, but by one formed out of the fibrillated substance of the cartilage. So far as I comprehend the growth of the bones of the Salmon, they appear to countenance this opinion. We find that the cartilage, prior to its conversion into what in other allied fishes would become membraniform bone, assumes in a very marked manner a fibrous character, without entirely losing its characteristic cells. At the same time, the *external* growths, which in other allied osseous

fishes consist of similar membraniform bone, here exhibit the same areolar structure as the plates of the vertebral centrum, whilst they are surrounded by a periosteum which preserves all the characteristics of a genuine fibro-cartilage. The same remark applies to the osseous laminæ which encroach upon the receding cartilage along the ossifying extremities of the bone, which laminæ are of course also formed in the fibro-cartilage. These circumstances leave little doubt upon my own mind, that in the case of the Salmon, the structure about to be calcified, whether lining a cancellus or existing as a periosteum, is merely a fibrillated modification of the primary cartilaginous matrix, and it becomes increasingly probable that the same is the case with the homologous portions of other osseous fishes. The subject is too important in its bearings upon some branches of physiological science to allow of rash or premature conclusions, and consequently I would throw out the above suggestions in a spirit of due caution, being well aware of the vast amount of detailed observations yet requisite before the general principle can be established.

This structure of the bones of the Salmon also teaches us, that though the differences between membraniform and chondriform bone are usually clear and well-marked, yet the extreme forms of each may be connected by inosculating links, which break down the artificial distinctions that our imperfect philosophy would set up. We have learnt the same lesson from the snout of the Saw-fish.

It will scarcely be necessary to prolong this memoir by recapitulating the various conclusions which the observations detailed in the preceding pages have enabled me to deduce, since they have been recorded as we proceeded with the details of the several topics. There are however two or three questions upon which the preceding inquiries have some bearing, and which require a passing notice.

We have already found that the non-cellular, fibrous membranes of Cycloid and Ctenoid scales, the fibro-cartilages of the Salmon and Saw-fish, and the true cartilages of various other fishes, appear to be calcified in one common way, viz. by the formation of small calcareous granules, which are not interposed between, but actually incorporated with, the several tissues in which they are developed. In many of the examples which I have recorded, these granules are so large as to be easily examined, whilst the degrees to which their size is capable of being increased by the external addition of calcareous atoms, which are not themselves discernible, vary in different tissues and species of organisms. Analogical reasoning renders it probable that the calcification of such tissues as human teeth and bones may have been accomplished in the same way; only through the agency of granules which rarely attain a sufficient size to become visible even under high magnifying powers.

Another still more important inquiry suggested by the growth of the various forms of kosmine, has reference to the relation which they sustain to true dentinal structures. We have seen that a great variety of kosmine tissues exist, which exhibit the closest resemblance to several of the modifications of dentine described by Professor Owen. Thus in the areolæ of the *Megalichthys*, the dermal teeth of the rostrum of the

Saw-fish and numerous other Plagiostomes, as well as of the scale of the Siluroid, we have what closely resembles ordinary dentine, if not its homologue. In the elaborate structures covering the scales and occupying the interior of the spines of *Ostracion* and of *Cœlorhynchus*, we have beautiful examples of osteo- and vaso-dentine. The tubes which enter from the external surface of the same structures, remind us again of the very similar tubes, which, as Mr. Tomes has demonstrated, penetrate in like manner the enamel of the teeth of most marsupial animals. The many varieties of kosmine which I already possess, leaves little doubt on my own mind that when we are as well acquainted with the numerous modifications of this tissue as we already are with those of dentine, it will be found that every variety of the latter has its representative amongst the former class of structures. When we remember the undoubted fact that all these forms of kosmine have been produced by the calcification, not of a cellular pulp, like that to which the growth of dentine has been attributed, but of successively added lamellæ of purely fibrous membrane, we are bound to admit the *possibility* that dentine *may* have been formed in the same way. So long as the old opinions respecting the direct influence of cells on the production of the Haversian systems of mammalian bones were recognized, nothing was more natural than to account for the growth of teeth by a similar process; especially since a cellular structure, to which to refer the process, was so conveniently at hand. But now that the growth of bone is almost universally regarded by intelligent physiologists in a different light, and bearing in mind that a series of tissues closely resembling dentine have been formed without the intervention of any pulp-cells, the question naturally suggests itself, whether the cells of the pulp have any direct connection with the calcification of a human tooth. I have already pointed out that the "Pulp-cavity" of the spine of *Ostracion*, the recent representative of the fossil *Cœlorhynchus*, is chiefly occupied by a reticular fibrous membrane, such as is seen in the scales of the same fish. The dermal defence bones of the common Picked Dog-fish (*Spinax acanthias*, Cuv.) contain a "pulp" of true cartilage, exactly like that with which the bone of the endoskeleton is associated. Both these tissues, existing in the interiors of structures which bear a very close resemblance to true teeth, are very different from the pulp-cells of a mammalian tooth.

There are some points in the structure of a mammalian tooth, which suggest a doubt as to the correctness of the pre-existing hypotheses.

In Dr. Muller's Physiology, a representation is given of the external portions of some of the dentinal tubes, from the incisor of a full-grown Horse (Dr. Baly's Translation, 2nd edition, p. 428, fig. 41), in which many of the lateral twigs of each tube terminate in well-defined lacunæ with numerous canaliculi; a dense series of similar lacunæ is also represented as existing at the extremities of the main tubes, immediately below the enamel. I have carefully examined this portion of the tooth, both in the Horse and Ox, and, with Professor Owen, I differ from the interpretation given by Dr. Muller. I am disposed to think that the dentinal portion immediately subjacent to the ex-

ternal enamel, consists of an aggregation of very minute spherical granules, similar to those existing in the Cycloid scales of fish, and still more closely resembling those seen in the *external* half of each osseous rod or prism in the rostrum of the Saw-fish. I am unable to detect any well-defined lacunæ, with regular radiating canaliculi, such as are represented by Dr. Muller. It appears to me, that owing to the incomplete coalescence of the minute granules, an irregular network of minute angular interspaces is left, varying in their size, and typically resembling those seen in some scales, and in the portion of the Saw-fish just referred to. From these small interspaces, the twigs of the dentine-tubes take their rise. The same granular aspect of the calcareous intertubular structure continues to be visible for some distance along the course of the tubes, but nearer the pulp-cavity the amalgamation of the granules has been so complete, excepting where the dentinal tubes have been left open, that all traces of what I have just described completely disappears. I suspect that this is the structure which caught the eye of Mr. Nasmyth, and led him, at the sixth meeting of the British Association, to declare that the intertubular portions of teeth were cellular.

In the fœtal condition of the tooth, according to Purkinje and Raschow, the pulp is surrounded by a membrane, between which and the pulp, these observers believe the formation of the dentine to be carried on. It may be worthy of inquiry whether the calcareous structure is not developed *in* rather than *beneath* this membrane; a continual growth of the latter at the pulp-surface, however thin and inappreciable, would supply all that was wanting to carry on the corresponding growth of the calcareous tissues of the tooth. But even should there be no evidence that this has been the case, and we are compelled to fall back upon the pulp itself as the matrix in which the calcifying process is carried on, it does not necessarily follow that the cells of the pulp are involved in the process. We have already pointed out the probability that the new membrane, lining the cavities produced by absorption of the cartilage in many fish, and designed to be ultimately converted into osseous cancelli, is probably nothing more than an altered condition of the intercellular portion of the cartilage. A similar state of things exists at the surface of the cartilage forming the pulp of the defence spines of the Picked Dog-fish, where some new lamellæ are successively added to the interior of the kosmine structure. This suggests a second subject for investigation, viz. whether the soft blastema, in which the cells, vessels and nerves of the tooth-pulp are distributed, may not undergo a similar change prior to the calcification of its outermost portions. Whatever may be ultimately proved to be the true rationale of this process, there is no question whatever, that the various peculiarities attending the growth and development of kosmine, indicate the propriety of a careful review of the evidence upon which the generally received hypotheses respecting the production of dentinal tissues are based.

Somewhat allied to the above subject is the collateral one respecting the true nature of what I have designated Haversian canals. The canals permeating the bones of

various fishes, are usually very different in their aspect from those with which the name of Havers has been connected by the anthropotomists. Similar complicated concentric systems of osseous lamellæ, with their parallel arrangements of lacunæ, have a definite existence in the scales of *Megalichthys*, and it is easy to trace a series of links which connect the latter with the more simple canals existing in the bones of the Pike; a further transition from these to the similar passages seen in the scale of *Dactylopterus* (fig. 14 *a*, *b* and *c*), and in the ganoin of *Balistes* (fig. 13 *f*, *f'*), is equally easy. Much as the extreme forms differ, the whole appears to represent one homologous series.

Another moot question, which derives some light from the preceding observations, is that of the nature of the opercular bones. Reference has been already made to the conflicting views of Professor Owen and M. Agassiz on this point; the former gentleman regarding them as portions of the endoskeleton, whilst the latter considers them to be enlarged and modified scales belonging to the dermal exoskeleton. The distinctions between the structure and mode of growth of cycloid scales, and of opercular bones, are clear and obvious. The characters are wholly dissimilar; whilst the genesis of the latter appears to be invariably connected with a primary development of cartilage, we have not the least ground for supposing that this tissue is ever connected with the former of these structures. In the case of the *Macropoma Mantelli* from the chalk, the distinction is even still more obvious than usual, owing to the large development of beautiful lacunæ, which has taken place in the true osseous opercula, and which correspond exactly with those seen in other portions of the endoskeleton, whilst nothing resembling them occurs in the interior of the true dermal scales. The existence of small points of kosmine on the surface of the operculum, resembling those covering the scale, does not invalidate this conclusion, since we find that closely analogous structures occur within the mouths of many osseous fish, connected with bones which are unquestionably portions of the endoskeleton.

The time has not yet arrived when any attempt can be safely made to arrange the various processes of ossification existing in fishes, so as to enable us to deduce from them any general law. Why one kind of bone should exist in the Salmon and another in the Pike; why the membraniform osseous tissues of the Eel, the *Lepidosteus osseus* and the *Loricaria* should abound in lacunæ, whilst they are absent from the corresponding tissues of the Cod, the Haddock and the Perch; and why the same tissues in the *Lepidosteus* and the fossil *Saurocephalus* should have in addition a large number of narrow tubes which also occur in the scales of *Lepidosteus* and *Lepidotus**, whilst these appendages are absent from the vast majority of ichthyal bones, are questions which we are not yet able to decide. The degree to which these several types prevail amongst the various families of fish is yet to be ascertained, and nothing but widely-extended observations will obtain for us the necessary information. Still it is very desirable that we should possess accurate knowledge on

* Philosophical Transactions, *ut supra*, tab. xl. figs. 2 *b* and 3 *a*.

these various topics, since it would not only be valuable in a physiological point of view, but also in its practical application to geology. We already obtain faint glimpses of the prevalence of some general law, through the operation of which a number of varied products appear to be developed out of one primitive blastema. Chondriform and membraniform bone, kosmine, dentine, ganoin, enamel, cartilage, fibro-cartilage and fibrous periosteum, appear to be mysteriously linked together, and to possess some community of nature which is not yet fully revealed. The present attempt to add to the existing stock of facts, has only shown to me how wide a field still remains unexplored, which field must be worked out before we can fully comprehend the laws regulating the osteo-genesis of fishes.

In conclusion, I have again to acknowledge the kind assistance which I have received from Sir Philip de Malpas Grey Egerton, who has facilitated my investigations in many ways, especially in supplying me with specimens for examination, which I could not otherwise have obtained.

Description of the Plates.

PLATE XXVIII.

Fig. 1. Scale of a Ctenoid fish from the Bay of Dulse (Mexico). Nat. size.

Fig. 2. Vertical section of the same scale, made in the direction of the dotted line, fig. 1 *a b*. Magnified 5 diameters.

Fig. 3. Still more highly magnified view of a portion of fig. 2 from *a*. Magnified 80 diameters.

Fig. 4. Similarly enlarged view of a portion from fig. 2 *b*. Magnified 80 diameters.

Fig. 5. Horizontal section of the lower or membranous laminæ of the same scale. Magnified 70 diameters.

Fig. 6. Horizontal section of the middle layer of the same scale. Magnified 100 diameters.

Fig. 7. Superficial aspect of a small portion taken from fig. 1 *c*. Magnified 25 diameters.

Fig. 8. Corresponding view of a portion removed from fig. 1 *d*. Magnified 25 diameters.

Fig. 9. Vertical section of part of the anterior portion of the scale of a Carp, made parallel to the mesial line. Magnified 160 diameters.

Fig. 10. Horizontal section of the uppermost of the membranous laminæ of the scale of the Perch. *Inverted* and magnified 200 diameters.

Fig. 11. Vertical section of the posterior margin of the scale of the Perch. Magnified 75 diameters.

Fig. 12. Horizontal aspect of the corresponding portion of the same scale. Magnified 50 diameters.

Fig. 13. Vertical section of a scale of a species of *Balistes* (File-fish), made parallel to the lateral line. Magnified 18 diameters.

PLATE XXIX.

Fig. 14. Vertical section of a scale of the Flying Gurnard (*Dactylopterus*), parallel to the lateral line. Magnified 20 diameters.

Fig. 15. Vertical section of the scale of *Loricaria cataphracta*, Linn. (*L. setigera*, Lacepede). Magnified 14 diameters.

Fig. 16. One of the dermal teeth from the surface of the same scale. Magnified 65 diameters.

Fig. 17. Dermal plate from a species of *Ostracion*, superior surface. Magnified 3 diameters.

Fig. 18. Inferior surface of the same.

Fig. 19. Vertical section of the same along the dotted line, 18 *a a'*. Magnified 12 diameters.

Fig. 20. Portion of a similar section. Magnified 36 diameters.

Fig. 21. Horizontal section of one angle of the above scale, in the plane of the canals, fig. 19 *h* and 20 *b*. Magnified 12 diameters.

Fig. 22. Vertical section of the margin of a scale of another species of *Ostracion*. Magnified 60 diameters.

Fig. 23. Central portion of a vertico-transverse section of the last scale, subsequent to its decalcification, and made in the direction of the lines *a' b* in fig. 18. Magnified 40 diameters.

PLATE XXX.

Fig. 24. Vertical section of a portion of the same decalcified scale. Magnified 55 diameters. This section is made nearly at right angles to the last, and near one of its extremities.

Fig. 25. Bundle of fibres from the same decalcified scale. Magnified 250 diameters.

Fig. 26. Horizontal section of part of the same decalcified scale, a little inferior to the plane of fig. 21, and showing the fibrous bundles (fig. 25), and the reticular ones of fig. 24 *d* in their natural positions. Magnified 300 diameters.

Fig. 27. One of the calcareous septa, figs. 19 *b* and 20 *e*, after the animal matter has been removed by exposure to a boiling solution of caustic potass. Magnified 150 diameters.

Fig. 28. Vertical section of the vertebra of a Ray (*Raia clavata*), made at right angles to the spinal axis, midway between the two concave articular cones. Magnified 6 diameters.

Fig. 29. Horizontal section of the ossified surface of the neural spine of the same fish. Magnified 25 diameters.

Fig. 30. Enlarged view of one of the osseous plates of fig. 29. Magnified 160 diameters.

Fig. 31. Vertical section of one of the same osseous plates with its subjacent cartilage. Magnified 125 diameters.

Fig. 32. One of the small interspaces between the contiguous osseous plates, fig. 29 *d*. Magnified 130 diameters.

PLATE XXXI.

Fig. 33. Vertico-transverse section of one of the vertebræ of *Carcharias vulgaris*, midway between the two terminal cones. Natural size.

Fig. 34. Small portion of the internal structure of the same. Magnified 250 diameters.

Fig. 35. Small portion of the margin of fig. 36 *c*. Magnified 200 diameters.

Fig. 36. Vertical section of the vertebra of *Carcharias vulgaris*, parallel with the vertebral axis. Slightly enlarged.

Fig. 37. Longitudinal section of the carpal bone of a young Pike, (*Esox lucius*). Magnified 20 diameters.

Fig. 38. Longitudinal section of the extremity of the opercular condyle of the epitympanic bone of the Pike. Magnified 16 diameters.

Fig. 39. Vertical section of the proximal half of one of the condyles of the same bone in a younger fish. Magnified 24 diameters.

Fig. 40. Vertical section of the articular extremity of the opercular bone of a Pike. Magnified 16 diameters.

Fig. 41. Vertical section across the centre of the vertebra of a Pike, at right angles to the vertebral axis. Magnified 8 diameters.

Since the preceding memoir was written, I have had the opportunity of examining the scales of the Eel, to which my attention was directed by my friend Dr. Carpenter. I find that in them the calcareous portion consists *of a single layer, of detached calcareous granules*, which have not coalesced so as to form a continuous tissue. This is interesting, since it exhibits a rudimentary modification of the type, which becomes so much more fully developed in the scales of the higher osseous fishes.

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XXX. *Report of further Observations upon the Tidal Streams of the North Sea and English Channel, with Remarks upon the Laws by which those Streams appear to be governed.* By Captain F. W. Beechey, *R.N., F.R.S.* *Communicated to Sir* Francis Beaufort, *K.C.B., Hydrographer, and presented to the Royal Society by* G. B. Airy, *Esq., F.R.S., and Astronomer Royal, &c.*

Received March 27,—Read May 15, 1851.

Sir, London, March 11th, 1851.

In pursuance of directions from the Lords Commissioners of the Admiralty that I should make further inquiry into the phenomena of the tidal streams of the Channel, I commenced a systematic investigation of the whole feature of tidal-streams throughout the strait which separates England from the Continent, and I have now the satisfaction to lay before you for their Lordships' information the result of these labours.

In order to prosecute this inquiry with as much advantage and as little delay as possible, I have combined with the information furnished by the vessel appropriated to this service as many observations from various authentic sources as were recommended by the known reputation of their authors.

Authorities consulted.

These consisted of various observations upon the streams of the English Channel and the North Sea by Captain Martin White, R.N., Captain Washington, R.N., Captain Hewitt, R.N., and Captain Bullock, R.N. Also by Monsieur Monnier, M. Beautems Beaupres, and by the Engineurs Hydrographiques, published in the 'Pilote Française.' To these have been added some by Mr. Græme Spence about the Scilly Islands, and a few by Captain Anderson, as given in the Philosophical Transactions; numerous observations along the shore by the Coast Guard, and a series of observations kept on board the light-vessels by order of the Trinity Board, which are particularly valuable as being made simultaneously, and, like those of Captain Washington, continued throughout several days at each station.

Method pursued in making the observations.

The method pursued in making the observations, was to anchor the vessel at each of the stations* for twelve hours and upwards, and to observe the direction of the tide, ebb and flood, every half-hour. The rate of the stream, in addition to the usual method by the common log, was detected by current logs, constructed for the purpose by Mr. Massey, and which registered feet; and that it might be ascertained whether the stream was confined to the surface or extended to the depth of a vessel's hull

* See Chart of the Stations, Plate XXXII.

in the water, another of these logs was occasionally sunk 12 feet, and registered simultaneously with the one which was 2 feet beneath the surface.

Method of making the observations.

The time of slack water, as well as the times of the cessation and commencement of the stream, were noted, as closely as such observations are capable of being made; and to render the times more certain, I have taken the mean interval between the time of the cessation of one stream and the time of the commencement of the next. The times of high water at Dover have been taken from the Admiralty Tide Tables, which were tested by observations made at Dover during some portion of the observations in the Channel, and the agreement being found sufficiently near to render a special register unnecessary, it was discontinued.

All the times of observation being referred to the *meridian of Dover*, and compared with corresponding times of high and low water at that place, tables of differences were obtained which gave *the interval between the time of slack water at each station, and the time of high water on the shore at Dover*, and also the direction and rate of the stream at each hour of the tide before and after the same standard. By this method of comparison observations made at various periods, and even in different years, were brought as nearly as possible to a simultaneous record.

To exhibit these results tables were drawn out, and twelve charts were constructed, showing the position of the stations and the direction of the stream at each hour of the tide at those places respectively, in order that the eye might at once detect the movement of the stream throughout both channels at any instant of time*.

In these charts the direction of the stream is indicated by arrows, which are connected by curved lines empirically drawn through them; these lines are continuous when the water is flowing towards Dover, and broken when running in the reverse direction. Lastly, all those tides which have a rotatory motion are indicated by spirals barbed in the direction in which they revolve.

Results.

All the observations having been subjected to this arrangement, some highly important results were obtained. Instead of these channels having, as has been hitherto supposed, a stream which turns progressively later as the tide advances up the strait, these observations have shown that the progressive changes of stream cease at a certain point near the mouth of the Channel, and that beyond that spot there is a tide peculiar to the Channel, and quite distinct from that of the seas on either side of it; so that there may be said to be two distinct streams on each side of the Strait of Dover,—the stream of the Ocean or outer stream, and the stream of the Channel, or that which is contained between the oceanic stream and the Strait of Dover. These streams are always running in contrary directions, and in the North Sea meet between the Texel and the estuary of Lynn; and in the English Channel between the Start and the Gulf of St. Malo.

Oceanic and Channel streams.

Mixed tides.

In the localities where these streams meet, the tide is ever varying its direction according as the strength of one stream prevails over that of the other; giving to the

* Eight of these charts only have been published.

water a rotatory motion, and scarcely admitting of any interval of slack water; whilst in the space between these rotatory tides and the point of meeting of the tides in the Strait of Dover, the stream is free from all rotatory motion, and sets steadily throughout the tide in a direction *towards* Dover, while the water is *rising there* and *away* from it while it is *falling at* that place. Rotatory streams. True Channel stream.

I have designated the last-mentioned the "*true channel stream*," and its extent is, as nearly as it can be measured, 180 miles in either direction from the point of union of the tides in the Strait of Dover to the region of rotatory tides off Lynn, and off the Start and St. Malo. Extent of the Channel stream.

As the true Channel streams are always running in opposite courses, there is necessarily a point where they meet and separate; and this occurs in the Strait of Dover. But *in this strait*, the stream, although it first obeys one tide and then the other, does not slack with the Channel streams, but is found to be still running at high and low water on the shore, at which times those streams are at rest, so that the Strait of Dover never has slack water throughout its whole extent at any time. I have in consequence called this an *intermediate* tide. Meet in the Strait of Dover. Intermediate tide.

The limits of neither of these streams appear to be stationary, but *range to and fro as the tide rises and falls at Dover*, travelling to the *eastward* on *both* tides, and at high and low water suddenly shifting sixty miles to the westward to recommence their easterly courses with the next tide; and although so far apart, they possess the remarkable peculiarity of shifting together; so that the Channel streams preserve, as nearly as possible, the same relative dimensions. Place of meeting of the tides changes with advance of the tide.

In the Strait of Dover this line of meeting and of separation oscillates between Beachy Head and the North Foreland, a distance of about sixty miles. When the water on the shore at Dover begins *to fall*, a *separation* of the Channel streams begins off Beachy Head. As the fall continues, this line creeps *to the eastward*. At two hours after high water it has reached Hastings; at three hours Rye; and thus it travels on until at low water, by the shore, it has arrived nearly at the North Foreland on one side of the strait, and at Dunkirk on the other. At this time the *Channel streams* on both sides slack, but in that portion which I call *the intermediate stream* in the Strait of Dover the water is still running to the westward; and when the new *Channel* streams make as the water rises on the shore, this *intermediate* portion is found to unite with, or to oppose one or the other of these streams, according as it was before the reverse; so that, as before mentioned, the line of meeting at low water appears off Beachy Head to recommence its easterly course. This *intermediate* stream forms a remarkable feature in the tidal system of the Channel; it is well established, as the line of meeting and of separation occupies a very limited space, and it seems to be entirely due to the contracted form of the Channel in this immediate locality, preventing the free escape of the water. Peculiarity of the stream in Dover Strait.

Captain Bullock, in order to test the point of separation, anchored two vessels a mile apart between Beachy Head and Dungeness, and found both vessels at the same

moment to ride with their heads in opposite directions, in obedience to the streams which were then running opposite ways.

Channel stream.

The *Channel stream*, which I have described as running between the intermediate stream and the rotatory or mixed streams at the outer extremities of the Channel, pursues a steady course along the main trunk of the strait, slacking only towards high and low water at Dover, when it is preparing to invert its course; and contrary to the generally received opinion of *a progressive slack water* in a strait having *a progressive establishment*, this stream has the peculiarity of slacking throughout its whole extent at *nearly the same time*, and this time, as was anticipated in my former paper, corresponds nearly with the time of high and low water on the shore at Dover, the site of the combined wave and of the *virtual head of the tide*.

Stream slacks at nearly the same time throughout.

Such is a general description of the phenomena of the streams of the English Channel and the North Sea; and I shall now endeavour to describe more in detail the several peculiarities of these features, and to remove various erroneous impressions which at present exist as to the rotatory motion of our tides, the direction in which the streams turn, and also as to the time of the stream attaining its maximum rate, &c., and then to lay before you such data as it appears to me will help to explain the cause of the tidal streams of these channels differing from those peculiar to ordinary tide-waves.

Simultaneous turn of the stream.

As the simultaneous turn of the stream is a point of considerable interest, I may be permitted to dwell a few minutes upon the data from which it has been derived. But I wish it to be clearly understood, that minute accuracy on such a point is scarcely possible, and could only be obtained by a long series of simultaneous observations at the extremities of the tide-wave; for the period of slack water is almost always spread over an interval of half an hour, and not unfrequently at the neaps, of an hour and upwards; and moreover winds are found to prolong the stream in the Channel according to the direction in which they blow. In estimating the time of slack water therefore from the observations, we must not expect a very close agreement*.

Time of high water at Dover, and the time of the turn of the stream compared.

It will be seen, on a very cursory inspection of the Charts which accompany this paper, that for a period of six hours after high water at Dover, and for five hours before that time, the stream maintains a steady direction from and towards Dover, so that any delay which may exist between the change of the stream and the time of high water is shown by these charts alone to be confined within very narrow limits, but a still closer agreement will be found on referring to the observations themselves*; especially if we take those which were made on board the light-vessels at a distance from the shore, which are particularly valuable from the circumstance of their being spread over intervals of several days at each station.

* The times of observed slack water at the stations marked in the Plans, as compared with the times of corresponding high waters at Dover, as shown by the tide tables, together with all other particulars relating to these observations, will be found entered in a book which accompanies this report, since deposited at the Hydrographic Office of the Admiralty.

Referring then to these observations, and taking them in the order in which the light-vessels are placed with respect to the progress of the tide-wave, we shall have the intervals between the turn of the stream at those places, and the time of high water at Dover (the standard), as follows:— Comparison continued.

At the Newarp L.V., the stream is slack twenty-two minutes before Dover.

At the Cockle L.V., the stream is slack at the same time as Dover.

At the Stanford L.V., the stream is slack nine minutes before Dover.

At two stations nearer Dover, the stream is slack twenty-four minutes after.

At the Shipwash L.V., the stream is slack twenty-four minutes after.

At the Galloper L.V., the stream is slack the same time as Dover.

At the Kentish Knock L.V., the stream is slack nine minutes after Dover.

Lastly, off the North Foreland, the stream is slack the same time as Dover.

Between these stations and the meridian of 3° 0′ E. we find the following intervals.

In the parallel of Cromer, twenty-four minutes before Dover; at the next station, southward, twenty-four minutes after; at the next, four minutes after; then, thirty-six minutes after; at the next, nineteen minutes after; at the next, three minutes after; and lastly, off the North Foreland, the same time as Dover.

In the first set of comparisons the differences are not greater than might be attributed to the ordinary irregularity of the tides; in the second we discover a progressive increase, and then the phase of the stream seems to be inverted, until at the end of the series we find the interval vanish, and the stream to turn with the high water on the shore.

Retardation of the turn of the stream towards the coast of Holland.

To the eastward of 3° of longitude, the times of slack water get considerably later in that part of the Channel which lies between the northern limit of the Thames and the Texel, and the phase of the stream in all this portion of the North Sea appears to be inverted; the intervals also to get *longer* as the Dutch coast is approached. The occurrence of this *inversion of the phase of the stream with the inversion of the phase of the establishments on the coast of Holland, and its position occupying the site of the node of the tide in Professor* WHEWELL'S *chart of cotidal lines*, is a coincidence which ought not to be overlooked, although I am not able to discover any connection between the phenomenon, which is evidently the cause of the unusual retardation of the stream above-mentioned, and the phenomena of the cotidal chart.

Cause of the retardation of the turn of the stream towards the coast of Holland.

There seems to be very little doubt that the retardation of the stream, and the consequent inversion of phase now noticed in all this part of the Channel, is entirely owing to *the outer stream setting round the Texel*, at a time when the stream is slack in all other parts of the Channel and preparing to go round*.

An inspection of the Plan for one hour after high water will clearly show this to be the case; it will there be seen that the stream of the Channel is making an effort

* See Chart one hour after high water, Plate XXXIII.

to run to the northward, and on the western side of the Channel has succeeded, the North Sea (or outer) stream being in that part favourable to it; but on the *eastern* side of the Channel, the stream coming round the Texel forces its way up the Channel, and creates a temporary disturbance, which is no doubt the cause of the retardation of the turn of the stream in this vicinity.

The two consecutive charts strengthen this supposition, by exhibiting the Texel stream receding as the Channel stream gains strength.

Comparison of time of slack water in the North Sea with the time of high water at Dover.

If, then, we except such observations as are shown to be due to the disturbance from the Texel tide, we shall find that *in the whole extent of channel from Cromer to the North Foreland, there is not half an hour of retardation in the time of slack water from the time of high water at Dover, while in the establishments there is an increase of at least five hours.* That we may appreciate this very small difference of the time of slack water throughout the Channel and the time of high water at Dover, let us inquire what takes place in parts of the North Sea, which exhibit the ordinary phenomena of tides under the influence of a progressive wave, such for instance, as are comprised in the space between Kinnaird's Head and the estuary of Lynn. There is in this space an increase of tidal establishment of five hours, an amount about equal to that which exists between Lynn and Dover. By the Admiralty Tide Tables, it will be seen that in this space there are nearly five hours' retardation in the time of slack water in the offing, making at least, *hour for hour in the change of the stream with the change of establishment*; whilst in the North Sea, in the same change of establishment, there is scarcely *half an hour difference of time in the change of stream.*

Comparison of time of slack water in the English Channel with high water at Dover.

If now we take the English Channel, we shall find half-way between the Start and Alderney, that the stream *turns with the high water at Dover.* Off Portland, *nine minutes before*; off St. Albans, *three minutes before*; between the Isle of Wight and Cape Barfleur, *fifteen minutes after*; and so we run on, the differences varying from *eight minutes one way to an extreme of forty-two minutes the other**. In the Baie de la Seine, the turn of the stream is earlier in the western part than towards its eastern extremity, but the formation of the coast about Cape Barfleur will fully account for this.

In the eastern part of the Baie de la Seine a retardation occurs towards the Somme; in that direction the turn of the stream, on the south side of the Channel, *is an hour later than* on the north side opposite; the observations, however, upon that coast are not given in a manner sufficiently clear, and they were made many years ago, and compared with the establishments of ports, which might not at that time have been very accurately determined; therefore I would not dwell much upon them*.

Upon the whole, therefore, judging from the observations which have been collected, there seems to be sufficient evidence to warrant the conclusion, that, with the excep-

* The vessel now about to sail to continue this inquiry, will, it is hoped, get some observations in this quarter.—November 5. Recent observations have shown these intervals to be in excess.

tions arising from extraneous causes, which are now known, and for which an allowance can be made, the streams of the Channel turn sufficiently near to the times of high water on the shore of Dover to be considered as simultaneous, without occasioning any error that can be of the smallest consequence to shipping*.

On the rotatory motion of the stream.

I shall now pass to the next point of interest,—the supposed rotatory motion of the stream in the English Channel and the North Sea. It will be seen by the Charts, that wherever any circular motion of the water appears, it occurs between the outer extremities of the Channel tide and the stream of the oceanic or parent wave; and is clearly to be accounted for by the streams acting obliquely upon each other.

Erroneous ideas respecting this motion.

Captain Martin White and Mr. Græme Spence, who have written upon the tides of the English Channel, were of opinion that this rotatory motion was not only applicable to the whole of the Channel, but common to the tides of all the world. Captain White, in his remarks upon currents, &c., observes, at p. 35, "Tides are governed by a regular periodical reciprocation in most parts of the world, making the round of the compass almost everywhere during the twelve hours. This rotatory motion," he continues, "is perfectly symmetrical during and after a series of moderate weather, and the curves it assumes resemble the form of *spiral curves*." It was Mr. Græme Spence, however, who originated this idea of a revolving tide†. It is a singular circumstance that the observations of these officers should not have extended to parts of the Channel where the stream runs true, but have been confined to the mixed tide. Mr. Græme Spence obtained his observations in the vicinity of the Scilly Islands, where the streams of the Atlantic and of the English Channel unite; Captain White's observations extended further up the Channel, but still not far enough to be clear of the effect of the Gulf of St. Malo. M. Monnier‡, who has often been quoted as the well-known author of a pamphlet on the tides of the English Channel, derived opinions similar to those above-mentioned, from observations made about the Channel Islands also; and was confirmed in his opinions by some observations made about the same time by Captain Hewitt in the North Sea, near the Lemon and Ower sands, which, curiously enough, are on the border of the mixed tide of the

Cause of this erroneous opinion accounted for.

* In my paper upon the tidal phenomena of the Irish Sea, I particularly referred to the nearly simultaneous turn of the stream in that channel which is also under the influence of a combined wave. Since that time observations have been made at the Smalls Lighthouse, at the Kunibeg and at other light-ships; all of which have confirmed in a very satisfactory manner those remarks.

At the Smalls, for instance, situated at the entrance of the Irish Channel, and differing from Liverpool nearly five hours in its establishment, the time of slack water occurs *fourteen minutes* only before high water at Liverpool; at the Kunibeg, *twenty-one minutes before*; then the mean of the intervals in the first page of observations, between Smalls and Bardsey, gives *four minutes after Liverpool*; the mean of the next page of observations thence to Holyhead, *thirteen minutes after*; going northward, the mean of the next page, *twelve minutes after*; off the Isle of Man, *at the same time*; and in the North Channel, *twenty-four minutes after*.

So that throughout the Irish Sea, as in the Channel which separates England from the Continent, the stream slacks throughout at nearly the same time as the combined wave is matured; or as the time of high water at *the virtual head of the tide*, as already stated in my first report.

† See his observations on the tides of the Scilly Islands. ‡ Mémoire sur les Courants de la Manche.

On the rotatory motion of the stream.

North Sea, as those of Mons. Monnier were on the border of the mixed tide of the other sea. These authors have been followed by other talented men, until a common opinion was gaining ground that streams in general had a rotatory motion.

Had the observers above-mentioned made their observations anywhere between the Start and Beachy Head, or between Cromer and the North Foreland, a different result would have been obtained; for in all this space, occupying nearly 360 miles in extent, the stream runs steadily throughout both tides, scarcely changing its direction for four hours and upwards, and then only preparatory to going round to the new coming tide.

Explanation of Plan 1, Plate XL.

Upon Plan 1, Plate XL. I have given examples of the streams in question. The upper figures represent the revolving tides, exactly in the order in which the observations were made, on a direct line between the Texel and Cromer. The circular arrows show the direction in which the stream revolves, and the figures against the arrows the times before and after high water in which the streams run in those directions. The second line of revolving tides was observed in the mixed tides off the Start, and is similar to those off the Texel. The lower lines are samples of the steadiness of the stream which prevails throughout the ordinary tides of the Channel; the stations are given exactly as they occurred on a parallel of 52° 15′ N., which nearly intersects the position Professor Whewell has assigned to the node of the North Sea tide, and as they were observed in the English Channel between Portland and Beachy Head.

It was in the upper or revolving streams that the late Captain Hewitt's observations were made, and in the second row, or similar tides, that M. Monnier and Captain White made their observations; and there can no longer be any doubt that the rotatory tides are the exceptions to the general courses of the stream, and not the rule.

On the time when the stream attains its maximum rate.

The time at which the stream attains its maximum rate in the Channel is another point of interest and of importance to the seaman. In the 'Pilote Française' we find numerous observations carefully drawn out, and all giving nearly the same result, which is to place the strongest part of the stream at high and low water, and the slack water at half-tide. In 'Mémoire sur les Courants de la Manche,' before referred to, we read at p. 15, "On conclura de ce qui vient d'être dit que l'heure de la haute mer doit coincider avec celle ou le courant de flot acquiert sa plus grand vitesse..... On concevra de la même manière que le courant de jusant doit atteindre sa plus grand vitesse au moment de la basse mer.....:" and in a work of high authority published in this country, it is stated, doubtless from the authorities beforementioned, that "in mid channel the motion of the water will be flowing most rapidly up the Channel at the time of high water, and its motion upwards will cease when the water has dropped to its mean height."

This idea of M. Monnier, viz. of the strongest part of the stream occurring at high water on the shore, is part of the same error which the author before quoted had

fallen into respecting the turn of the stream in the Channel, viz. that of comparing the observations with a false standard. Taking the same observations and comparing them with the high water at Dover, the times in question all give the half-tide as the period at which the stream attains its greatest strength; and by referring to another table, in which our own observations are compared, a precisely similar result is obtained. The question is a very simple one, and rests entirely upon the truth of the observations, a specimen of which I have given in the subjoined Table for the purpose of consultation.

The time at which the stream attains its maximum rate.

Table showing the time at which the stream attains its greatest strength, and also its rate per hour at that time.

English Channel.						North Sea.					
No. of station.	Time from high water, Dover.	Rate of stream.	No. of station.	Time from high water, Dover.	Rate of stream.	No. of station.	Time from high water, Dover.	Rate of stream.	No. of station.	Time from high water, Dover.	Rate of stream.
	h m	Knots.		h m	Knots.		h m	Knots.		h m	Knots.
12	3 15	1 81	G	2 50	5 00	3	4 40	1 18	41	2 30	2 35
	2 30				4 50					2 10	2 35
13	3 00	2 27	H H	2 34	2 30	4	3 00	2 07	42	3 00	2 30
	3 30			2 00	2 30		2 45			2 30	2 45
14	2 15	2 05	E E		2 00	6	2 30	2 53	43	3 00	2 70
	2 00				2 00		2 15			2 30	2 90
15	3 15 E.	3 80	D	3 00	3 70	8	3 00 E.	2 84	44	3 15	2 40
16	3 15	3 14		2 40	3 70	10	3 00	2 25			
	3 00		O	2 45	4 50		3 00		45	3 00	2 60
17	2 30	3 30		3 50	3 60	12	3 30	1 65		3 00	2 21
	3 00	2 75	48	3 00	3 2		3 30			3 00	1 85
18	2 15	2 20		3 40	3 0	13	4 00	1 60	46	2 00	2 00
	3 00	1 60	79	3 00	3 15		3 00			2 35	2 60
19	2 45	2 75		3 00	2 90	14	2 45	1 37	100	3 00	2 60
	3 00	2 00	47	3 00	3 4		3 30			2 45	2 00
20	2 30	2 25		2 15	3 2	15	3 45	1 54	101		
	3 30	1 80	46	2 10	3 6	17	2 45 E.	2 91	102	3 00	1 60
A A	3 20 E.	1 30		3 30	3 3	18	3 30	2 44		1 45	1 85
A	3 20	1 20	49	2 10	3 3		3 30		102A.	3 00	
	2 10	2 00		3 30	3 1	19	2 45 E.	2 33		2 30	
B	3 20	2 50	58	In the Baie de la Seine.	3 2	39	2 30	2 25	103	2 45	1 70
C	2 20	3 80				45	4 00	2 10		1 35	2 15
	3 00	3 80	77	2 30	3 6		2 30		104	3 00	2 60
D	2 45	3 86		3 00	3 15	37	4 00	3 00		3 15	3 00
	3 15	4 33	61	2 5	3 2		2 45	2 40	107	3 30	1 40
E	2 40	2 80		3 5	3 0	38	2 40	2 50		3 20	3 00
	3 10	2 80	75	3 00	3 5		2 45	2 18			
Ea	2 30	2 80		2 30	2 9	39	2 30	2 50			
	3 30	2 80	76	2 50	3 6		2 45	2 35			
				3 00	3 20	40	3 00	2 40			
							2 45	1 64			

In the 2nd, 5th, 8th and 11th columns of this Table are given the times before and after high water at Dover at which the stream attains its greatest strength, which is clearly seen to be about the time of *half-tide*, Dover, and not, as supposed, at high and low water on the shore.

Direction in which the stream turns.

The *direction in which the stream turns* will next engage our attention. This is a question more curious than useful to the navigator; it is nevertheless one of interest, and I have given it my attention.

Upon Plate XXXII., I have shown, by curved arrows, the direction in which the streams in all parts of the Channel pass through their various rhombs, and I cannot discover that there is any general rule with respect to *particular sides of the Channel.* On the contrary, the direction appears to be wholly dependent either upon the course of that stream which commences first or continues the longest, or upon that which becomes the most powerful, and which, acting obliquely upon the weaker stream, compels it to partake of its own direction. A good example of this may be seen in the chart of the streams at three hours after high water (Plate XXXV.), where the incoming Ocean stream off the Lemon and Ower is dividing the Channel stream and causing all the tides to the eastward of that spot to turn *with the sun,* and those to the westward of it in a *contrary direction.*

On a supposed tide and half-tide.

I have now only to mention a popular error which still exists with reference to the streams of our channels, viz. a belief in a "*tide and half-tide*;" by which it is understood that the stream runs half a tide longer near the centre of the Channel than it does near the coast. So general an opinion requires substantial proof to invalidate it, and I must refer once more to the diagrams which accompany this report. It will be seen there that in no part of either channel does the change of stream in the centre differ even an hour from the change of stream as near the land as a vessel would be safe in navigating, except in that part of the North Sea which is affected by the tide sweeping round the Texel; and then, at the most, it amounts only to a quarter-tide. It is not difficult to discover that this error has arisen from the confusion of high water on the shore with the time of slack water in the offing.

I have now described the principal features of the tidal streams of this strait, and it is evident from the observations which have been collected that the laws by which the streams of these channels are governed, differ materially from those which regulate the streams of ordinary tides.

It will be interesting to see in what this consists, and to endeavour to account for the cause.

Peculiarity of the wave of the English Channel and North Sea.

Under ordinary circumstances, such as those which attend the passage of a *single tide*-wave up an estuary or deep gulf, or even along an open coast, we find *a progressive turn of the stream to accompany a progressive increase of establishment.* But in the channels under discussion the stream is found to turn *nearly simultaneously throughout the strait,* wholly regardless of the *order of its establishments,* and to reverse *with the time at which a wave peculiar to such channels is matured.*

This wave, which exercises so singular an influence over the streams of these Channels, is occasioned by one portion of the oceanic tide-wave of the Atlantic passing round the north end of the British Islands, while the other finds its way up the English Channel and meets it in the Strait of Dover. In consequence of this meeting or opposition, the character of the wave is changed; its dimensions become

diminished just one half in extent, its rate of travelling is reduced a like quantity, and its streams are almost always in opposition to those of the outer or parent wave.

Formation of a combined wave in the Strait of Dover.

Plans 3 and 4 exhibit the forms of this wave at full and change, as they appear at the hours respectively marked against them at the side of the Plan. They are constructed principally from the establishments and ranges of tide published in the Philosophical Transactions, 1836, Part II., by drawing a line through the fairway of the Channel, and setting off upon it the times of high water and the ranges of tide as they would exist at the stations selected for this purpose.

In these Plans we can distinctly trace the formation of a combined wave at Dover. In Plan 4, for instance, at V hours, the crests of two waves are seen to be formed a little beyond the Deadman on one side and at the Spurn Point on the other, and the foot of the wave or hollow which they create is resting at Dover. Following the progress of these two crests down the page, we find them gradually approaching each other and wearing out the depression in advance of them, until between VIII. and IX. o'clock the hollow has vanished; at X. o'clock there is a decided convexity of surface, which gradually increases until at XI. o'clock, Plan 3, the crest has attained its zenith: the combined wave is perfected, and it is high water at Dover.

If now we follow the wave down the same page, Plan 3, as the tide falls, we find the crest to become more oblate, and finally, to be obliterated without there being any material progress of the wave in either direction. In fact, the wave on this section appears to descend nearly perpendicularly, and the progress of the derivative waves to be so far destroyed that we can scarcely trace any indication of their advance. On the contrary, we see a new wave on each side preparing to roll up the Channel to renew the wave thus clearly common to both. So far then we perceive, from these data, that the wave in the Strait of Dover is due to the combined action of two waves derived from the parent or Atlantic wave, and which advance from opposite quarters. These waves, it may be seen, are materially different in their dimensions and rate of travelling from those of the oceanic wave from which they are derived. The dotted line drawn through the crests of the wave upon the Plan 3, will sufficiently discover the material change it undergoes on approaching the channel in which the branch wave exists.

The same result, nearly, as regards the formation of the combined wave, is arrived at if we take a mean between the sections of the wave as it would appear from the ranges and establishments along the coasts of England and of France and Belgium. In this mean section, however, we trace the formation of a combined wave at IX. o'clock off Selseabill, and can detect a progressive advance of a small wave thence to the eastward, making high water along the Dutch coast in the inverse order of the high water resulting from the wave upon the opposite side of the channel, and then merging into the wave preparing to renew the order of the tides in the North Sea*.

* This merging of the waves into each other may possibly be the cause of the whole rise of the tide at the Texel occurring in the first half of tide, as stated in "Ariel," Remarks upon the Tides of the Texel, published at Amsterdam.

This wave is not, however, of sufficient magnitude to affect *the streams* which traverse its surface, which seem to be under the entire control of the combined wave. The inverse order of the establishments upon the coast of Belgium and of Holland is a point of considerable interest, and I greatly regret that the season has been so unfavourable that no observations have been made which throw any additional light upon the subject.

If we place the mean water-level of the middle sections of the wave together, we shall form the figures traced in Plan 2, Plate XLI., and shall be struck with the very small limits off Harwich through which the large wave at Dover is transmitted; this contracted portion of the sketch is nearly in the situation in which Professor WHEWELL has drawn the node of the North Sea tides.

Powerful influence of the combined wave over the streams of the Channel.

The influence of the combined wave over the tidal phenomena of the channels on both sides of the Strait of Dover is very remarkable. The streams of both it will be seen reverse together as the wave is matured; they extend to the same distance from the apex of the wave, where they are met by rotatory streams; in short, the whole tidal feature of one-half of the strait bears so close a resemblance to that of the other as to leave no doubt of their being both due to a common impulse. For instance, it may be seen in the Plans which accompany this report, that in the North Sea the streams of the oceanic and Channel wave meet off the estuary of Lynn, as they do off the estuary of St. Malo in the English Channel, at the same hours and at the same distances nearly from the head of the combined wave; that there is an increased rise of the tide at Lynn as in the Gulf of St. Malo, although not to the same extent, owing perhaps to a different conformation of coast, but sufficiently large to establish the similitude, being in both cases nearly double that of the offing rise; that in these localities, on both sides, the streams are all of a rotatory character, and that from the meeting of the streams off Lynn to the meeting of the streams off Dover, there is, as in the English Channel, from the meeting of the streams off St. Malo, to the meeting of the aforesaid streams off Dover, a stream which flows steadily on both sides towards Dover, whilst the water is *rising* at that place and sets *away from it*, whilst the water is *falling* there; each portion having the remarkable peculiarity of reversing its stream throughout nearly simultaneously with the time at which the combined wave is matured.

Streams of the Irish Channel under a similar influence.

I may here observe, that in this respect the Irish Channel is precisely under the same circumstances as the English Channel and North Sea. The tide-wave enters that sea in like manner by opposite routes, and forms one vast wave, having all the peculiarities of a combined wave. On each side of the apex of this wave the streams reverse throughout at nearly the same time; that time corresponding with the time at which the combined wave is perfected, as occurs with the streams in the English Channel.

In the Irish Channel the streams are even more regular than those of the North Sea and English Channel, and suffer less disturbance from the ocean or offing streams. It is true that in the Irish Sea there is no intermediate stream where the tides meet,

but, on the contrary, a large tract of still water which the streams scarcely ever disturb, but this arises from the different conformation of the inner part of the strait. The Irish Sea, for instance, may be said to be contracted at its entrances, and to expand into a large inland sea in its centre, whereas the other channel has wide entrances and a very contracted centre. They are, however, both canals in which the waves from two tidal seas enter at opposite ends and meet in the centre, where they form a combined wave, and hence they have the property of reversing their streams simultaneously.

Tidal phenomena of the Irish Sea and of the English Channel compared.

There cannot therefore be any doubt that the waves which are formed in the centre of these straits exert a powerful influence over the course of the waters on both sides of them, and in fact that they both govern their movements and regulate the limits of their streams.

It was the consideration of this influence that determined me to refer all the movements of the streams of these Channels to the time when the combined wave was matured. In the Irish Sea I adopted Liverpool as the standard*, and as Dover was situated so near to the apex of the wave in the English Channel, and had tables of its tides already printed and in general circulation, I adopted the time of high water at that place as a standard to which all the observations there should be referred.

I shall now compare the general inclination or slope of the surfaces of the combined wave on both sides with the direction of the streams passing along the Channel at the moment, for the purpose of showing the mechanical action of the water, and of tracing the intimate connection that exists between these slopes and the simultaneous turn of the stream.

Comparison of the inclination of the surface of the wave with the direction of the stream along it.

If we direct our view to the inclination of the surfaces of these waves, Plans 3 and 4, Plates XLII. XLIII., we shall see that the directions in which the streams run do not always correspond with the *existing* slopes of the surfaces, as they might be supposed to do from the known law of gravitation, but that during the last half of the tide they are to be traced to the effect of *a previous* and contrary depression. The maximum rate of the stream may be seen to occur when the surface has its smallest depression†, and that there is no stream at all at the moment of the greatest elevation and depression. It will be seen that the rate of the stream depends upon the amount and continuance of the inclination; and also that a stream, when once produced, will continue its progress for a length of time, although the inclination that produced it may have been reversed by the passage or descent of the crest of the wave. Consequently from the time of the passage of the wave, or the reversal of the inclination of its surface, until the stream finally ceases to flow, the water will be seen to run up an inclined plane, and will continue to do so for nearly as

* See my paper in Philosophical Transactions, Part I. 1848.

† The maximum rate of the stream is shown in the Table at page 711 to occur at half-tide, at which time the wave is seen to have the smallest depression.

long an interval before it be brought to rest as it did to acquire its momentum, the relative inclinations at the beginning and end being about the same.

The reversal of the stream throughout the strait at the same time will not therefore be a matter of surprise, as it will be seen to be only the effect of gravitation due to *the general slope* of the surface. That it is the *general slope* of the surface of the combined wave that occasions the stream throughout these channels, is even more manifest in the North Sea than in the English Channel, from the fact of the stream there pursuing a steady course between two shores, of which the order of their tidal establishments is inverted without reference to either.

Cause of the stream explained.

If it be required to trace the streams of the Channel through their gradations, it may be done on Plan 3, thus: at XI. o'clock, it is high water at Dover, the wave has consequently attained its zenith, and the depressions of the surfaces on either side are at their maximum. There is now no stream, because the momentum acquired during a former depression has only now ceased, notwithstanding the increasing obstruction for two hours which has been presented by the reversed inclination of the wave. The water at Dover now falls, the stream begins, and as the depression of the surface continues, the stream gradually acquires strength until about half-tide, when the depression is at its minimum and the strength of the stream at its maximum: after this the inclination of the surface is unfavourable to a continuation of the stream. From this time, therefore, the rate of the stream begins to diminish, because the reversed inclination is progressively becoming more adverse to its progress, and at low water at Dover, after having run up an elevation for two hours, its effort is exhausted and there is slack water throughout the wave, as at first*.

Conclusion.

In conclusion, I trust it will appear to their Lordships, that the means which they have been pleased to place at my disposal have been advantageously employed; and I venture to express a hope, that whilst science will be benefited by the inquiry, the navigation of our channels will be so far improved that the seaman will in future find his course through these moving waters rendered simple and plain. Hitherto the numerous and perplexing references which he has been compelled to make to establishments of ports, with some of which he was not even furnished, and which, under any circumstances, rendered a calculation necessary, have in too many instances, it is feared, caused the set of the tides to be wholly disregarded, or what is worse, misapplied, the consequences of which, it is now seen, are very likely to have been attended with disastrous results. For instance, the meeting of the streams off the Casquets and the Start in the English Channel, and the direction there given to the water at a particular time of the tide, will fully account for the numerous wrecks about the Channel Islands; whilst near the Strait of Dover an unexpected set of the stream directly down upon the Somme, and in a part of the Channel where from its narrowness a true stream might be expected, is evi-

Beneficial results of this inquiry to the seaman.

* In my report upon the streams of the Irish Sea, Philosophical Transactions, Part I. 1848, at figures A and B the effect is there shown to be precisely the same.

dence of the danger of approaching this part of the Channel if ignorant of the set of the stream; and most singularly this unsuspected evil occurs exactly in the spot where those disastrous wrecks of the Conqueror and Reliance took place, and where the Curaçoa, one of Her Majesty's frigates, so narrowly escaped a similar fate.

Henceforward a simple reference to a Dover tide table will enable the mariner to determine in which direction his vessel is being carried, assured that whilst the water is rising at Dover he will have a fair stream from the Lemon and Ower to the North Foreland in one channel, and from Alderney to Beachy Head in the other, and *vice versâ*; whilst the times and places of the meeting of the streams will be apparent upon the Plan, so that it is hoped, when the contents of the present paper are sufficiently known and circulated, they will be the means of diminishing the number of those losses of both life and property with which the annals of Lloyd's abound, and of advancing our knowledge of the tides by the practical illustration of the phenomena of the tidal streams of straits under the influence of a combined wave.

In closing this report, I particularly wish to observe that my thanks are due to the officers whose names have been mentioned as having contributed to this inquiry, and especially to the Trinity House and Captain Bax, to whose exertions in training the light-vessel keepers the observations made by them owe their value; and to Captains Washington and Bullock and the officers, who have been engaged in both making the observations and drawing the plans.

To you, Sir, I am indebted for the unremitted assistance you have rendered this cause by the exercise of a zeal which is ever forward in promoting whatever tends to the improvement of navigation or to the advancement of science.

I am, Sir, your humble Servant,

F. W. Beechey, Captain R.N.

To Rear-Admiral Sir Francis Beaufort, K.C.B.,
&c. &c. &c.,
Admiralty.

Note.—Since this report was forwarded, observations have been made between the Spurn and Helgoland, and it appears that in all that part of the sea lying between the rotatory streams off the Texel and Helgoland, or rather between 2°·30 E. and 7°·30 E., the stream *turns nearly simultaneously* with the time of high water at Helgoland and at Dover, the establishments of those places being nearly the same. We may therefore look for a combined wave at the mouth of the Elbe, and for a succession of rotatory streams extending to a considerable distance in a north-easterly direction from Lynn. A few observations have been made in the English Channel also which confirm our former results, and show that such of the observations of the French surveyors referred to in p. 708 as have been tested, were in excess, as had been anticipated.—F. W. B.

Explanation of the Plates.

Chart of the places of observation	. . . . .	Plate XXXII.
Chart of the stream at one hour after H.W.	. . .	Plate XXXIII.
Chart of the stream at two hours after H.W.	. . .	Plate XXXIV.
Chart of the stream at three hours after H.W.	. .	Plate XXXV.
Chart of the stream at six hours after H.W.	. . .	Plate XXXVI.
Chart of the stream at five hours before H.W.	. .	Plate XXXVII.
Chart of the stream at three hours before H.W.	. .	Plate XXXVIII.
Chart of the stream at one hour before H.W.	. .	Plate XXXIX.
Plan 1. Rotatory and true streams compared	. .	Plate XL.
Plan 2. Mean section of Channel wave	. . . .	Plate XLI.
Plan 3. Hourly section of the rising tide-wave	. .	Plate XLII.
Plan 4. Hourly section of the falling tide wave	. .	Plate XLIII.

XXXI. *On the* Megatherium (*Megatherium Americanum*, Blumenbach).

By Professor Owen, *F.R.S.* &c.

Part I.—*Preliminary Observations on the Exogenous Processes of Vertebræ.*

Received November 8, 1850,—Read January 9, 1851.

Before entering upon the description of the skeleton of the Megatherium, it is requisite to premise some remarks on the vertebræ of the Mammalia in general. Hitherto these parts have been described by means of the terms supplied by Human Anatomy. But the skeleton of Man is one which deviates most from the common archetype: some parts are developed in excess; other parts, which are present as a general rule in the Mammalian series, are either rudimental or absent. The latter is more particularly the case with those processes of the vertebræ which are developed in relation to the attachment and force of the muscles; and which, being connected with particular modes and media of motion, become eminently significative of the habits and affinities of the species. The consideration and comparison of these processes, therefore, most of which have received no names in Human Anatomy, are essential to a right determination of the habits and affinities of the Megatherium; and in order to make intelligible the terms in which the vertebral peculiarities of that great extinct animal will be described, I propose first to illustrate them by giving a comparative survey of the principal modifications of the exogenous processes in the vertebrate series.

The 'exogenous' processes of a vertebra are those which grow out of the previously ossified parts, and are so classified and named in contradistinction from the 'autogenous' parts or elements of a vertebra which are developed from their own proper centres of ossification*.

In the sixth cervical vertebra, for example, of the human fœtus, the part which Soemmerring, the most exact and classical author on Anthropotomy, calls "radix prior seu antica processus transversi vertebræ†," is developed from a separate centre and for some time continues to be a distinct bar of bone; whilst the part called "radix postica processus transversi vertebræ‡," grows out of the base of the "radix arcus posterioris§," as Soemmerring denominates the autogenous part which I have called 'neurapophysis:' and from this element, or from the arch formed by its coalesence with its fellow, when it constitutes the "arcus posterior vertebræ" of Soemmerring, other processes grow out; as, for example, the "processus obliqui seu

* On the *Plesiosaurus macrocephalus*, Geological Transactions, vol. v. 2nd series, p. 518, 1838.

† De Corporis Humani Fabricâ, 8vo. 1794, tom. i. pp. 239, 241. ‡ Ib. § Ib. p. 236.

articulares superiores et inferiores." These different processes offer many modifications of size and shape, and a certain change of position and direction, when traced through the vertebrate series; in the course of which comparison the anatomist meets with other exogenous processes, traceable with equal certainty from species to species and determinable under all their various modifications: whence it becomes necessary for the purpose of brief and clear description to indicate each of such processes by a single definite name, capable of being formed into an adjective to express the properties or appendages of such processes*.

The convenience of such terms will be admitted by the Human Anatomist who considers that the processes in question have, as yet, received no distinct substantive names in Anthropotomy; and who knows, from Osteogeny, how vaguely some of the descriptive terms, as 'transverse process,' for example, are applied to parts obviously very different in their nature and constitution. The 'transverse process' of a dorsal vertebra is a simple exogenous growth from the neurapophysis, commonly affording an articular surface for a rib; the 'transverse process' of a sacral vertebra† is an autogenous part, which continues distinct long after birth; the 'transverse process' of a cervical vertebra is partly exogenous, partly autogenous, consisting in fact of two distinct processes and a rudimental rib, but distinguished in Anthropotomy from other transverse processes only by 'being perforated.'

But if such common phrase of 'transverse process,' with a note of distinction, as that "it is perforated in the cervical vertebræ," be sufficient for the exigencies and applications of Human Anatomy, it becomes quite inadequate and often totally inapplicable to the answerable parts in the lower animals: in the Crocodile, for example, the cervical rib is much more developed and maintains constantly a free articulation by a 'head' with a well-developed inferior *transverse* process ('radix antica,' Soemm.), and by a 'tubercle' with an equally distinct superior *transverse* process ('radix postica,' Soemm.). In the Wombat (Plate XLIV. fig. 12), the Hare, the Pig, and most other quadrupeds, the *transverse* process of the lumbar vertebræ consists of an exogenous base and an autogenous apex, this apex being, as Theile and Muller‡ have shown, a rudimental rib like the autogenous part of the transverse process in the neck.

In entering upon the descriptions of the extinct species of the Reptilian class§, the most varied, extraordinary and heteroclite, as Cuvier justly remarks, of any of the

* This I have done for some years past in my Lectures at the Royal College of Surgeons, in the 'Catalogue of the Human and Comparative Osteology in the Museum of the College,' and in some of my published memoirs: as *e. g.* On the *Plesiosaurus macrocephalus*, Geological Transactions, 4to, 1838. On the Anatomy of the Male Aurochs (*Bison europæus*), Proceedings of the Zoological Society, November, 1848, p. 131.

† "Laterales partes, quæ processus transversos coalitos repræsentant," Soemm. *op. cit.* t. i. p. 271.

‡ Archiv für Anatomie und Physiologie, Jahrg. 1839. Vergleichende Anatomie der Myxinoiden, in 'Berlin Abhandlungen,' 1836, p. 304.

§ Reports of the British Association, 1839.

lost forms of animal life, I found that a very large proportion of their fossil remains consisted of vertebræ or parts of vertebræ, and that a definite idea of these complex bones, with a substantive nomenclature for their parts, was indispensable in order to give a clear and intelligible description of them. In a memoir on the *Plesiosaurus macrocephalus* read before the Geological Society, April 1838, I therefore defined the parts of the typical vertebra, distinguished the autogenous elements from the exogenous parts, and proposed the nomenclature which I have since used and further illustrated in subsequent works. Referring more particularly to my 'Lectures on the Comparative Anatomy of the Vertebrate Animals*,' and my work 'On the Archetype of the Vertebrate Skeleton†,' for the fuller exposition of the views entertained by anatomists on the nature of the vertebra, it is sufficient for the present purpose to remark, that the term 'parapophysis' was proposed for the 'inferior transverse process,' or 'anterior root of the perforated transverse process of the cervical vertebræ'; the term 'diapophysis' for the 'superior transverse process,' or 'posterior root of the perforated transverse process'; and the term 'pleurapophysis' for that element, which, essentially the same in different regions of the spine, is short, styliform, and speedily anchylosed in the neck of Man, is remarkable for its length and freedom of motion in the back, and is as remarkable for its breadth, thickness, and wedged fixedness in the sacrum; which is hatchet-shaped and free in the neck of the Crocodile and Plesiosaurus; and is as broad and flat, in the first dorsal segment of a Whale, as the ordinary mammalian scapula or any of the 'flat bones' in Anthropotomy.

The necessity for a definite technical term for this vertebral element was the greater because the name 'costa' or 'rib' has not only been withheld in Human Anatomy from the element in question, under every modification save that in which it retains its individuality and mobility with a certain length and slenderness; but it has been extended to another and quite distinct element, viz. the 'hæmapophysis,' which from being commonly cartilaginous in the thorax of Man, has obtained the name of 'cartilage of the rib,' or 'pars cartilaginea costæ.' As this part however is most commonly a distinct ossified part in the air-breathing Vertebrata, it has been called in Comparative Anatomy the 'sternal rib,' in contradistinction to the 'vertebral rib' or 'pars ossea costæ' of Anthropotomy. Both, however, are distinct elements of the 'vertebra' in its true or wider anatomical sense; each may exist independently of the other, and the hæmapophysis is often present in other regions besides the sternal or thoracic one.

The subject, however, of the present communication being the 'exogenous parts of the vertebræ,' I shall return to it by remarking, that, in Human Anatomy, other processes have been recognized besides the 'spine,' the 'zygapophysis' (oblique or articular process), the 'diapophysis' (transverse process and 'radix postica' in the neck) and the 'parapophysis' (part of the 'radix antica' in the neck). Monro, for

* 8vo. Longmans, 1846. † 8vo. Van Voorst, 1848.

example, the first exact describer of Human Osteology, when treating of the lumbar vertebræ, observes,—"5. Betwixt the roots of the superior oblique and transverse processes a small rising may be observed, where some of the 'musculi erectores trunci corporis' are fixed*." And Soemmerring bears testimony to the accuracy of Monro's observation of this 'small rising' as a constant part, by calling it "processus accessorius processui transverso et articulari superiori interpositus†." There are, in fact, two such accessory processes, as Soemmerring, indeed, seems to indicate. One of these 'small risings' or 'accessory processes,' although it seems so insignificant, and is so small, as to be commonly passed over without notice in our modern anthropotomical compilations‡, is not only so constant as to merit unfailing notice in the human subject; but, in extending our comparisons from Man to the lower mammalia, we find it speedily, in the Quadrumanous Order, for example, gaining a development which makes it, in the dorsal and lumbar regions, more conspicuous than the articular processes themselves: whence Cuvier, observing its relation to the anterior of these processes, calls it "une seconde apophyse articulaire§;" and M. de Blainville, struck rather with the shape of the process, calls it "apophyse styloïde‖;" whilst M. Straus-Durckheim, regarding it as a sort of accessory transverse process, prefers to call it "apophyse plagienne¶:" I have termed it the '*anapophysis***.' The second tubercle is usually a somewhat larger process, situated between the diapophysis and the upper or anterior zygapophysis; it is equally constant in certain vertebræ in Man, and attains in some mammals, the Armadillos for instance, as great a length as the spinous process itself. The part in Man to which I allude is that which the accurate Bourgery has depicted in his beautiful plates, and indicated as the 'face rugueuse de l'apophyse articulaire supérieure††.' Straus-Durckheim mistakes it for the superior articular process itself, in the feline quadrupeds, calls it 'apophyse antoblique,' and denies the existence of anterior oblique processes in the third to the ninth dorsal vertebra inclusive; which do, in fact, possess the articulating surfaces so called, but want the accessory process which becomes so markedly developed in the posterior dorsal and lumbar vertebræ. For this accessory process I have proposed the name of '*metapophysis*‡‡,' and have noticed some of its modifica-

* The Anatomy of the Humane Bones, 12mo, 1726, p. 202. In later editions he calls it a 'small protuberance.'

† *Op. cit.* tom. i. p. 268. He speaks of two such processes, but whether he meant the pair of metapophyses, or both met- and an-apophysis, is uncertain.

‡ I may instance Bell's Anatomy, vol. i. 'On the Bones,' &c., ed. 1826, p. 20, and the Anatomist's Vade Mecum, by Erasmus Wilson, 12mo, 1842.

§ Leçons d'Anat. Comp., ed. 1835, t. i. p. 197. ‖ Ostéographie, *passim*.

¶ From πλάγιος, *transversus*. Anatomie du Chat, t. i. p. 95.

** From ἀνὰ *retro*, ἀπόφυσις *processus*, because it generally projects more or less backwards.

†† Traité Complet de l'Anatomie de l'Homme, fol. t. i. pl. 9. figs. 2 and 4 *k*. This appears to be indicated, in the 5th edition of Quain's 'Anatomy,' as a tubercle projecting from the superior articulating process.

‡‡ From μετὰ *inter*, ἀπόφυσις *processus*, because it is usually between the transverse process (diapophysis) and the anterior oblique process (prozygapophysis).

tions in my 'Anatomy of the Male Aurochs,' in which, as in most other Ruminants, the anapophysis is not present*. The process which is developed, sometimes singly, sometimes in pairs, from the under part of the centrum in some mammals and in most birds and reptiles, I have called '*hypapophysis*†.'

I propose in the first place to trace the homologies and to indicate the chief modifications of the metapophyses and anapophyses in the mammalian class, and thereby to vindicate their title to distinct names, and demonstrate the utility, if not the necessity, of such signs for parts so constant in existence and so variable in character in a great proportion of that class. I may remark, also, that as I shall frequently have to allude to the anterior as distinct from the posterior zygapophyses, I shall call the former 'prozygapophysis,' and use the term 'zygapophysis' simply to signify the posterior pair.

* "With regard to the vertebræ of the trunk of the Aurochs, I may remark, that the only accessory process in addition to the ordinary zygapophyses and diapophyses is the 'metapophysis,' which appears as a stout tubercle above the diapophysis in the middle dorsals, and gradually advances and rises upon the anterior zygapophyses in the posterior dorsal and lumbar vertebræ. This process is developed to an equality of length with the spinous processes in the Armadillos. It is commonly associated with another accessory exogenous process, to which I have given the name 'anapophysis' in the Catalogue of the Osteological Series in the Royal College of Surgeons. This process, which in most of the *Rodentia* rises, at first, in common with the metapophysis, as a tubercle above the diapophysis, separates from the metapophysis as the vertebræ approach the pelvis, and in the lumbar series the anapophysis is seen projecting backwards from the base, or a little above the base of the diapophysis, its office being usually that of underlapping the anterior zygapophysis of the succeeding vertebræ and strengthening the articulation, whence Cuvier has alluded to it as an accessory articular process; but its relation to the zygapophysial joint is an occasional and not a constant character. The tenth dorsal vertebra of the Saw-toothed Seal, *Leptonyx serridens*, affords a good example of well-developed metapophyses; they are also large in most of the trunk-vertebræ of the Tapir. The anapophyses are well-developed in the anterior lumbar vertebræ of the Hare and Rabbit."—Proceedings of the Zoological Society of London, November 1848.

In a volume of Swedish Transactions, for 1848, Prof. Retzius has a paper, illustrated by woodcuts, on the modifications of some of these processes in the Class *Mammalia*. I have been recently favoured with a copy of the memoir by the learned author, bearing date 'Stockholm, 1850,' on the title-page, and received since the greater part of this paper was written and all the materials for it collected, which has been a work of some years past; but I regret that my ignorance of the Swedish language has prevented my profiting by the remarks of the estimable anatomist. In the cuts the metapophyses are called 'processus mammillares,' the anapophyses 'processus accessorii.'

[In a German translation of this memoir, kindly transmitted to me by Prof. Retzius since the present paper was in type, the following passage is added to the original :—

"(Zusatz des Verfassen; aus brieflicher Mittheilung desselben an den Uebersetzer vom 5 August 1850.—Nachdem diese Abhandlung bereits gedruckt war, hat der Verf. erfahren, dass Prof. Richard Owen in London schon i. J. 1848 (Proceedings of the Zoological Society) den *Pr. mammillares* sowohl als den *Pr. accessorii*, eigene, einfache Benennungen gegeben hat. Er nennt nämlich die ersteren oder Pr. mammillares 'Metapophyses' und die letzteren oder die Pr. accessorii 'Anapophyses,' von *μετὰ inter*, und *ἀνὰ retro*, die ersten als zwischen den Diapophyses und Zygapophyses *Owen* liegend; die letzteren als mehrentheils nach hinter gerichtet.") p. 449.]—R. O. October 27th, 1851.

† From *ὑπὸ sub*, *ἀπόφυσις processus*.

Order Bimana. Genus *Homo*.
Var. *Caucasica*, seu *Indo-Europæa*.

In the skeleton of a Frenchman, from which the figures 1 and 2 of Plate XLIV. are taken, the metapophysis first appears as a prominence (*m*) from the fore-part of the diapophysis (*d*) of the tenth dorsal vertebra: it becomes a distinct and well-marked process (*m*) from the upper and fore-part of the diapophysis of the eleventh dorsal vertebra, where the rudiment of the anapophysis (*a*) may be discerned. In the twelfth the metapophysis has increased in size and advanced nearer to the prozygapophysis (*z*, 12, fig. 2), and the anapophysis (*a*), though small, becomes distinct on this vertebra. Both processes are reduced in size in the first lumbar vertebra, in which the metapophyses (*m*) have advanced to the outer side of the prozygapophyses (*z*); while the anapophyses have descended to the back part of the base of the diapophysis. An anapophysis (*a*) becomes well-developed behind the base of the diapopophysis of the fourth lumbar: the metapophyses are reduced to tuberosities in the last four lumbar vertebræ.

In the skeleton of an Englishman, in the Museum of St. Bartholomew's Hospital, the metapophyses begin to be developed on the diapophyses of the tenth dorsal, are intermediate between the diapophyses and zygapophyses in the eleventh, are so placed in the twelfth as to overlap the posterior zygapophyses of the eleventh, and are large round tubercles upon the prozygapophyses of all the lumbars save the last.

In the skeleton of an Englishman with six lumbar vertebræ, in the Hunterian Museum, the metapophysis rises in common with the anapophysis, as a tubercle upon the diapophysis of the eleventh dorsal vertebra; the met- and an-apophyses become separated and of greater length on the twelfth dorsal, and continue equally distinct and well-developed on the first lumbar, which, from the persistent mobility of one of its pleurapophyses, might be equally regarded as a thirteenth dorsal. The metapophyses, however, subside as usual to mere tuberosities on the second lumbar, but the anapophysis continues to the fifth lumbar vertebra, in which it is most developed. It is obsolete on the last lumbar.

In the skeleton of the Irish giant, in the same museum, the metapophyses become distinct processes on the eleventh dorsal, and are repeated on the twelfth dorsal and first lumbar vertebræ: the anapophyses appear on the twelfth dorsal, and are repeated on the three first lumbar vertebræ*.

* The common subsidence of the accessory processes on the last lumbar vertebra seems to have attracted the notice of Galen, who writes in his chapter 'De lumborum vertebris,'—"Et sane alia quædam declivis in ipsis apophysis est ab insigni exortu nervi constituta, hæc quæ interdum quidem inest omnibus, interdum vero in postremis aut exigua plane, aut prorsum nulla, sed superposita perpetuo ipsam habeat, quemadmodum et ultimæ dorsi duæ."—De Ossibus, Cap. x. Opera, fol. ed. by Renatus Charterius, Lutetiæ Parisiorum, 1679, p. 18. The *processus declives* here described by Galen in the last two dorsal and the upper lumbar vertebræ, would seem, from their defined direction, to be the 'anapophyses:' but the origin assigned to them—παρὰ τὴν ἔκφυσιν τοῦ νεύρου—near the outlet for the nerve, does not exactly apply. Accordingly Vesalius, commenting on this passage, writes,—"Ego hunc processum in humanis vertebris nunquam reperi." And he adds,—"In

Var. *Malayana*.—In the skeleton of a male Polynesian, the metapophyses are well developed and project midway between the diapophyses and prezygapophyses on the eleventh dorsal: they rise above the prezygapophyses on the twelfth, in which there is a rudimental anapophysis, as well as a small diapophysis. The anapophysis subsides to a ridge in the lumbar vertebræ, where the diapophysis suddenly increases in length and thickness.

Var. *Papuana*.—In the skeleton of an Australian, the metapophysial tubercle, which is slightly indicated in the tenth dorsal, becomes more distinct in the eleventh, and is well developed in the twelfth dorsal. In this vertebra the anapophysis is first distinctly recognizable. In the first lumbar the metapophyses have increased in size, but do not project so freely by reason of the extension of the articular surfaces of the prezygapophyses upon the inner sides of their base. The anapophyses continue distinct. Both metapophyses and anapophyses, though small, are distinct on the second as well as the third lumbar vertebræ: the metapophyses subside on the fourth lumbar, but the anapophyses are distinctly developed in it. In the last lumbar the anapophyses appear like a part of the upper border of the base of the diapophyses, pinched up and produced backwards.

In the following observations from the lower Mammalia, *d* signifies the dorsal vertebræ, and *l* the lumbar vertebræ, in the formulæ in which their numbers are given.

Order Quadrumana.

In the Chimpanzee (*Troglodytes niger*), with *d* 13, *l* 4, the metapophysis, commencing at the eleventh dorsal, is very distinct on the twelfth dorsal, where it projects forward from above the diapophysis: in the thirteenth dorsal the metapophyses are thicker and longer than they usually are in the last dorsal of Man. In all the lumbar vertebræ they project from the upper and outer part of the anterior zygapophyses, from which they are separated by a narrow groove. There is a feeble rudiment of an anapophysis from the back part of the long and depressed diapophysis: it is longer and more distinct at the back part of the diapophysis of the last lumbar.

In a young of the great Bornean Orang (*Pithecus Wurmbii*), with *d* 12, *l* 4, the metapophysis begins to project from the anterior angle of the diapophysis in the seventh dorsal, progressively increases in size, and is advanced in position close to the anterior zygapophysis in the last dorsal, which resembles that of Man in the distinct

simiarum igitur lumborum vertebris ad radicem transversi processus in inferiori ipsius sede, acutus conspicitur processus, rectà deorsum protensus, et sinus qui nervi nomine illic incisus est, externum latus quodammodo constituens, ac veluti intervallum cum descendente processu efformans, in quod ascendens inferioris vertebræ processus subintrat . . . . . Peculiari enim cuidam extruitur musculo, quem simiæ cum canibus communem obtinent, et quo homines inferiorem dorsi partem non æque ac illa animalia in circulum flectentes, non minus destituuntur quàm omni presentis processus signo."—De Humani Corporis Fabricâ, fol. 1555, p. 96. In the latter statement, however, Vesalius is in error; as will be seen by whoever compares the human spine with that of the Ape or Dog, in reference to the existence of the *signa* or rudiments of the anapophyses.

development of its metapophyses; but differs in the relatively smaller size of its body and its shorter neural spine.

In a mature Sumatran Orang (*Pithecus Abelii*), with *d* 12, *l* 5, the metapophysis appears as a tubercle near the base of the prozygapophysis of the twelfth dorsal vertebra. It is equally distinct on the first lumbar, but subsides to a slight eminence on the succeeding vertebræ. The anapophysis is only distinguishable from the diapophysis upon the first lumbar vertebra; where, however, it well illustrates the homotypal relation of the diapophysis to the same processes in the antecedent dorsal and the succeeding lumbar vertebræ.

In the *Hylobates syndactyla*, with *d* 13, *l* 5, the last dorsal shows well the distinct diapophyses, metapophyses, anapophyses and zygapophyses, more especially the distinction between the prozygapophysis and the now superadded metapophysis. The diapophyses progressively increase in the first three lumbar vertebræ, whilst the anapophyses diminish and disappear on the third lumbar. The metapophysis recedes from the prozygapophysis in the last lumbar vertebra, and becomes quite distinct from it on the first sacral, in which, nevertheless, the articular surface has a nearly vertical position.

In the Silvery Gibbon (*Hylobates Leuciscus*), with *d* 13, *l* 5, both metapophysis and anapophysis become distinct on the twelfth dorsal, and diverge from each other with increase of size on the thirteenth. The anapophysis disappears in the lumbar vertebræ, but the metapophysis is retained.

In the Papas Monkey (*Cercopithecus ruber*), with *d* 12, *l* 7, the metapophyses and anapophyses are separate and distinct upon the eleventh dorsal vertebra, and also in the succeeding vertebræ to the penultimate lumbar, where the anapophysis, which is remarkable for its length in the preceding vertebræ, disappears.

In the Wrinkled Monkey (*Macacus rhesus*), the common rudiment of the meta- and an-apophysis, situated upon the diapophysis in all the dorsal vertebræ, progressively increases and elongates as the dorsal vertebræ recede from the neck, and divides into the two distinct processes upon the ninth dorsal. The metapophysial tubercle ascends upon the prozygapophysis, and is continued throughout the lumbar series; the anapophysis increasing, as in *Cercopithecus*, in the anterior lumbars so as to underlap the prozygapophysis of the succeeding vertebra, disappears in the last lumbar.

In the Entellus Monkey (*Semnopithecus Entellus*), with *d* 12, *l* 7, the metapophysis exists, as an elongated tubercle, outside the anterior zygapophysis from the eleventh dorsal to the last lumbar: the anapophysis is present from the tenth dorsal to the sixth lumbar, it is long and styliform in the intermediate vertebræ.

In the *Macacus niger*, with 13 *d* and 7 *l*, the metapophysis is developed pretty clearly on the back part of the diapophysis of the tenth dorsal, and an anapophysis projects from its lower edge; on the eleventh these parts become still more distinct; in the twelfth the metapophysis is situated upon the prozygapophysis, and the

short but strong diapophysis developes from its lower edge the anapophysis; on the thirteenth the diapophysis has disappeared, but both metapophysis and anapophysis remain, the anapophysis being developed as a long pointed process. In the lumbar vertebræ the diapophyses reappear, but both the metapophyses and anapophyses are present, although they gradually diminish in size and distinctness to the sixth, and the anapophysis disappears on the last lumbar vertebra.

In the Brown Monkey (*Macacus nemestrinus*), with *d* 12, *l* 7, metapophysial and anapophysial tubercles are developed on the second and succeeding dorsal vertebræ, increasing in distinctness and size to the tenth (Plate XLV. fig. 3, $_{10}$, *m*, *a*). In the eleventh they suddenly become separate processes, and the metapophysis (*m*) developes a facet for the accessory articular surface of the posterior zygapophysis in the tenth dorsal. This additional interlocking is continued to the antepenultimate lumbar vertebra, the joint being further strengthened by the underlapping of the long-pointed anapophysis: this 'processus acutus' of Vesalius disappears on the last lumbar. The diapophysis is a rudimental ridge in the last dorsal (*d* 12), but becomes a distinct depressed sharp plate in the first lumbar and progressively increases in size, with an antroverted direction in the succeeding lumbar vertebræ.

In a Mandril (*Papio Mormon*), with *d* 12, *l* 6, an anapophysial tubercle is developed from the diapophysis of each dorsal vertebra, progressively increasing in length and distinctness to the lumbar vertebræ, in which it gradually diminishes, and is reduced in the last to a mere ridge on the upper part of the base of the diapophysis: a corresponding ridge may be recognized on the first sacral vertebra. The metapophysis is suddenly developed from the tenth dorsal, and presents an articular surface to a second articular facet on the outer side of the posterior zygapophysis.

In another Mandril the accessory joint between the zygapophysis and the parapophysis begins between the tenth and eleventh vertebræ.

In the *Papio porcarius* a rudimental metapophysis and anapophysis may be recognized in all the dorsal vertebræ after the second: they gradually increase in size to the ninth, and more suddenly in the tenth, where the metapophysis articulates with the accessory facet of the posterior zygapophysis of the ninth. The metapophysis, anapophysis and diapophysis are quite distinct processes in the last dorsal; in the lumbar vertebræ the pointed anapophyses progressively decrease, and they disappear in the last of this series.

In a Spider-Monkey (*Ateles paniscus*), with *d* 14, *l* 3, the metapophyses begin to curve inwards from the second dorsal, assume the form of a distinct plate in the tenth and eleventh, and separate themselves from the anapophyses in the thirteenth and fourteenth dorsal vertebræ: the latter processes are suppressed in the last lumbar.

In the Marimonda (*Ateles Beelzebuth*), with *d* 14, *l* 4, the tuberous rudiment of the accessory processes above the diapophysis of the middle dorsals becomes a ridge, which is produced forwards into an angular metapophysis in the eleventh dorsal. In the thirteenth it is produced to the same extent backwards into an anapophysis: in

the fourteenth these processes are distinct and well-developed, but the diapophysis has disappeared; it reappears, however, in the lumbar vertebræ, from the first and second of which, nevertheless, the anapophysis continues to be developed; but this subsides in the last.

In the Capuchin (*Cebus capucinus*), with *d* 13, *l* 6, in the fifth dorsal the tubercles representing the met- and an-apophyses project distinctly, the one from the fore-part, the other from the back part of the diapophysis: they progressively increase in size and distinctness, and in the thirteenth dorsal the metapophysis has risen upon the anterior zygapophysis. The anapophysis continues to be developed to the penultimate lumbar vertebra.

In the Marmoset (*Callithrix sciureus*), with *d* 13, *l* 7, the tubercle above the diapophysis divides into a met- and an-apophysis in the ninth dorsal; in the tenth these processes diverge, and in the eleventh become quite distinct; they continue to increase in length to the fifth lumbar; the anapophysis then diminishing, and being suppressed in the last lumbar.

In the Black-fronted Lemur (*Lemur nigrifrons*), with *d* 12, *l* 7, the metapophysis begins to be developed in the middle dorsal vertebræ, and in the tenth projects from above the diapophysis: this disappears in the eleventh dorsal, and the metapophysis is on the outside of the anterior zygapophysis. From this vertebra a well-marked anapophysis is developed, and is continued from all the succeeding vertebræ. The diapophysis reappears upon the first lumbar, and increases in length and breadth as these approach the sacrum.

In a Ring-tailed Lemur (*Lemur Catta*), with *d* 13 and *l* 7, and in a Ruffed Lemur (*Lemur Macauco*), with *d* 12, *l* 7, the division of the 'transverse process' into di-, met- and an-apophyses is well shown in the tenth dorsal, and in the succeeding vertebræ the three processes are distinct and remote.

In the Indri (*Lichanotus Indri*), with *d* 12, *l* 9, the diapophyses of the dorsals support a metapophysial tubercle, which grows as the diapophyses diminish, and seems to take their place in the eleventh and twelfth dorsals: in the twelfth the metapophysis rises from above the prozygapophysis and is continued backwards upon a well-developed anapophysis which commences at once in that vertebra, and continues to be developed though decreasing in length to the penultimate lumbar inclusive. The metapophyses, which are prominent in the anterior lumbar vertebræ, gradually subside as they approach the sacrum. The diapophysis has a low rough tubercle in the first lumbar vertebra, which is developed into a depressed plate, increasing in length and breadth as it approaches the sacrum.

In the Slow Lemur (*Stenops gracilis*), with *d* 14, *l* 9, a metapophysial tubercle is developed from above the diapophysis from the first to the twelfth dorsal vertebra: in the thirteenth it takes the place of the diapophysis, and in the fourteenth it extends forwards and presents an articular surface to a facet on the outer side of the post-zygapophysis of the thirteenth vertebra. The metapophysis has the same disposition

and connections throughout the lumbar series, where it is above and distinct from the diapophyses, which are serial repetitions of the base supporting the anchylosed ribs on the first lumbar vertebra.

Order Carnivora.

In the Lion (*Felis Leo*), with *d* 13, *l* 7, the common base of the met- and an-apophyses appears as a strong ridge above the diapophysis of the tenth dorsal; the ridge increases in the eleventh, divides into a large and strong metapophysis and anapophysis in the twelfth; both of these increase in size in the thirteenth dorsal. The metapophyses are continued throughout the lumbar region: the anapophyses diminish to short and thin styliform processes, underlapping the prozygapophyses of the first four lumbars, and becoming rudimental in the rest. The prozygapophyses are distinctly developed in all the dorsal vertebræ anterior to that in which the metapophyses are superadded to them.

In the Puma (*Felis concolor*), with the same vertebral formula, the common accessory tubercle indicates the anapophysis by its backward production from above the diapophysis of the ninth dorsal (Plate XLV. figs. 5 and 6, *a*): the anapophysis grows longer in the tenth (fig. 5, *a*); and in the eleventh the metapophysis (*m*) suddenly shoots upwards and forwards, the anapophysis (*a*) diverges from it backwards, underlapping the metapophysis of the next vertebra, and both processes far surpass the diapophysis (*d*) in size, which clearly, however, coexists with them. It disappears in the thirteenth vertebra, in which both ana- and met-apophyses have increased in length; and the only question that remains with regard to the 'transverse process' (*d*) of the following vertebræ is, whether it includes the diapophysis at its base, or consists solely of the connate pleurapophysis. The eleventh vertebra permits of no doubt as to the distinctness of the diapophysis (*d*) from the two larger superadded processes (*m* and *a*), and, in fact, a rudiment of the diapophysis may be recognized above the costal pit in the last dorsal. Fig. 6 shows the prozygapophyses (z) existing, as in all the anterior dorsals, independently of the metapophyses*: fig. 7 shows them coexisting (z) with metapophysis (*m*), anapophysis (*a*), and diapophysis (*d*). The neural spine in this vertebra, which is the centre of motion of the spine, is low and nearly vertical; the longer spines of contiguous vertebræ converge to it.

In the *Hyæna crocuta*, with *d* 15, *l* 5, the anapophyses begin to be distinct, and to project backwards in the thirteenth dorsal, underlap the prozygapophyses in the first three lumbar, and subside on the penultimate lumbar vertebra.

In the Wolf (*Canis Lupus*) and Dog, with *d* 13, *l* 7, the metapophyses begin to be developed on the eighth dorsal, and are continued throughout the lumbar series: the anapophysis projects backwards in the eleventh dorsal, and subsides to a mere ridge in the penultimate lumbar. In the Fox they begin to be developed on the seventh

* '*Apophyses antobliques*,' Straus-Durckheim.

dorsal, and are reduced to mere tubercles on the third lumbar. In the Civet (*Viverra Civetta*) and the Genette (*Viverra Genetta*) the anapophyses begin to be developed on the tenth dorsal, and continue to the penultimate lumbar vertebra.

In the Otter (*Lutra vulgaris*), with *d* 14, *l* 5, the metapophyses begin to be developed on the twelfth dorsal vertebra, and are continued throughout the lumbar series; they are low and obtuse. The anapophyses commence at the eleventh vertebra, and are continued to the penultimate lumbar.

In the Sable (*Mustela zibellina*), with *d* 14, *l* 6, the anapophyses are present from the ninth dorsal to the penultimate lumbar inclusive.

The Kinkajou, *Cercoleptes caudivolvulus*, with the same number of true vertebræ as the Sable, has the same disposition of the anapophyses.

In the *Mydaus meliceps*, with *d* 14, *l* 6, anapophyses are present only on the two last dorsal and first lumbar vertebræ: whilst in the Badger (*Meles Taxus*), with *d* 15, *l* 5, they are present on the first three lumbar vertebræ.

In the labiated Bear (*Ursus labiatus*), with *d* 15, *l* 5, the met- and an-apophyses are distinct on the twelfth dorsal, diverge and increase on the succeeding dorsals, the metapophyses continuing throughout the lumbar series, the anapophyses, after underlapping the prozygapophyses of the first and second lumbar, rapidly subsiding.

In the Saw-toothed Seal (*Leptonyx serridens*), with *d* 15, *l* 5, the metapophyses commence as tubercles outside the prezygapophysis on the second dorsal, are distinct on the third dorsal, pass on the fore-part of the diapophysis in the fourth, and continue rudimental as far as the tenth dorsal, on which they are well and distinctly developed (Plate XLVI., fig. 8, *m*); they again pass upon the outside of the prozygapophysis in the eleventh and twelfth dorsals, and so continue throughout the lumbar, sacral, and anterior caudal vertebræ. The anapophyses are mere rudimental projections from the back part of the diapophysis (fig. 8, *a a*).

Order Rodentia.

In the skeleton of the common Squirrel (*Sciurus vulgaris*), with *d* 12, *l* 6, may well be discerned the progressive metamorphosis, in the last six dorsal vertebræ, of the common accessory tubercle upon the diapophysis into the metapophysis and anapophysis, which distinctly coexist with the diapophysis in the ninth dorsal: the diapophysis subsides to a feeble ridge in the three following dorsals, but reappears and rapidly increases in the lumbar series; in the last two of which the anapophyses are suppressed.

In the Malabar Squirrel (*Sciurus maximus*), with *d* 13, *l* 6, the ridge above the diapophysis of the seventh dorsal rapidly expands in the succeeding ones, divides in the tenth into metapophysis and anapophysis, which become more distinct in the eleventh, and are so continued throughout the lumbar series.

In the Marmot (*Arctomys Marmotta*), with *d* 12, *l* 7, the accessory tubercle appears upon the diapophysis of the ninth dorsal, and divides in the tenth, in which the di-

apophysis is suddenly shortened: the metapophysis and anapophysis are distinct in the eleventh dorsal, and so continue to the penultimate lumbar.

In the *Hydromys chrysogaster*, with *d* 14, *l* 7, the common tubercle divides on the twelfth dorsal, and the met- and an-apophyses are distinct on the five following vertebræ.

In the Rat (*Mus rattus*), with *d* 13, *l* 6, the common tubercle, which first appears on the sixth dorsal, divides into metapophysis and anapophysis in the ninth. The anapophyses become obsolete in the last two lumbar vertebræ.

In the Cape-Jerboa (*Helamys capensis*), with *d* 12, *l* 7, the diapophyses of the first dorsal vertebra are unusually long and strong: the anapophysis begins to be developed from the back part of the diapophysis of the eighth dorsal, and the metapophysis from the front part of that of the ninth; in the tenth dorsal (Plate XLVI., fig. 9) both metapophysis (*m*) and anapophysis (*a*) are distinct from the diapophysis (*d*). In the twelfth dorsal the metapophysis ascends upon the prozygapophysis, the anapophysis has increased in length, and the diapophysis is retained. Both ana- and met-apophyses are of considerable length in the lumbar region, except in the two last vertebræ.

The progressive development of the accessory tubercle, and its transformation into metapophysis and anapophysis, are well exemplified in the last five dorsal vertebræ of the Beaver (*Castor fiber*).

In the Porcupine (*Hystrix cristata*), with *d* 15, *l* 4, the accessory tubercle begins to be developed upon the fourth dorsal, and, progressively increasing, it divides in the eleventh into metapophysis and anapophysis. The distinction of these from the diapophysis which supports the last floating rib is well-marked in the last dorsal vertebra. In the lumbar vertebræ the anapophyses strengthen the joints by underlapping the metapophyses, but both processes become rudimentary in the last lumbar vertebra.

In the Coypu (*Myopotamus Coypus*), with *d* 13, *l* 6, the common tubercle appears upon the sixth dorsal; progressively expands, and in the twelfth divides into metapophysis and anapophysis: the latter subsides in the last lumbar vertebra, but the metapophysis is continued along the sacral and a great part of the caudal region, continuing long after the true zygapophyses have disappeared.

In the Paca (*Cœlogenys Paca*), with *d* 13, *l* 6, the common rudiment of the met- and an-apophyses extends above the diapophysis of the seventh dorsal as a broad ridge; in the eighth the ridge is bilobed, in the ninth the lobes diverge; in the tenth and eleventh they become distinct processes: the anapophysis disappears in the last lumbar, but the metapophysis is retained in all the lumbars.

In the Hare (*Lepus timidus*), with *d* 12, *l* 7, both metapophysis and anapophysis appear abruptly on the ninth dorsal: the metapophyses progressively increase to the second lumbar, and are continued, remarkably developed, throughout that region: the anapophysis becomes a low long ridge in the last dorsal, and continues, as such,

with a slight posterior projection to the antepenultimate lumbar; it is rudimental on the penultimate one, and disappears on the last lumbar. The last dorsal and the three first lumbars are remarkable for their long hypapophysis.

In the *Lagotis Cuvieri*, with *d* 12, *l* 7, the common potential base of the dia-, ana- and met-apophyses divides more equably than usual into the three processes; and perhaps no better example can be had of their real distinctness from one another and from the zygapophyses, than that which the ninth dorsal (Plate XLVI. fig. 10, 9, *a*, *m*, *d*, *z*, *z'*) affords; the neural arch seems studded with processes, the nature of which is perfectly comprehended by the comparisons which are given in the present Memoir. In the tenth dorsal the anapophysis (*a*) and metapophysis (*m*) increase in size, especially the latter, which is now closer to the prozygapophysis. The anapophyses are remarkable for their length and slenderness in the lumbar vertebræ (*ib.* 2 and 3, *a a*). The long and slender anapophyses are equally remarkable in the *Lagostomus*, to judge from the figure of the skeleton in the Linnæan Transactions for 1828, vol. xvi. pl. 9, but as those processes had not at the time been defined in the Rodentia no notice is taken of them in the text.

The Capybara (*Hydrochærus capybara*), with *d* 14, *l* 6, gives one of the best illustrations of the distinction of the metapophyses (*Apophyses antobliques* of Straus-Durckheim) from the true anterior oblique processes. The metapophysis may be distinguished as a tubercle above the diapophysis from the third to the eighth dorsal vertebræ inclusive: at the eighth and ninth it (Plate XLVII. fig. 11, 8 and 9, *m*) begins to project forwards; in the tenth (fig. 11, 10, *m*,) it is longer than the diapophysis (*d*) that supports it; in the eleventh the metapophysis (*ib.* 11, *m*) begins to shift its position and rises half-way between the diapophysis and the prozygapophysis (*z*); in the twelfth dorsal it rises behind the prozygapophysis; in the fourteenth dorsal it has got above that process and the articular surface begins to ascend upon the inner side of its base: the change of place and aspect of that surface is completed in the lumbar series. The anapophysis separates itself from the metapophysis in the eleventh dorsal (*ib.* 11, *a*), and progressively increases to the penultimate dorsal, beyond which it decreases, and it disappears on the fifth lumbar. The diapophysis is suppressed in the last four dorsal vertebræ.

Order Insectivora.

In the Hedgehog (*Erinaceus europæus*) the metapophysis commences as a slight tuberosity above the diapophysis of the second dorsal, becomes a distinct process in the third, projects outwards and forwards freely above the diapophysis in the fourth and fifth dorsals, ascends to the outside of the base of the prozygapophysis in the sixth, and continues there, inclining a little more upwards, and with increased breadth in the remaining dorsals, and in the lumbar vertebræ: also along the caudal vertebræ as far as the tenth, where the zygapophyses disappear and the metapophyses alone are present. There are no anapophyses.

The vertebral characters of the Tenrees (*Centetes*) resemble in these respects those of the Hedgehogs.

In the Moles (*Talpidæ*) short anapophyses are developed from some of the anterior lumbar vertebræ.

In the Shrews (*Soricidæ*) they are somewhat better marked; the anapophyses are continued from the penultimate dorsal to the third lumbar vertebræ in the great Indian species (*Sorex myosurus*); whilst in the *Cladobates ferrugineus* they underlap the metapophyses of the succeeding vertebra in the last dorsals and in all, save the last lumbar vertebræ. The diapophyses in both *Talpidæ* and *Soricidæ* resemble those in the *Erinaceidæ*.

Order Marsupialia.

In the Thylacine (*Thylacinus Harrisii*), with *d* 13, *l* 6, the anapophysis appears first upon the ninth dorsal, as a pointed process projecting from the back of the diapophysis; it increases in size and ascends in position on the tenth; is large, obtuse, and underlaps the metapophysis of the succeeding vertebra in the last two dorsals; progressively diminishes in the lumbar vertebræ, and disappears on the fourth of that series. The metapophysis is developed abruptly on the tenth dorsal external to the prozygapophysis, increases in size in the following dorsals, diminishes in the lumbar vertebræ, but is present throughout the series as a strong obtuse process: it is continued, also, through a great part of the caudal series, in which the zygapophyses become obsolete at the eighth vertebra.

In the Ursine Dasyure (*Dasyurus ursinus*), with *d* 13, *l* 6, both anapophyses and metapophyses commence at the eleventh dorsal: the anapophyses increase to the second lumbar, diminish in the two following, and disappear in the fifth. They underlap the metapophyses of the first three lumbar vertebræ. The metapophyses are continued throughout the sacral and a great part of the caudal region, in which the zygapophyses cease to be developed at the eighth vertebra.

The diapophyses are not obliterated in the last dorsal vertebræ, which renders their serial homology distinctly traceable along the lumbar region.

In the Wombat (*Phascolomys vombatus*), with *d* 15, *l* 4, the metapophysis (*m*) rises suddenly from the outside of the prozygapophysis (*z*) of the twelfth dorsal (Plate XLVII. fig. 12, D 12); increases in length to the second lumbar (L 2), diminishes by degrees to the second sacral; and is rudimental in the following sacral and caudal vertebræ. A rudiment of the anapophysis (*a*) is first discernible on the eleventh dorsal: the process gradually increases to the last dorsal (*a* D 15), diminishes in the lumbar, and disappears in the last of that series. In the skeleton showing the above modifications, the sutures between the short straight pleurapophysis (*pl*) and diapophysis (*d*) of the first lumbar vertebra (L 1) still in a great degree remain; the anchylosis is only partial; and the proportion of the autogenous and exogenous elements

of the so-called 'transverse process' is plainly demonstrated. The diapophysis, moreover, is not suppressed on the last dorsal vertebra as in some of the *Quadrumana*, *Carnivora* and *Rodentia*. The serial homology of the transverse processes of the lumbar vertebræ is here, therefore, manifested in the most unequivocal way; both metapophyses (*m*) and anapophyses (*a*) coexist with diapophyses (*d*) in the last four dorsal and the first three lumbar vertebræ. So that, whether the metapophysis or the anapophysis be the part called 'tubercle' by some Anthropotomists, neither of them is in the lumbar vertebræ, the process named 'transverse' in the thoracic vertebræ; that process, to which I restrict the name 'diapophysis,' is continued distinctly into the lumbar region, and is there lengthened out by a superadded 'pleurapophysis,' which is ossified from a distinct centre in the Wombat, but is not so ossified in the human skeleton, in which only a mere epiphysis, like that at the extremity of the spinous process, is superadded to the end of the transverse process of the lumbar vertebræ*.

In the skeleton of the Great Kangaroo (*Macropus major*), with *d* 13, *l* 6, both

* The author of the article 'Skeleton' in the Cyclopædia of Anatomy and Physiology, Part XXXV., March 1839, cites, p. 628, the following passage from my work on the Archetype and Homologies of the Vertebrate Skeleton:—"Each of the five succeeding segments is represented by the same elements coalesced, that constitute the so-called dorsal vertebra; they are called lumbar vertebræ; they have no ossified pleurapophyses," with the following comment:—"Professor Owen's 'pleurapophysis' is the rib or costal appendage of his typical vertebra. While he states, therefore, that the lumbar vertebra has no pleurapophysis, he means that it has no rib or costal piece. This oversight (which, with all respect, I believe it to be) has arisen from the evident error of mistaking the lumbar transverse process as being the counterpart or homologue of the dorsal transverse process, which, if such were the case, would leave the lumbar vertebra without a rib." The passage so commented on is taken from a 'Section' of my work treating of the specialities of the Human Skeleton; in the 'Section' treating on lumbar vertebræ in general, I state,—"The lumbar vertebræ, which in some mammals show, in the fœtal state, distinct rudiments of pleurapophyses more minute than those in the neck, have them soon anchylosed to the extremities of the diapophyses, which are thus elongated," p. 94. No one proposing fairly to criticise my opinions on the transverse process of a lumbar vertebra in general, ought to have represented them by a quotation of a description of the lumbar vertebra of a particular species. But taking the passage upon the human skeleton, as cited above, the conclusion of the Critic that I "leave the lumbar vertebra without a rib," can only be sustained by the omission of a word in that passage. What I do state is, that, in them, "they have no *ossified* pleurapophyses." Now in a previous part of the work quoted from, it is shown, that a vertebral element may exist in three states, 'fibrous,' 'cartilaginous,' 'osseous,' and that its absence was not to be assumed merely because it had not passed into the bony stage. I have never met with a human lumbar vertebra showing the transverse process lengthened out by a distinct autogenous ossification, meriting, like that of the Wombat, Plate XLVII. fig. 12, L 1, to be regarded as a pleurapophysis: the whole is an exogenous outgrowing diapophysis, with a minute epiphysis, which begins to be ossified about the age of sixteen, and is comparable in my opinion with that at the extremity of the metapophysis and neural spine. As to the opinion that the transverse process of the dorsal vertebra is homologous with the 'tubercle' of the lumbar vertebra, whether by 'tubercle' is meant the metapophysis or the anapophysis, I need only refer to those skeletons of mammalia, in which, as in the Wombat, Kangaroo and Dasyure, the true diapophysis is not suppressed on the last dorsal vertebra, in order to ensure a conviction in every impartial observer that the exogenous part of the transverse process of the lumbar vertebra is the serial homologue or tallying part with the diapophysis, or part of the transverse process supporting the rib, of the dorsal vertebræ.

met- and an-apophyses commence, suddenly, on the twelfth dorsal, the former (*m*) as broad compressed plates from the anterior two-thirds of the side of the neural arch, the latter (*a*) as short processes from the hinder border of the same arch. The metapophyses (*m*) increase to the first lumbar, and after the second decrease to the sacral vertebræ: they again begin to increase in the caudals and attain great size in the third to the seventh of that series, in which latter vertebræ the zygapophyses have disappeared: but the metapophyses (see Plate LIII. fig. 62, A, C, *m m*) are continued to near the end of the tail. The anapophyses (*a*) increase to the second lumbar, and rapidly decrease after the third; they are obsolete in the last lumbar.

The smaller *Macropus Parryi*, with the same vertebral formula, resembles the Great Kangaroo in regard to the accessory processes; except that the anapophyses are relatively smaller and begin in the eleventh dorsal (Plate XLVII. fig. 13, D 11, *a*) upon the diapophysis (*d*); passing in the twelfth to the back part of the neural arch; they underlap the metapophyses (*m*) of the first four lumbar vertebræ and become mere rudiments in the two last (*ib.* L 5 and 6, *a*). In both species of Kangaroo the diapophyses (*d*) continue to be developed in the last dorsal vertebræ, rendering the homologies of those of the lumbar series unmistakeable.

In a *Perameles lagotis*, with *d* 13, *l* 6, the metapophysis begins at the ninth dorsal vertebra, rapidly increases to the first lumbar, and continues large, though slightly decreasing in the last two lumbars: they are continued along the sacrum and a great part of the caudal vertebræ. There are no anapophyses in this species.

Order Monotremata.

The metapophyses are double, one behind the other, in the form of low tubercles, on each side of the neural arch of the third, fourth, and fifth dorsal vertebræ of the *Ornithorhynchus paradoxus*; they become single on the sixth dorsal, and gradually increase in size to the twelfth: beyond this vertebra they increase in antero-posterior extent chiefly, and thus attain their maximum of size in the last dorsal and the two lumbar vertebræ; they are continued throughout the sacral and to near the end of the caudal region. A rudiment of the anapophysis is discernible on the fourteenth, the fifteenth and sixteenth dorsal vertebræ, projecting backwards over the hole for the nerve at the side of the neural arch.

Order Ungulata.

Suborder *Artiodactyla*.

Tribe *Ruminantia*.

In the Musk-Deer (*Moschus moschiferus*), with *d* 14, *l* 5 (Plate XLVIII. fig. 14), a distinct metapophysis (*m*) begins to be developed above the diapophysis of the second dorsal, is midway between this and the prozygapophysis in the tenth dorsal, rises upon the outside of the prozygapophysis on the eleventh, attains its greatest length

in the twelfth, and becomes thicker and more obtuse in the remaining dorsals and in the lumbar vertebræ. There are no anapophyses.

In the skeleton on which these observations were made, there is a peculiarity that well illustrates the serial homology of the diapophyses of the dorsal and lumbar regions. The tubercle disappears in the penultimate pair of ribs, and the diapophysis is reduced in the thirteenth dorsal vertebra to a short rough tuberosity (d'): but in the last pair of ribs the tubercle with its articular surface reappears, and the diapophysis (d) resumes its normal size and articulation with the rib, which inclines forwards at its commencement. In the first lumbar vertebra the diapophysis (d) suddenly increases in length and breadth, which increase is doubtless due to the ossified and coalesced rudiment of a rib (pl).

In the Meminna (*Moschus meminna*), with d 13, l 7, the metapophysial tubercle appears above the diapophysis of the anterior dorsals, projects as a distinct process midway between the diapophyses and prozygapophyses of the seventh to the tenth dorsals inclusive; gets upon the prozygapophysis of the succeeding dorsals and anterior lumbars, and subsides in the penultimate lumbar vertebra. The tubercle has disappeared on the tenth rib, and does not reappear in those that follow: but the diapophysis continues on the last four dorsal vertebræ, increasing in length upon the last two, and plainly showing the nature of the longer diapophyses in the lumbar region.

In the Elk (*Alces Americana*), with d 13, l 6, the metapophysial ridge becomes distinctly developed upon the diapophysis of the eighth dorsal, is a tubercle on the ninth, forms a process exceeding in length the diapophysis in the tenth and eleventh dorsals, and subsides to a tuberosity again in the twelfth and succeeding vertebræ, where its position has changed from the diapophysis to the zygapophysis. There is a slight trace of an anapophysis in the last lumbar vertebra.

In the Equine Antilope (*Antilope equina*), with d 13, l 6, the metapophysis is developed from the fore-part of the diapophysis of the second to the ninth dorsals inclusive, where it begins to be transferred to the outer side of the prozygapophyses, from which part it projects in the last four dorsals, and in all the lumbar vertebræ. There is a short anapophysis in the last two dorsals, but not in any of the lumbar vertebræ.

In the Gnu (*Catoblepas Gnu*), with d 14, l 6, there are no anapophyses; the metapophyses resemble those in the Equine Antilope.

In the Common Ox (*Bos Taurus*), with d 13, l 6, the metapophysis, beginning as a rudimentary tubercle at the anterior dorsals, increases in size in the tenth, eleventh and twelfth dorsals, in which it is transferred from the diapophysis to the prozygapophysis.

In the Aurochs (*Bison Europæus*), with d 14, l 5, the metapophysis is most distinct on the ninth, tenth and eleventh dorsals, in which it ascends to the prozygapophysis.

In the Giraffe (*Camelopardalis Giraffa*), with d 14, l 5, the metapophyses com-

mence as distinct and well-marked tuberosities above the diapophyses of the second dorsal; subside at the eighth, and are rudimental to the thirteenth; they recommence as distinct processes at the fourteenth dorsal, where they project forwards on the outer side of, but quite distinct from, the prozygapophyses; ascend upon those processes in the first lumbar, and so continue throughout the lumbar series. There are no anapophyses.

In the Camel (*Camelus Bactrianus*), with *d* 12, *l* 7, the metapophysial tubercle is developed from the diapophysis in the eleven anterior vertebræ, and passes upon the prozygapophysis in the twelfth, continuing in that position throughout the lumbar series. There are no anapophyses.

In the Vicugna (*Auchenia Vicugna*) the metapophysis is between the diapophysis and prozygapophysis, from the fourth to the tenth dorsal, and passes upon the prozygapophysis at the eleventh dorsal. There are no anapophyses.

Tribe *Non-Ruminantia*.

In an Indian Wild Boar (*Sus scrofa*), with *d* 13, *l* 6, the metapophysis commences as a tuberosity upon the diapophysis of the middle dorsal vertebræ, projects forwards midway between the dia- and prozyg-apophysis in the tenth, passes upon the prozygapophysis of the eleventh dorsal, and is continued in that position throughout the lumbar series. There are no anapophyses.

In the Peccari (*Dicotyles labiatus*), with *d* 14, *l* 6, the metapophysis begins to be developed at the third dorsal, and increases in length to the eleventh, beyond which it is transferred from the dia- to the prozyg-apophysis.

In the Hippopotamus (*Hippopotamus amphibius*), with *d* 15, *l* 4, a metapophysial ridge is developed above the diapophysis of the eighth dorsal, changes its position and shape in the succeeding vertebræ, and in the eleventh has passed, as a tuberosity, upon the prozygapophysis. It retains these characters throughout the rest of the dorsal and the lumbar series. There are no anapophyses.

Suborder *Perissodactyla*.

In the Horse (*Equus caballus*), with *d* 19, *l* 5, the metapophysis, commencing as a tuberosity above the diapophysis, passes gradually from that part to the outer side of the prozygapophysis, which it finally attains in the seventeenth dorsal vertebra, and continues in the same place throughout the lumbar series. There are no anapophyses.

In the Sumatran Rhinoceros (*Rhinoceros Sumatranus*), with *d* 19, *l* 3, the metapophysis begins, as a tuberosity, above the diapophysis of the fourth dorsal, projects distinctly forward from the eighth, and continues so projecting, and slightly increasing in length, to the seventeenth; in the eighteenth it begins to approach the zygapophysis, and gets outside of it in the nineteenth; and so continues, reduced to

a tuberosity, in the lumbar vertebræ. There are no anapophyses. The back part of the diapophysis of the last lumbar presents a distinct articular process for a corresponding one on the fore-part of that of the first sacral vertebra.

In the Sumatran Tapir (*Tapirus Malayanus*), a very distinct metapophysis projects upwards and forwards from the fore-part of the transverse process of the third dorsal; it becomes a thicker and lower tuberosity above the diapophysis on the four succeeding vertebræ: then again becomes prominent, with a rounded anterior edge in the succeeding dorsals, projecting from the fore-part of the diapophysis in the four last dorsals, where that process is raised above and is distinct from the rib, and projects upwards and outwards. In the lumbar region the metapophysis is very distinct, and does not pass upon the prozygapophysis until the last lumbar*. There are no anapophyses.

Tribe *Proboscidia*.

In the Asiatic Elephant (*Elephas Asiaticus*), with *d* 19, *l* 4, the metapophyses present a character different from that in all other *Ungulata*; they are first recognizable as an obtuse point projecting forwards from the fore-part of the diapophysis of the sixteenth dorsal vertebra; in the seventeenth the metapophysis has assumed a subdepressed quadrate shape, and projects obliquely outwards and forwards from the outside of the prozygapophysis: in the eighteenth dorsal it extends forwards, and overlaps an anapophysis developed from the back part of the diapophysis of the seventeenth dorsal, and these accessory joints continue between the succeeding vertebræ as far as the first and second lumbars. The anapophyses have disappeared in the third and fourth lumbars, in which the metapophyses are reduced to short thick tuberosities, still projecting, not from above, but quite external to, the prozygapophyses. The metapophyses are continued along the sacral and caudal vertebræ, and in a large proportion of the latter supersede the prozygapophyses.

Order Sirenia.

In the Dugong (*Halicore indicus*) there is a feeble rudiment of a distinct metapophysis on the outer side of the prozygapophysis of the fifth dorsal; but the homotypes of the metapophyses are so little developed in the preceding vertebræ, that the change of aspect of the articular surface seems to be due exclusively to a gradual change of position in the anterior zygapophyses themselves.

Order Cetacea.

In the small Bottlenose Whale (*Delphinus Tursio*), the metapophysis (*m*) begins to project from the fore-part of the diapophysis (*d*) of the third dorsal, increases in length

* Straus-Durckheim's term 'anapophyse antoblique,' applied to these processes in the Cat, would be quite inappropriate and unintelligible if applied to their homologues in the Tapir.

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in the fourth (Plate XLVIII., fig. 15, D 4), and is gradually transferred in the sixth (D 6) and seventh (D 7) dorsals to the outer side of the prozygapophyses (*z*): in the following vertebræ it seems to take their place, and to occasion a reversing of the usual relative position of the zygapophysis; for whereas in the cervical and anterior dorsal vertebræ the anterior zygapophyses are overlapped, as in other mammals, by the posterior zygapophyses, in the succeeding dorsals, beginning with the seventh, the posterior zygapophyses seem to be overlapped and concealed by the anterior zygapophysis. But this is not the case; the appearance is due to the place of the prozygapophyses (*z*) being taken by the metapophyses (*m*). These latter processes, in fact, continue after the articular surface has ceased to be developed, and after the entire disappearance of the posterior zygapophyses, to project forwards from the thirteenth dorsal to the sixth lumbar vertebra inclusive; beyond which the neural arch is devoid of all exogenous processes, save the spine, until the middle caudal vertebræ, where rudiments of the metapophyses again reappear.

In the common Dolphin (*Delphinus Delphis*), Plate XLVIII. fig. 16, the metapophysis (*m*) begins abruptly, as a long well-marked process, from the fore-part of the diapophysis (*d*) of the fourth dorsal (D 4); progressively approximates and attains the outside of the prozygapophysis in the eighth dorsal (D 8), performs the function of an articular process as far as the sixth lumbar, clamping, as it were, the sides of the back part of the base of the spine of the antecedent vertebra, disappears in the next dozen lumbar vertebræ, and reappears in the caudal vertebræ at the fore-part of the base of the spine.

There are no anapophyses in the *Cetacea*. With respect to the metapophyses, Cuvier, in his description of the vertebræ of the Dolphin, confounds them, as Straus-Durckheim has done in the Cat, with the true zygapophyses. He writes,—"The last cervical and the six first dorsals have their articular processes joined together by horizontal facets, of which the anterior looks upwards. At the sixth they begin to be oblique, at the seventh they are nearly vertical, the anterior looking inwards*." But in the figure which he refers to, of the fourth dorsal vertebra of the *Delphinus Delphis*, the accurate artist, M. Laurillard, represents the metapophyses as distinct from the prozygapophysis or anterior articular process, although less so than it is in nature; and it is incontestably the progressive development of this superadded process which gives rise to the change of position of the articular surface of the connate prozygapophysis: and the metapophysis continues to be developed, as the figures in the 'Ossemens Fossiles' demonstrate, long after the articular process or any articular surface has ceased to exist†.

* Ossemens Fossiles, Ed. 1823, tom. v. pt. i. p. 303, pl. xxiii. figs. 25—29.

† This fact is clearly recognized by Professor Stannius, in his 'Vergleichende Anatomie der Wirbelthiere,' 8vo., p. 345, where he describes the 'metapophyses' as 'processus accessorii.'

Order Bruta (*Edentata*, Cuv.).

The exogenous processes of the vertebræ present their greatest and most complex developments in this order, but by no means with that degree of uniformity which renders their modifications characteristic of some of the preceding Orders. The two extremes in the degree of development are, in fact, presented by the much-diversified families of the present extraordinary group of mammals.

The spinous processes are almost obsolete in a great part of the unusually long dorso-lumbar region of the Two-toed Sloth (*Bradypus didactylus*), and the metapophyses would be hardly recognizable if their modifications had not been previously traced in other mammals. The first indication of them is seen in the penultimate dorsal vertebra, where they appear as angular productions from the upper and fore-part of the diapophyses: in the last dorsal they have advanced clear of the diapophysis to the outer side of the prozygapophysis, the articular surface of which begins to ascend upon them; and they continue, as low and long tuberosities, above the prozygapophyses, throughout the lumbar region. In the sacral vertebræ they are obsolete. No anapophyses are developed in this species of Sloth*.

* In assigning, with Cuvier, this formula to the dorsal region of the spine, I have been guided by characters which have appeared to me more demonstrative of the homology of the eighth and ninth vertebræ of the Ai, with the last two cervicals of other mammals, than the sometimes persistent freedom of their short pleurapophyses are of their homology with the first two dorsals. The penultimate cervical vertebra, for example, has a well-marked and constant character in most mammals in the greater antero-posterior extent of its anchylosed pleurapophysis; and I find this character repeated in the eighth vertebra, counting from the atlas, in the Three-toed Sloth. Subjoined are the chief particulars which are noticeable in the cervical region of the skeleton of the mature specimen of that species in the Museum of the Royal College of Surgeons.

The hind-part of the bodies of the second to the sixth cervical vertebræ inclusive, are produced backwards and underlap the fore-part of the body of the succeeding vertebræ. The transverse process of the atlas is imperforate, but the base of the neural arch is pierced by the vertebral artery anteriorly and by the cervical nerve posteriorly. The spines of the cervical vertebræ are moderately and more equally developed than in other mammals; that of the dentata being little larger than the rest. The pleurapophysial part of the transverse process of the eighth cervical is anchylosed, and is more extended antero-posteriorly than in the preceding cervicals, in which respects it resembles the sixth cervical vertebra in ordinary quadrupeds. The pleurapophysial part of the transverse process of the ninth cervical is free, and is more extended in the direction of its length, but is very short compared with the homologous part of the succeeding vertebra. The slender neck and head of this little rib joining the fore-part of its centrum occasions the perforated character of the ninth, as in the antecedent cervical vertebræ. In the fourth cervical, however, the vertebral artery perforates the right transverse process, but only grooves the left on its anterior part. The transverse processes of the second and third cervical vertebræ are both imperforate.

The homology of the vertebra succeeding the ninth with the first dorsal of the Unau and of other mammals, is demonstrated by the junction of its rib with the manubrium by the superadded hæmapophysial element, which is here, however, ossified and anchylosed with both its pleurapophysis and the manubrium. Nine pairs of ribs directly articulate to the sternum, which consists of eight bones; these are compressed and progressively increase in depth. The hinder ones are divided into a larger posterior and a smaller anterior part, between which are four articulated facets on each side for the bifurcated extremities of two of the ossified hæmapophyses†.

† Proceedings of the Zoological Society, 1849, p. 146.

In the Ai (*Bradypus tridactylus*), with *d* 16, *l* 3, both met- (*m*) and an- (*a*) apophyses, Plate XLIX. fig. 17, are present, and the former are more distinctly developed than in the Unau: they are, however, restricted, as in that species, to a very few vertebræ. The metapophysis (*m*) is first developed as a distinct, though small obtuse conical process from the fore-part of the diapophysis (*d*) of the penultimate dorsal vertebra (D 15); it increases in size and approaches the prozygapophysis (*z*) of the last dorsal (D 16); ascends upon the outside of that process, and becomes more obtuse in the first (L 1) and following lumbar vertebræ.

In the *Orycteropus capensis*, with *d* 13, *l* 8, an accessory tubercle is developed upon the diapophysis of the seven anterior dorsal vertebræ, which divides near the eighth into metapophysis and anapophysis. These progressively increase and diverge from one another in the succeeding dorsals, and in the first lumbar vertebra the metapophysis projects upwards, outwards and forwards upon the outside of the anterior zygapophysis; whilst the anapophysis extends backwards from the back part of the diapophysis, which it equals in length. The anapophysis decreases in size in the following lumbar vertebræ and disappears in the last*; the metapophysis also decreases in size, but is continued throughout the lumbar series and along part of the sacral. The transverse processes of the three anterior sacrals join the ilia; those of the three posterior ones coalesce to form a broad depressed plate, with the posterior angles produced, but not joining the ischia. A long and strong process is sent out from above the tuber ischii. Metapophyses are developed from the outside of the anterior zygapophyses, as far as these extend along the caudal series, viz. to the eighth vertebra; beyond these the metapophyses are developed, independently of the zygapophyses, to near the termination of the tail.

In the Long-tailed Armadillo (*Dasypus longicaudus*, Pr. Max.), with *d* 11, *l* 5, the metapophyses commence, abruptly, as long slender obtuse processes, on the seventh dorsal (Plate XLIX. fig. 18, D 7, *m*) rising midway between the diapophyses (*d*) and prozygapophyses (*z*), and projecting obliquely upwards and forwards: they pass upon the outside of the base of the prozygapophyses of the eighth dorsal (D 8, *m*), develope an articular surface from the inner side of their base for an extension of the surface of the zygapophysis upon its outer side in the ninth and succeeding dorsals; in the eighth dorsal they also develope a second articular surface from the under part of their base for articulating with the anapophysis of the seventh dorsal. The metapophyses in the following vertebræ, retaining their two articular surfaces, progressively increase in length until they exceed the neural spines in this respect in the last three lumbar

* I have ascertained that the vertebral column of the so-called 'Ant-eater' exhibited by Dr. Melville at the meeting of the Zoological Society, on December 12, 1848, in order to demonstrate "that the Edentata had no posterior or backwardly projecting processes from the diapophyses," is not of a true Ant-eater, but of an *Orycteropus*, in which, although the development of the anapophyses is much less than in the species of *Myrmecophaga*, they are nevertheless sufficiently recognizable. In no order of Mammals, as I shall presently show, are the 'anapophyses' or 'backwardly projecting processes' more extraordinarily developed than in the *Edentata*.

vertebræ. The anapophysis (*a*) commences on the seventh dorsal, projecting backwards behind the base of the diapophysis, and presenting on its upper side an articular surface for that beneath the base of the succeeding metapophysis. The anapophysis increases in length, but more so in vertical thickness in the succeeding vertebræ. The diapophysis (*d*) subsides to a mere ridge in the last dorsal and disappears in the lumbar vertebræ, where the transverse processes are formed by parapophyses. In the penultimate dorsal vertebra the anapophysis is notched posteriorly, and developes a small surface from below its inferior division for articulating with the head of the last rib. The succeeding anapophysis developes a similar surface for articulating with a short and thick exogenous process or 'parapophysis': and this complex condition of the anapophysis is continued throughout the lumbar region, the last vertebra having a longer and narrower anapophysis. Thus on each side of the fore-part of each lumbar vertebra there is *one* articular surface on the prozygapophysis, a *second* on the inner side of the base of the metapophysis, a *third* on the outer and under side of the same base, and a *fourth* on the upper side of the base of the parapophysis; and on each side of the back part of the same vertebræ there is *one* articular surface on the under side, and a *second* on the outer side of the posterior zygapophysis, a *third* surface on the upper, and a *fourth* on the under side of the anapophysis; making sixteen synovial joints, four on each side of both fore and back parts of the vertebræ, which joints may be called, beginning from above downwards, the 'zygapophysial,' 'metapophysial,' 'anapophysial' and 'parapophysial' articulations respectively*; and the power of inflecting the substantive names for the several processes and applying them adjectively to such modifications as I have described is not one of the least advantages of such substantive names to the descriptive Anatomist. These different articulations form double tenon-and-mortise joints on each side of the vertebræ. But their comparison with these joints in artificial carpentry, affords but a meagre idea of their true nature and complexity, whilst the supposition of their homology with the similar tenon-and-mortise joints of the backbone of Serpents is, as I shall presently demonstrate, unsupported by exact comparison.

In the *Dasypus tricinctus*, the true or moveable vertebræ present the following, amongst other, characters:—

The spine of the third cervical has completely coalesced with that of the dentata, which is thick and high, but more extended forwards than backwards. The spine of the fourth cervical is applied to its back part. The neural arches of the succeeding cervicals have no spines, but form thin transverse bars of bone above, which are incomplete in the middle at the fifth and sixth cervicals, upon which the rest are strongly bent backwards. Their bodies are extremely broad in proportion to their length or antero-posterior diameter. The articular bed for the head and tubercle of the first dorsal rib is contributed to in equal shares by the last cervical and first

* The zygapophysial and metapophysial synovial sacs communicate and form a common joint in the posterior dorsal and lumbar vertebræ.

dorsal vertebræ. Ten vertebræ show the impressions of the articulation of the head of the rib in addition to the first dorsal, and the neurapophyses of these eleven vertebræ are directly perforated by the spinal nerves, Plate XLIX., fig. 19, *n*, *n**. The articulation for the last rib (*pl*) is as equally divided between the two contiguous vertebræ as is that of the first rib. The prominence (*p*) supporting the articular surface for the head of the rib answers to the parapophysis, just as the prominence for supporting the articulation for the tubercle of the rib, represents the diapophysis. The prominence in the first lumbar vertebra (*p*) which articulates with the under part of the anapophysis (*a*) of the last dorsal, repeats the prominence in that dorsal, which articulates with the head of the last rib; it is therefore a parapophysis. The diapophysis (*d*) projects, as in the dorsal vertebræ, from the upper and outer part of the base of the short and thick anapophysis (*a*), and this anapophysis presents, as in other Armadillos, two articular surfaces, one above, for the under part of the metapophysis (*m*), another below, for the upper part of the parapophysis (*p*). Thus the vertebræ are interlocked by tenon-and-mortise joints, as Cuvier has described, but it is by distinct parts of the vertebra from those which form the corresponding joints in the backbone of serpents.

The metapophysis (*m*, 7) begins in the seventh dorsal as a distinct process: the anapophysis is first developed in the sixth (*a* 6), and articulates with the under part of the first metapophysis; the subsequent metapophyses (*m*, *m*) present the usual progressive and great development, as also the two joints, one on the inner and the other on the under side of their base. Nothing can be more distinct than the anterior and posterior zygapophyses of the sixth dorsal vertebra: but equally distinct are the metapophyses which coexist with them in the seventh dorsal; to confound these two processes, or to describe the longer metapophyses in the subsequent vertebræ as developments of the anterior zygapophyses, is to confound two things manifestly distinct.

In the *Dasypus sexcinctus*, some modifications of the accessory processes are present, which render a notice of its vertebral characters desirable.

The vertebral formula is seven cervical, eleven dorsal, three lumbar, nine sacral, sixteen caudal. The second and third cervicals have coalesced together, and they develope a strong confluent neural spine. The last four cervicals are without those spines, and are equally devoid of zygapophyses. The diapophysis of the last cervical contributes to the articular cavity for the tubercle of the first dorsal rib, which is enormously expanded; and the anchylosed pleurapophysis of the seventh cervical projects backwards beneath the head of the first dorsal rib. The neurapophyses of the middle dorsal vertebræ are directly perforated for the spinal nerves. Anapo-

* This is one of so vast a series of exceptions to the "Loi de conjugaison" of Prof. Serres, as to deprive it of any claim to be considered as the law of formation of foramina, and to demonstrate the abuse of the term 'law,' in cases like that in which it has been made to express the idea that every foramen is formed by the approximation of two notches in distinct bones.

physes are developed from behind the diapophyses of the first three dorsals; they decrease in size in the succeeding dorsals, but again increase in the posterior dorsals and in the lumbar vertebræ, where they present an articular surface to the metapophysis of the succeeding vertebræ. The metapophyses commence as ridges on the upper surface of the diapophyses of the second and third dorsals; they form prominent tubercles on the seventh and eighth dorsals, and rapidly elongate in the succeeding vertebræ, where they surpass in length and equal in thickness the neural spines. They are much diminished in length in the first sacral vertebra. Each of these elongated metapophyses presents a distinct articular surface to an accessory posterior zygapophysis; and on its opposite side an articular surface to the subjacent anapophyses of the preceding vertebra, producing two additional pairs of joints to the normal ones formed by the anterior and posterior zygapophyses.

The parapophyses of the lumbar vertebra do not materially increase in length, but chiefly in antero-posterior extent, overlapping each other, the back part of the anapophysis of the last dorsal resting on the fore-part of the parapophysis of the first lumbar vertebra, and the parapophyses of the last lumbar vertebræ similarly articulating with a prominence of the iliac bone; an additional pair of articulations being thus formed externally to those between the anapophyses and metapophyses.

The long sacrum has coalesced with both the iliac and ischial bones. The pubic bones complete a wide arch by their confluence at their slender symphysis. The ischiatic notches are converted into foramina, which are inferior in size to the foramina obturatoria. The spines of most of the sacral vertebræ form by their confluence a continuous ridge. The strong tuberosity of the ischium is bifid. The zygapophyses are developed and coarticulated in the first five caudal vertebræ. The anterior ones support short and thick metapophyses, and these are continued in the succeeding caudal vertebræ, after the anterior zygapophyses have disappeared.

In the Pangolin (*Manis pentadactyla*), with *d* 13, *l* 4, the metapophyses commence as tubercles on the first dorsal vertebra and rapidly increase in size. Anapophyses are not developed. The metapophyses continue to be developed from the sacral series. The transverse processes of the last sacral suddenly expand both in length and breadth, and articulate with the tuberosities of the ischium. Well-developed hæmal arches are articulated to the inferior interspaces of the caudal vertebra as far the penultimate one. The anterior zygapophyses cease upon the fourteenth vertebra of the tail, but the metapophyses are continued as far as the penultimate caudal.

In the Tamandua (*Myrmecophaga Tamandua*), with *d* 17, *l* 2, all the cervical vertebræ have spinous processes except the atlas; that of the dentata is produced forwards, not backwards, and those of the sixth and seventh cervicals are as long as the spine of the first dorsal, which exceeds in length that of any of the other vertebræ. The pleurapophysial part of the transverse process of the dentata is broad, and produced downwards and outwards: it increases in length and breadth in the four suc-

ceeding vertebræ, in the three last of which the diapophysial element of the process stands out distinctly and strongly. In the seventh cervical this element alone is developed, and the transverse process is accordingly said to be imperforate. A metapophysial tubercle is developed from the outer side of the prozygapophysis of the five last cervicals. The diapophyses of all the dorsals present an articular surface for the tubercle of the rib. The metapophysial tubercle is continued from the cervical to the dorsal region, appearing there upon the upper part of the diapophyses of the first dorsal; it continues increasing in size and length to the thirteenth dorsal, where it begins to shift its position, and in the remaining dorsals projects midway between the diapophysis and zygapophysis. In the fourteenth dorsal the metapophysis also increases suddenly in breadth, and developes an articular surface from its inner side to join an accessory posterior zygapophysis, as well as an articulation upon its fore and under part for the anapophysis of the preceding vertebra. In the fourteenth dorsal the anapophysis suddenly acquires increased length and breadth, with a distinct articular surface upon both its upper and under part, the upper one articulating with the metapophysis, the under one with the parapophysis of the succeeding vertebra. Thus there are not fewer than sixteen co-adapted articular surfaces, in addition to those for the head and tubercle of each rib and the articulations between the ends of the centrum of one and the same vertebra. In the first lumbar vertebra the diapophysis increases in thinness and decreases in length, presenting the form of a de pressed plate; the other processes with thin articular surfaces are retained in both lumbar vertebræ. Parapophyses and metapophyses are also developed from the fore-part of the first sacral vertebra, together with another accessory process extending to the parapophyses, projecting from the fore-part of the diapophyses, and presenting an articular surface to a corresponding accessory articulating process for the anapophysis of the last lumbar. The neural spines of the five sacral vertebræ have coalesced into a continuous ridge, on each side of the base of which are the tubercular representations of the metapophyses. The transverse processes of the last sacral are enormously expanded, and develope from their under part a broad rough prominence for syndosmosis with the anterior tuberosity of the ischium. The posterior and ordinary tuberosity of that bone projects freely outwards beneath the transverse processes of the first caudal. The metapophysial tubercles begin to be developed from above the prozygapophyses from the first to the eighteenth caudal, beyond which the metapophyses exclusively represent the articular processes. The broad transverse processes have an accessory tubercle near their extremities, as far as the sixth caudal; at the seventh they are notched at their extremities, and the notch deepens until it divides the diapophyses into two in the eighteenth vertebra, and so on till they disappear at the end of the tail. Hæmal arches are articulated between the vertebral interspaces of most of the caudals.

In the Great Ant-eater (*Myrmecophaga jubata*), with *d* 15, *l* 3, the transverse processes of the atlas are pierced in two places obliquely at the fore-part of the neural

arch on each side. The axis has a transverse perforation on each side the neural arch anterior to the transverse process, which is imperforate. The transverse processes of the three succeeding cervicals are imperforate, the vertebral artery entering the neural canal behind and perforating obliquely the base of the neurapophysis anteriorly, as in the *Camelidæ*. In the sixth cervical, the canal for the vertebral artery runs through the base of the transverse process. These processes are much extended antero-posteriorly in all the cervicals and overlap each other. The diapophysial and pleurapophysial portions are very distinct in the fifth and sixth cervicals. The spines of the third and sixth cervicals inclusive are triangular and pointed; that of the seventh is longer than the rest and truncate above; it is much exceeded in antero-posterior diameter by the spine of the first dorsal, but not in height. A metapophysial tubercle is developed from the outer side of the anterior zygapophysis in all the five posterior cervicals. It is placed more outwardly in the first and second dorsals, and gets upon the top of the diapophyses in the succeeding dorsals. In the eleventh dorsal the metapophysis (Plate XLIX. fig. 20, *m*) begins to resume its former position, and developes an articular surface (Plate L. fig. 21, *front*, *m a*) from its under part, which joins the upper articulating surface (Ib. *back*, *a m*) of the anapophysis (*a*) of the preceding vertebra. In the thirteenth dorsal, the metapophysis (*m*) is half-way between the diapophysis and anterior zygapophysis, and repeats the same articulation with the anapophysis. In the last two dorsal vertebræ, the base of the metapophysis developes a second articular surface (Plate L. fig. 22, *front*, *m z*) from its inner side, which joins a new or accessory articular surface (ib. *back*, *z'*) on the outside of the posterior zygapophysis of the antecedent vertebra. This tenon-and-mortise articulation of the metapophysis with the zygapophysis on the inner side and with the anapophysis on the outer side, is repeated throughout the whole lumbar series. The anapophysis (*a*) begins to be developed from the anterior dorsal vertebra, and even there presents an articular surface (Plate L. fig. 21, *back*, *a p*) at its under part to join a corresponding surface (ib. *front*, *p a*) on a parapophysis (*p*) developed from the fore and outer part of the neural arch of the succeeding vertebra. In the tenth dorsal a second articular surface (ib. *back*, *a m*) is established in the upper part of the anapophyses for the inferior metapophysial one (*m a*) of the succeeding vertebra; here, therefore, the anapophysis begins to be mortised between the parapophysial and metapophysial articular surfaces, which surfaces continue to the antepenultimate lumbar vertebra, from which, forwards, to the eleventh dorsal, there are sixteen joints between each pair of vertebræ, as in the Armadillo. But this complication goes further in the Great Ant-eater, for, in the penultimate lumbar vertebra, a third articular surface (Plate L. fig. 23, *back*, *a d*) is developed from the under and outer part of the anapophysis (*a*), which joins an articular surface (fig. 23, *front*, *d a*) on the upper and fore-part of the diapophysis of the last lumbar; and this vertebra is united in a similarly complex manner with the first sacral vertebra, which would make eighteen synovial joints, in addition to those at the ends of the centrum, but

that those between the normal articular processes, or zygapophyses, are suppressed. The true serial homology of the processes (*p. p.*) as 'parapophyses,' developed from the fore-part of the base of the neural arch to articulate with the under part of the anapophyses, is well illustrated by the vertebræ of the Great Ant-eater, and, as will be afterwards shown, in the Megatherium, in which the true diapophyses are better developed than in the Armadillos.

Definite terms for those several processes that undergo such remarkable modifications in the great *Edentata* become indispensable; they could scarcely be rendered intelligible by the ordinary descriptive periphrases. But, by means of single substantive names, all their modifications can be defined, and, by adjectival inflections of those names, the articular surfaces can be distinguished.

In figures 21, 22 and 23, Plate L., initial letters indicate the processes on one side, and the articular surfaces on the other side of each vertebra, thus:—

m is the metapophysis, with *m a* its 'anapophysial,' and *m z* its 'zygapophysial' articulations.

d is the diapophysis, with *d pl* its 'pleurapophysial,' and *d a* its 'anapophysial' articulations.

p is the parapophysis, with *p a* its 'anapophysial' articulation.

a is the anapophysis, with *a m* its 'metapophysial,' *a p* its 'parapophysial,' and *a d* its 'diapophysial' articulations.

z and *z'*, in figs. 21 and 22, indicate the ordinary anterior and posterior zygapophyses; they become very small in the middle dorsal vertebræ, and are suppressed in the posterior dorsals and lumbars: the process *z'* in figs. 22 and 23 is not a gradual modification of the posterior zygapophyses, *z'* in fig. 21, but is a substitution for them, called out, as it were, by the new surface *m z* upon the metapophysis.

As the complex structure of the vertebræ of the Armadillos and Ant-eaters has, hitherto, been sought to be illustrated by reference to the vertebræ of Serpents, it becomes necessary, after the foregoing analysis, to ascertain how far the analogy holds good.

The Ophidian reptiles so far resemble the *Mammalia* in their vertebral characters, that all the autogenous elements, except the pleurapophyses (Plate LI., fig. 31, 32, *pl*), coalesce with one another in the trunk, and the pleurapophyses (*ib.*, fig. 28, *pl*) themselves become anchylosed to the diapophyses (*ib. d*) in the tail. But the exogenous processes present many differences from those in *Mammalia*. The diapophysis (figs. 24 and 25, *d*) extends from the fore-part of the side of the centrum, and is large, vertically oblong or hemispherical (fig. 38, *d*), and very short: it is covered by the articulation for the head of the rib*. The base of the neural arch

* This may be the 'head' of the rib in the same sense in which it would be said of that end of the simple rib which articulates with the diapophysis in the hinder dorsal vertebræ of the Crocodile. Guided by this modification of the ribs in that reptile, the Anatomist would be led to the homology of the simple ribs, and of the

swells outward from its confluence with the centrum, and developes from each angle a transversely elongated zygapophysis; that (z) from the anterior angle looking upwards, that (z′) from the posterior angle looking downwards, both surfaces being flat and almost horizontal. A wedge-shaped process, which I have termed 'zygosphene*' (fig. 25, z s), is developed from the fore-part of the base of the spine; the apex of the wedge being truncate or emarginate, and the converging sides presenting two smooth, flat, articular surfaces. This wedge is received into a cavity, which I have called 'zygantrum†' (fig. 26, z a), excavated in the posterior expansion of the neural arch, and having two smooth articular surfaces to which the zygosphenal surfaces are adapted. Thus the vertebræ of Serpents articulate with each other by eight joints, in addition to those of the cup and ball on the centrum; and interlock by parts reciprocally receiving and penetrating one another. This is the most conspicuous, but is not, as has been affirmed‡, the peculiar characteristic of the Ophidian vertebra; the zygosphenal (z s) and the zygantral (z a) surfaces being developed in certain Lizards, as *e.g.* the Iguana (Plate LI., figs. 40–42, z a, z s); where the zygosphene (z s, fig. 41) is deeply notched anteriorly, and the zygantra (z a, fig. 42) are shallow and separated from each other behind.

In the vertebræ of Reptiles and Mammals with double 'tenon-and-mortise' joints, the only articulating processes which are truly homologous are the zygapophyses (z, z′ in all the figures): the superadded processes in the Reptiles are, the zygosphene (z s) and zygantra (z a). A median 'tenon' and two lateral 'mortises' at the fore-part of the vertebra, and a median 'mortise' and two lateral 'tenons' at the back part of the vertebra, all of them *above* the true zygapophyses, are the result of the superaddition in Serpents; whilst in the Edentates a median mortise and two lateral tenons in front, and a median tenon and two lateral mortises behind the vertebra, are all of them *below* the true zygapophyses, and are the result of the additional processes called 'metapophyses' and 'anapophyses;' whilst a second pair of 'tenons

sustaining transverse processes in the Lacertians and Ophidians, which is indicated by the term 'diapophysis' applied to those processes. If, however, the lower articulating process in the bifurcated anterior ribs of the Crocodile be not regarded as the superadded part, but as answering to the head of the simple rib in the Snake, then the transverse process to which this is attached will be a 'parapophysis,' and the position of such transverse process in certain parts of the vertebral column, would lead to that supposition. But the modifications of the processes in question, in the Ophidian series, would indicate them to include, as in the *Batrachia* and in the neck of the *Plesiosaurus*, both diapophysis and parapophysis: and it is in this extended sense that I apply, for the sake of convenience, the term 'diapophysis' to the entire process in Ophidia and Sauria.

* Ζυγὸν, *a yoke*, σφὴν, *a wedge*.

† Ζυγὸν, and ἄντρον, *a cavity*.

‡ Cuvier, Leçons d'Anat. Comp. Ed. 1835, t. i. p. 216; where, after the double articular processes are described, it is stated, "Ces apophyses sont agencées de manière qu'il resulte, comme pour les vertèbres lombaires de certains Edentés, que deux vertèbres sont articulées entre elles par un double tenon entrant dans une double mortaise. La seule différence, c'est que les facettes de tenon et de la mortaise supérieurs sont continues et forment entre elles un angle aigu." The complex joints are, however, formed by different processes in the *Reptilia* and *Edentata*, as is explained in the text.

and mortises' is superadded below the others, in some Edentates, by the special development of the anapophyses and of parapophyses.

I know not how these complex but constant characteristics differentiating the Ophidian and Mammalian vertebræ could be intelligibly made known without the use of 'names' for the parts; and, as I trust, that I have shown the parts to be not mere subdivisions of the diapophyses, but constant, characteristic, and independent processes, definite names for them cannot be reasonably deprecated as unnecessary*.

Before leaving this part of my subject, I may point out a few of the modifications of the exogenous processes of the vertebræ, by which different genera of *Ophidia* may be distinguished.

In the genus *Coluber* (Plate LI., fig. 29–32), the diapophysis, besides developing the articular tubercle (*d*) for the rib, sends a long process (d'') outwards above that tubercle, between it and the prozygapophysis (*z*), and a very short process (d'), like a parapophysis, downwards below that tubercle: the neural spine (*n s*) is low in proportion to its antero-posterior extent: the zygosphenal surfaces (*z s*) are nearly vertical, and the zygantra (*z a*) are wide apart: the hypapophysis (fig. 32, *hy*) beyond the anterior fourth part of the vertebral column is reduced to a straight ridge, and is not produced posteriorly.

In the Sea-Serpent (*Hydrus*) (fig. 33.), the diapophysis also sends out a process above and below the articular tubercle; but that which seems to underprop the zygapophysis, does not extend so far out as in the *Coluber*. The neural spine (fig. 33, *n s*) is higher in proportion to its antero-posterior extent: a very small hypapophysis projects below the articular ball of the centrum, and a low ridge is continued from it along the under part of the centrum.

In the *Naja* or Cobra di Capello, the neural (fig. 34, *n s*) spine is lower, but the hypapophysis (*hy*) is longer than in either of the preceding genera. The diapophysis presents a well-marked articular tubercle (fig. 35, *d*) upon its upper part, and both the upper (d'') and lower (d') processes are well-marked, although the former is shorter than in the genus *Coluber*.

The Rattle-Snake (*Crotalus*) (fig. 36–39) is remarkable for the development of the hypapophysis (*h*) to an equal length with the neural spine (*n s*) throughout the vertebræ of the trunk; such development being restricted to the anterior vertebræ in most other genera: and any single vertebra might be distinguished from an anterior trunk-vertebra of a Boa or Python, for example, by the following characters:—The diapophysis developes a small hemispheric tubercle (*d*) from its upper convexity; and a process (d') from its under part extending downwards and forwards below

* See the criticism in 'Proceedings of the Zoological Society,' December 12, 1808, in which the author,—"Understanding that Professor Owen had proposed names for these mere subdivisions of the diapophyses, strongly deprecated the overloading this difficult part of anatomy with unnecessary names," p. 146; and yet he proceeds "to animadvert upon Cuvier and M. de Blainville for having neglected to describe them," as if they could be intelligibly described without determinations and names. What is truly to be deprecated is the unnecessary change of names when once rightly applied.

the level of the centrum (*c*); the anterior zygapophysis (*z*) seems to be supported by a similar process (*d*'') from the upper end of the diapophysis, the point of which projects a little beyond the end of the zygapophysis (*z*, fig. 37): I refrain from giving names to these upper and lower processes, because they really are nothing more than mere subdivisions of the diapophyses, and characterize the vertebræ of a few genera of a single order of Reptiles.

In the Boas and Pythons (fig. 24–27) the divisions (*d*'' and *d*') of the diapophysis are wholly wanting; that process is entirely occupied by the articular surface for the rib, which is vertically oblong, convex in the axis of the centrum, and convex vertically at its upper half, but concave vertically at its lower half. The base of the neural arch swells outward, from its confluence with the centrum, and developes from each angle a zygapophysis, which is much longer transversely than in any of the preceding genera of Serpents (fig. 27, *z*, *z*'). The zygosphene (fig. 25, *z s*) is narrower in proportion to its height, and the zygantrum (fig. 26, *z a*) is correspondingly deeper and more circumscribed. In an African Constrictor (*Python regius*), which measured 15 feet 6 inches in length, and had 348 vertebræ, the first seventy had long hypapophyses, which afterwards subsided into the obtuse ridge and tubercle shown at fig. 27, *hy*.

Of the value of the minute and apparently insignificant modifications of the vertebral processes above described, some idea may be formed by their application to determine the nature and affinities of the great Serpent, of which the vertebræ have been found fossil in the eocene tertiary deposits of Kent and Sussex. The veritable Ophidian nature of the fossils in question is demonstrated, not only by the superadded zygosphenal (*z s*, fig. 45) and zygantral (*z a*, fig. 46) articulations, but by the solidity of the zygosphene, by the size and form of the centrum, by those of its articular cup (*c*, fig. 45) and ball (*c*', fig. 46), and of its hypapophysis (*hy*); and also by the size and prominence of the diapophysis (*d*). The largest vertebræ, probably from about the middle of the body, as compared with the vertebræ from the same part of the skeleton of a *Python Sebæ*, 20 feet in length, are longer in proportion to their breadth, and the cup and ball of the centrum are larger; the hypapophysis (*h*) is more produced, and there is a second smaller hypapophysis close to the anterior part of the under surface of the centrum, which in most of the large vertebræ is connected by a ridge with the hinder and normal hypapophysis (fig. 47, *hy*), but in a few vertebræ is not so connected. The articular cup and ball are less obliquely placed upon the extremities of the centrum, being nearly vertical. The rim of the cup is sharply defined, and is more produced from between the bases of the diapophyses; a deeper and narrower chink intervening than in the Python. The transverse diameter of the cup (*c*, fig. 45) is greater than that of the zygosphene (*ib.*, *z s*),—a proportion which I have not found in the vertebræ of any existing genus of Serpent, in which the base of the zygosphene always equals at least the parallel diameter of the articular cup. The articular part of the diapophysis (*d*) is more produced out-

wards, and less extended vertically in *Palæophis* than in *Python*, and it is uniformly convex; a ridge is continued from its upper end obliquely forwards; as far as, but not beyond, the apex of the anterior zygapophysis (*z*), forming the angle between the lateral and anterior surfaces, whilst the horizontal articular facet forms the third surface of that three-sided conical process. In the *Python* the non-articular part of the same zygapophysis is convex, and the process is much more extended outwardly; the proportions of the zygapophysis in the *Palæophis* more resemble those in the *Coluber* and *Hydrus*, but differ from these, as also from *Naja* and *Crotalus*, in the non-extension of the diapophysial point beyond the articular surface.

A ridge or horizontal rising of the bone extends from the anterior to the posterior zygapophysis, but is more or less blunted or subsides midway, and is by no means so produced outwards as in Python; in this respect more resembling that in *Coluber* and *Hydrus*. Below the middle of this ridge, on a level with the upper surface of the centrum, there is a short, nearly parallel rising in *Palæophis* (fig. 44). The zygosphene (fig. 45, *z s*) is slightly excavated anteriorly, and shows no trace of the tubercle which characterizes the middle of that surface in the *Python* (fig. 25); it is also broader in proportion to its height. But perhaps the most characteristic feature of the vertebræ of the *Palæophis*, is the peculiar production of the posterior border of the neurapophysis into an angle (*n*, fig. 44) directed upwards, outwards, and backwards: there is no trace of this process in the *Hydrus* (fig. 33), or the other existing genera of Serpents. The posterior zygapophysis resembles, of course, the anterior one in its much less extent, especially transversely, as compared with that in the *Python*; and the posterior border of the neurapophysis (fig. 44, *z'*, *n*) rises from its apex vertically, or a little inclined outwards and backwards, giving a squarish form to the surface of the neural arch in which the zygantra (*z a*, fig. 46) are excavated; these cavities, in proportion to the articular ball beneath, are smaller and less deep than in the *Python*, or any other existing genus of Serpent. The sloping sides of the neural arch above the zygapophysial ridge are more concave than in *Python*, and so resemble those parts in *Coluber* and *Hydrus*. The latter genus (fig. 33) and *Crotalus* (fig. 36) most resemble *Palæophis* in the proportions of the neural spine (*n s*); this part, however, in *Palæophis* differs from that of *Hydrus* in having its base coextensive with the supporting arch, springing up from the fore-part of the zygosphene, whilst the zygosphene entirely projects forwards, clear of the base of the spine in *Hydrus*, as in *Python*, *Coluber*, and *Naja*; but in *Crotalus* the base of the spine has the same antero-posterior extent as in *Palæophis*. The neural spine had been more or less fractured in every detached specimen of the brittle crumbling vertebræ of the *Palæophis Typhœus* from the Bracklesham Clay;—only one specimen, which I carefully worked out in relief, from a mass of matrix, after imparting some of its original tenacity to the substance of the bone, afforded a true idea of the peculiar character of these Ophidian vertebræ, which is afforded by the great height of the neural spine, as shown in figure 44.

Of the Hypapophysis.—Some idea of the nature and importance of this process may be formed by its constancy and range of variation in the Ophidian Reptiles, and especially by its degree of development in the genus *Crotalus* (Plate II., fig. 36–39, *hy*), which is maintained throughout the vertebræ of the trunk. It is present, under various modifications, in certain regions of the vertebral column, in most other reptiles; as likewise in Birds, and in a certain proportion of the mammalian class.

Cotter seems first to have noticed this process in the Hare, and subsequent comparative anatomists have pointed out this peculiarity of certain lumbar vertebræ of the *Leporidæ*, as 'an inferior spinous process.' It is not, however, the inferior 'homotype,' or answerable part on the lower or ventral aspect of the vertebra, to the 'spinous process' above; this is properly repeated by a true inferior or hæmal spine developed from the apex of the arch on the ventral aspect (Plate LIII., fig. 60, *h*, *h s*), which arch answers to the neural arch (*ib. n*, *n s*) on the dorsal aspect of the vertebra. The hypapophysis is a process commonly exogenous and always developed from the body of the vertebra, not from the hæmal arch. Cuvier applies the same term 'épine inférieure' to the exogenous processes from the centrum of the vertebræ of the Rattlesnake and Hare, and to the autogenous hæmal spines in Fishes.

In the Hare (*Lepus timidus*) the hypapophysis appears as a ridge in the posterior dorsal vertebræ, is developed into a process in the last dorsal, rapidly increases in length in the first and second lumbar vertebræ, inclines forwards in the third, and is suddenly reduced again to a mere ridge in the fourth and following lumbars.

In the Rabbit (*Lepus caniculus*) it is a low ridge in the posterior dorsals: the ridge is produced into a low angle in the first lumbar; becomes a longer and more slender process in the second lumbar; inclines forwards at a more acute angle in the third lumbar than in the Hare; and subsides to a mere ridge in the succeeding lumbars. There is a short hypapophysis from the middle of the inferior part of the ring of the atlas, in both the Hare and Rabbit; and in most other Rodents. In the Guinea-pig (*Cavia porcellus*) a hypapophysis is developed from beneath both atlas and dentata.

The hypapophysial ridge is strongly marked upon the lumbar vertebræ in the genus *Dasyprocta*: but the most constant location of the inferior exogenous processes from the centrum is in the caudal region, where they are frequently associated with a hæmal arch and spine.

In the Cape Hare or Jerboa (*Helamys capensis*) the hypapophysis, in the middle caudal vertebræ, arises from the fore-part of the under surface of the centrum, and immediately bifurcates: in some of the vertebræ anterior to these, a hæmal arch is attached to the ends of the bifurcate hypapophysis: towards the end of the tail the hypapophysis appears as two short diverging processes.

The modification which consists in a single process becoming double, affects most of the exogenous processes in certain parts of the vertebral column in the vertebrate series. We have already seen that the metapophysis is double in the anterior

thoracic vertebræ of the Ornithorhynchus: the diapophysis, which is single in the trunk, not unfrequently becomes double in the tail; and the hypapophysis commonly undergoes the same modification in that region.

In the Australian Water-rat (*Hydromys chrysogaster*) there are two pairs of hypapophyses in the fifth to the tenth caudal vertebræ inclusive, in most of which the anterior pair, homologous with that in *Helamys*, are longer than the posterior pair. But in such an exceptional instance it is not necessary to multiply names for the superadded or rather subdivided processes; the purposes of correct and definite description being sufficiently answered, by distinguishing under the name '*hypapophysis*' those, commonly exogenous, inferior processes of the centrum, whether single or in pairs, from the autogenous hæmal arches and spines, with which they have usually been confounded.

In the anterior thoracic and the cervical region of Crocodiles and Lizards the hypapophysis is constant and commonly single, but I have met with fossil vertebræ of a Crocodile in which the cervical hypapophysis was divided in the median plane, so as to form a pair*. The basioccipital sends down a pair of hypapophyses in the *Iguana* and *Mosasaurus*†, as likewise in Ruminants. This bifid modification is more commonly found in the caudal vertebræ of Reptiles, as it is in Mammals; but in the *Iguanodon* some of the caudal vertebræ have the hypapophysis single, where, *e. g.* it supports the coalesced bases of the hæmapophyses of such vertebræ.

In the Saw-toothed Seal (*Leptonyx serridens*) the cervical and anterior dorsal vertebræ have a hypapophysial ridge, which, in the latter, is produced into a tuberosity: the lumbar vertebræ are characterized by a pair of hypapophyses from near the hinder end of the centrum.

In the *Phoca grænlandica* the strong hypapophysial ridge of the lumbar vertebræ also divides into two tuberous processes. These processes indicate the great development of the anterior vertebral muscles, *e. g.* the 'longi colli' and 'psoæ'; and relate to the important share which the vertebræ and muscles of the trunk take in the locomotion of the Seal-tribe, especially when on dry land, where they may be truly called 'gasteropods,' in respect of their peculiar mode of progression.

The hypapophysis is pretty constantly developed in the cervical region of the Ungulate quadrupeds, especially in the artiodactyle division. It is single in the atlas of the Hippopotamus, but divides into two ridges, diverging as they extend backwards, and terminating in tuberosities in the second and third cervical vertebræ: in the fourth they begin to subside, and disappear in the fifth cervical vertebra. A pair of hypapophysial tubercles reappear in the third lumbar vertebra of the Hippopotamus.

In the great extinct Irish Deer (*Megaceros Hibernicus*) the hypapophysis is a

* On Fossil Reptiles from New Jersey, Quarterly Journal of the Geological Society, November 1849, p. 380, pl. 10. fig. 2.

† *Ib.* pl. 10. fig. 5.

strong single ridge developed into a tuberosity in most of the cervical vertebræ. The atlas has a hypapophysis in most Ruminants.

In the Musk-deer the dentata has a sharp hypapophysial ridge, extending from below the base of the odontoid to beyond its posterior surface, where it underlaps the next vertebra. A similar ridge, ending in a backwardly produced process, is developed from the two succeeding cervicals; beyond which the hypapophysis gradually subsides to the seventh cervical.

In the Camel (*Camelus bactrianus*) the atlas has no hypapophysis; but this commences as a ridge from the hinder half of the centrum of the dentata, expands as it proceeds backwards, and divides into two tubercles. In the third and fourth cervicals it is principally represented by the pair of posterior tubercles, which subside in the fifth cervical vertebra.

The dentata of the Giraffe has a strong hypapophysial ridge from near the forepart of its body.

In the Monotremes the atlas is remarkable for a modification of the hypapophysis, similar to that which it undergoes in the caudal vertebræ of most Mammals; it forms, *e. g.* a pair of processes which diverge from one another: in the succeeding cervicals the hypapophysis is single.

As the aim of the present communication is to demonstrate the individuality and distinctness of the exogenous processes, and their title to distinct names, the modifications of each in any given class will not be traced further than appears requisite to attain the proposed end: and I shall conclude the present notice of the exogenous processes in the Mammalian class, by applying the principles and nomenclature above defined to the description of some caudal vertebræ.

Plate LIII., fig. 60, is the first caudal vertebra of the *Myrmecophaga jubata*, in which the hæmapophyses *h h* are superadded and articulated to the hypapophyses *hy hy*, forming a hæmal arch: in the succeeding vertebræ the hæmapophyses coalesce at their lower ends, and develope a true inferior or hæmal spine. *n, n*, are the neurapophyses; *n s*, the neural spine; *z*, the zygapophyses, here coexisting with *m* the metapophyses. The transverse process is long and single on each side, and consists of an exogenous base or 'diapophysis,' *d*, and an anchylosed 'pleurapophysis,' *pl*.

Fig. 61, A, is an upper view of the seventh caudal vertebra of a Beaver (*Castor fiber*), where *n n* are the neurapophyses, which have ceased to coalesce with each other and complete the neural arch: *m m* are the metapophyses; *d d* are the anterior diapophyses, and *d' d'* the posterior diapophyses. Fig. 61, B, gives an under view of *d d* and *d' d'*, and of the hypapophyses, *h h*. Fig. 61, C, is the front view of the same vertebra, showing *n*, *m m*, *d d*, and *hy*.

Fig. 62, A, B, C, is the ninth caudal vertebra of the Kangaroo; *n' n'* are the neurapophyses, which, coalescing at *n*, form the neural arch, now reduced to its smallest dimensions: *m, m* are the metapophyses; all trace of zygapophyses or articular processes has now disappeared from both the neur- and met-apophyses: *d d* are

the anterior, and *d' d'* the posterior diapophyses. These are, also, shown in the view of the under surface of the same vertebra, fig. 62, B, in which the hypapophyses *hy*, *hy* are figured. Fig 62, C, gives an anterior view of the centrum with *m m* the metapophyses, *d d* the anterior diapophyses, and *hy* the hypapophyses, diverging from it, as exogenous processes. The neurapophyses *n n* and the posterior diapophyses *d' d'* are also seen.

Many excellent figures of the caudal vertebræ of different species of Mammalia are given in the 'Ostéographie' of M. De Blainville, the characters of which may be readily understood by the above definition of the processes that diverge from them.

In the class *Aves*, the hypapophyses attain their maximum of development and range of variation in the Penguins (*Aptenodytes patachonica*, Plate LII., fig. 48). Single and long on the first four vertebræ of the neck, especially on the dentata, their place is taken by a pair of short processes in the sixth cervical, which processes increase in size in the four following vertebræ, converging in the last (*h h*, C 10) so as almost to complete a bony canal for the carotids: in the eleventh cervical (C 11) they have coalesced again into a single median plate, with its extremity extended both forwards and backwards; this plate decreases in the three following vertebræ. In the next, which may be regarded as the first dorsal, the hypapophysis again appears as a pair of processes (D 1, *h h*); both of these are broad plates, almost coextensive with the under surface of the vertebra and diverging from each other (fig. 49, *hy*). The common base of the diverging plates progressively lengthens in the second (fig. 50) and third dorsals, and the hypapophysis appears in the succeeding ones as a strong vertical plate, simply expanded at its end, and growing somewhat shorter to the last (fig. 51, D 4, *hy*).

In the *Sphæniscus minor*, the first dorsal vertebra, or the fourteenth vertebra from the head, presents three hypapophyses, which appear to be blended together in the next vertebra so as to form the common base of a pair of diverging plates. The hypapophyses in the Penguins have an analogous function to that in the Seals, extending the surface of attachment of the powerful muscles on the ventral aspect of the vertebral column, which so materially aid in the shuffling gasteropodal movements of both kinds of aquatic animals, when on land.

The hypapophysis is well-marked in the anterior cervical vertebræ, especially the dentata, in most birds: it usually reappears, as a pair of processes, from the parapophyses of the lower cervicals; which hypapophyses converge as they approach the dorsal region, and coalesce into a single process in the anterior vertebræ of this region, and in some birds in the lower cervicals.

In the Pelican, the parial hypapophyses coalesce at their lower extremities, and form a hæmal arch for the carotids in many of the lower cervicals.

The anterior trunk-vertebræ in many Siluroids are anchylosed by a hypapophysial development of bone continued from beneath one vertebra to another, and which is sometimes perforated lengthwise by a canal lodging the aorta*. The neck of the

* On the Archetype of the Vertebrate Skeleton, p. 92.

Ichthyosaurus derives additional strength and fixation from hypapophyses autogenously developed, at the inferior interspace between the occiput and atlas, and at those between two or three succeeding cervical vertebræ*. The so-called 'body of the atlas,' in recent Saurians, Birds, Mammals and Man, is the homologue of the first of these subvertebral wedge-bones, and represents only the hypapophysial or lower cortical part of such body. The odontoid process of the second vertebra is the central and chief part of the body of the atlas†, but not the whole body, as Cuvier describes in the fimbriated Tortoise‡.

The most interesting and instructive steps in the transition of this main part of the centrum of the atlas to the modified condition which it presents in Man, and which is signified by the term 'processus odontoïdes' in Human Anatomy, are exemplified in the order *Chelonia*. In the Australian Long-necked Tortoise (*Hydraspis longicollis*, Bell), I find the centrum (*c c*, figs. 57, 58, Plate LII.) completely anchylosed with the neural arch (*n*) of the atlas, with the exception of the hypapophysis (*hy*), which is autogenous in this vertebra and preserves its distinctness. In the Soft-turtles (*Trionyx*) (*ib.* figs. 52, 53 and 54) both the centrum and hypapophysis of the atlas maintain their primitive individuality: the centrum (*c*) is relatively shorter than in the *Chelys* and *Hydraspis*; it presents a subcubical form; has a small subcircular surface (*c*, fig. 55) on its lower and fore-part for articulating with the hypapophysis; above this it expands transversely and presents a convex articular surface adapted laterally to the bases of the neurapophyses, and, mesially, completing the cup for the occipital condyle, and filling up the vacancy shown at *c*, fig. 53: the back-part presents a concave surface (*c*, fig. 56) which articulates with the anterior convexity of the centrum of the dentata. The neurapophyses of the atlas (fig. 54, *n*) rest partly upon their proper centrum (*c*), partly upon its hypapophysis (*hy*): their bases nearly meet above the centrum, and their apices quite meet, uniting by suture above the neural canal (fig. 53). The degradation of the centrum of the atlas to the odontoid process of the axis is completed by its confluence with the centrum of the latter vertebra, and by the total transference of the neurapophyses of the atlas to the hypapophysis. Fishes, Batrachians and the extinct Enaliosaurians, manifest the typical state of the atlas, the Warm-blooded Vertebrates its aberrant state, the transitional modifications are traceable in the Chelonian, Lacertilian and Ophidian reptiles§.

When the hypapophysis attains so complex a form as to be perforated and thus to constitute an arch protecting a vascular canal, it may readily be mistaken for something more than the mere *analogue* of the 'hæmal arch' of the typical vertebra; and, in my earlier investigations on that subject, I was led into the error‖ of regarding it

* Sir Philip de Malpas Grey Egerton, Bart., F.R.S., in Geol. Trans. 2nd Ser. vol. v. p. 187, pl. 14.

† Annals of Nat. History, vol. xx. 1847, p. 92.

‡ Ossemens Fossiles, tom. v. pt. ii. p. 207.

§ See the description and figures of the atlas and axis of the Python in my 'History of British Fossil Reptiles,' Part III. p. 136, pl. 3 (*Ophidia*), figs. 38—40.

‖ Geol. Transactions, vol. v. 2nd Ser. p. 519, pl. 44, figs. 2 and 3.

as the *homologue* of that arch. The perforated hæmal arch in the cervical vertebræ of the Pelican, and the parallel descending processes similarly protecting the caudal artery in the Python (Plate LI., fig. 28), are alike exogenous growths from the centrum, and are serially homologous with the hypapophysis, not with the hæmapophysis. The total absence of true ossified hæmapophyses throughout the trunk and tail vertebræ, is a striking characteristic of the Ophidian reptiles*, as it is of fishes.

The determination of the homology of the autogenous hæmal arches in the tail carries with it that of the question of their relation to the inferior and sometimes perforated exogenous processes of the centrum in more advanced parts of the vertebral column.

The vast difference in point of size between the so-called 'chevron-bone' of the Crocodile and the bony arch formed by the sternal and vertebral ribs of a thoracic vertebra, seems at first sight to discountenance the idea of any homology between them. But the difference is not greater than that between the neural arch of the parietal segment of the skull in Man, and the arch formed above the middle dorsal vertebra: yet I think it may be now assumed as a settled point in anatomy, that those arches are homotypes or serial homologues. The difference in point of size is, indeed, much less in the cold-blooded and small-brained Vertebrates than in Man; but the transition from the abdominal to the caudal hæmal arches in fishes is much more gradual than that of the cranial to the abdominal neural arches in the same class, whilst the similarity of the neural and hæmal arches to one another is so close throughout the region of the tail, as to impress the mind with a conviction that they are alike the result of modified annular elements, in short, 'vertical homotypes.' Cuvier, accordingly, designates them by the same descriptive phrases†.

The general proposition that the 'chevron bones' or 'inferior annular part and spine' of the vertebræ in the tail of fishes, are modifications of the hæmal arches in the trunk, may be deemed to be unassailable. But the hæmal arch is a complex whole, and may include diapophyses or parapophyses with pleurapophyses, hæmapophyses and hæmal spine. In the abdomen of fishes the bony arch is incomplete below, and in its most complex state consists only of parapophyses and pleurapophyses. With the exception of the clavicular and pubic arches, bony hæmapophyses are developed in no part of the trunk of fishes. When, therefore, we rigidly scrutinize the composition of the hæmal arches in the tail, we find them composed of parts quite distinct from true hæmapophyses, and also composed of different vertebral elements in different species of fishes, a fact which adds to the proof of the essential serial homology of those arches in the tail with the hæmal arches in the trunk. In many osseous fishes the hæmal arches in the tail are formed by the gradual bending down and coalescence of the parapophyses: in some the union

* Cartilaginous appendages to the pleurapophyses in Serpents may be viewed as rudimental 'hæmapophyses.'

† "Les caudales ont une partie annulaire et une apophyse épineuse en dessus et en dessous." Cuvier, 'Leçons d'Anat. Comp.' Ed. 1835, t. i. p. 223.

is effected by a transverse bony bar prior to the actual confluence of those processes, and pleurapophyses continue to be attached to some of the parapophyses after their transmutation into the hæmal arches, as *e.g.* in the Tunny, the Dory and the Salmon*. Nevertheless, in the Salamandroid *Polypterus* and *Lepidosteus*†, and in the nearly allied *Protopterus* and *Lepidosiren*, the true pleurapophyses gradually bend down and unite to form the hæmal arches in the tail; whilst in the *Amia calva* the pleurapophyses coalesce to form the inferior spine, continued from the hæmal arch, formed by the coalesced parapophyses‡. In the Cod-tribe (*Gádidæ*) the progressively reduced pleurapophyses coalesce with the parapophyses to form the hæmal arches. In Ophidians, as in Fishes, the transition from the trunk to the tail is not interrupted by the modification of any of the vertebral segments to form a 'pelvis.' The ribs are supported by diapophyses, with which they become confluent in the tail, where some of the pleurapophyses are singularly modified, bifurcate or thickened at the extremity, but never bent down and united together to form the hæmal arch: as there are no bony hæmapophyses completing the hæmal arch in the trunk, the succeeding segments of the tail retain, or have been influenced by, the same modification of vertebral development, and the analogue of the hæmal arch is formed, as I have before observed, by a pair of parallel exogenous hypapophyses (Plate LII., fig. 28, *hy*), and is open below, like the incomplete hæmal arches throughout the trunk. In Lizards and Crocodiles a new vertebral element appears to be introduced to complete the hæmal arch in the chest; it is, however, only a repetition of an element which, under a more developed and modified form, completes the hæmal arches in the head, in both fishes and reptiles, viz. the 'hæmapophysis:' this element is called 'upper' and 'lower jaw' and 'horn of the hyoid' in the head, 'sternal rib' or 'cartilage of the rib' in the chest: the same element is retained to form the 'pelvis,' where it is called 'pubis' and 'ischium'; and it is continued as a distinct pair of bones in the beginning of the tail in the Great Ant-eater (Plate LIII., fig. 60, *h h*), and throughout a great part of the tail in the *Ichthyosaurus* and *Plesiosaurus*; whilst the pair of hæmapophyses become confluent at their distal ends, and form the so-called chevron-bones in the Crocodile, and most other Saurians. Under all these modifications the hæmapophyses coexist with pleurapophyses in the thorax, the pelvis and the tail; only in the latter region the pleurapophyses become anchylosed to the diapophyses or to the centrum, and stand out as 'transverse processes' (fig. 60, *d*, *pl*). The sudden diminution in the size of the hæmal arch in the tail, as contrasted with its expanse in the pelvis, is apt at first to excite some doubt as to the serial homology of the 'chevron bones' with the 'ischia' and 'pubes'; but when we compare them in a species, as *e.g.* the Mole (*Talpa*), in which the hæmal arch in the pelvis is limited to

* Cuv. l. c. p. 224.

† Müller, Ueber den Bau und die Grenze der Ganoiden, und über das naturliche System der Fische, 1844.

‡ "Omnes arcus inferiores spinam gerunt e costis concretis formatam."—Franke, 'Nonnulla ad Amiam Calvam accuratius cognoscendam.'

the same protective functions as it performs in the tail of other quadrupeds, we find it reduced to the same small area or vertical dimensions, and applied close to the centrum, and the difficulty disappears. As the hypapophyses usually coexist with and support the hæmapophyses, when these are present in the tail of a reptile or mammal, they cannot be 'homologous' in any sense, and the hypapophyses are only 'analogous' when they happen to be perforated by the carotid arteries, as in the cervical vertebræ of some birds, or when they include the caudal artery and vein, as in the tail of Serpents. No one can confound the parapophyses or inferior transverse processes (Plate LII., fig. 52, *p*) with the hypapophyses (ib. *h*), who has observed them as they coexist in the cervical vertebræ of a Crocodile; and the hypapophyses are equally demonstrated to be distinct from the hæmapophyses by the presence of both in the tail of the *Myrmecophaga jubata* (Plate LIII., fig. 60, *hy*, *h*).

Description of the Plates.

In each figure the same processes are indicated by the same letters.

m. Metapophysis.
a. Anapophysis.
z. Zygapophysis.
d. Diapophysis.
p. Parapophysis.
hy. Hypapophysis.

PLATE XLIV.

Fig. 1. The three last dorsal and three first lumbar vertebræ of a Frenchman. Side view.

Fig. 2. The three last dorsal, first and fourth lumbar vertebræ of the same, from the dorsal aspect.

PLATE XLV.

Fig. 3. The three last dorsal and four first lumbar vertebræ of the Brown Monkey (*Macacus nemestrinus*), from the side.

Fig. 4. The tenth and eleventh dorsal vertebræ of the same, from the dorsal aspect.

Fig. 5. The five last dorsal and first lumbar vertebræ of the Puma (*Felis concolor*). Side view.

Fig. 6. The ninth dorsal vertebra of the same, from the dorsal aspect.

Fig. 7. The eleventh dorsal vertebra of the same, from the same aspect.

PLATE XLVI.

Fig. 8. An oblique upper view of the tenth, eleventh and twelfth dorsal vertebræ, to show the distinctness of the metapophysis (*m*) from the zygapophysis (z) and the diapophysis (*d*), and the gradual change of position of the metapophysis (*Leptonyx serridens*).

Fig. 9. The last five dorsal and the lumbar vertebræ of the *Helamys capensis*. Side view.

Fig 10. The ninth and tenth dorsal with the second and third lumbar vertebræ of the *Lagotis Cuvieri*. Side view.

PLATE XLVII.

Fig. 11. The last seven dorsal vertebræ of the Capybara (*Hydrochærus Capybara*), from the dorsal aspect, chiefly to demonstrate the distinction between the metapophyses (*m*) and the prozygapophyses (*z*).

Fig. 12. The twelfth, fourteenth and fifteenth dorsal vertebræ, and the first and second lumbar vertebræ of the Wombat (*Phascolomys Wombatus*), from the dorsal aspect, to show the serial homology of the diapophysis (*d*), and the partially anchylosed rib (*pl*) of the first lumbar vertebra (L_1).

Fig. 13. The last four dorsal vertebræ, the lumbar vertebræ and sacrum of the *Macropus Parryi*, side view, showing the abrupt commencement of the metapophyses (*m*) on the twelfth dorsal, the comparatively small size of the anapophyses (*a*); and the distinctness and persistency of the diapophyses (*d*) in the posterior dorsal vertebræ.

PLATE XLVIII.

Fig. 14. The last eight dorsal vertebræ and first lumbar vertebra of the Musk-deer (*Moschus moschiferus*), showing the progressive development and change of position of the metapophysis (*m*); the serial homology of the dorsal and lumbar diapophyses (*d*); and the anomalous development of the pleurapophysis (*pl*) of the fourteenth vertebra from the neck. Side view.

Fig. 15. The fourth, sixth, seventh, twelfth and fourteenth dorsal vertebræ of the *Delphinus Tursio*, from the dorsal aspect, showing the gradual supercession of the prozygapophyses (*z*), by the metapophyses (*m*).

Fig. 16. The eleven anterior dorsal vertebræ of the *Delphinus Delphis*, showing the sudden beginning and progressive ascent of the *metapophysis* (*m*) in that part of the vertebral column; also the progressive elongation of the diapophysis (*d*).

PLATE XLIX.

Fig. 17. The last two dorsal and first lumbar vertebræ of the Three-toed Sloth (*Bradypus tridactylus*), from the dorsal aspect.

Fig. 17*. The last six cervical and first two dorsal vertebræ of the *Echidna hystrix*, showing the distinct short and thick pleurapophyses (*pl*) of the neck, still articulating with the diapophysis (*d*) and parapophysis (*p*).

The body of the atlas (*c a*) has not yet coalesced, as the 'odontoid process,' with the dentata.

Fig. 18. The eleven dorsal vertebræ and four of the lumbar vertebræ of the long-tailed Armadillo (*Dasypus longicaudus*), from the dorsal aspect, showing the abrupt commencement and great development of the metapophyses, *m*.

Fig. 19. The twelve dorsal vertebræ and three lumbar vertebræ of the Three-banded Armadillo (*Dasypus tricinctus*); a side view, showing the successive superaddition to the diapophyses (*d*), of metapophyses (*m*), anapophyses (*a*) and parapophyses (*p*); and the progressive development of the anapophyses and metapophyses.

Fig. 20. The last five dorsal and first three lumbar vertebræ of the Great Ant-eater (*Myrmecophaga jubata*), showing the progressive development of the metapophyses (*m*) and anapophyses (*a a'*), together with the diapophyses (*d*) and parapophyses (*p*). The pleurapophyses (*pl*) are left in outline. A side view.

PLATE L.

Fig. 21. Front and back views of the eleventh dorsal vertebra of the same Ant-eater, showing—

p. The parapophysis; *p a*, its articular surface.

d. The diapophysis; *d*, *pl*, its articular surface for the pleurapophysis.

m. The metapophysis; *m a*, its articular surface for the anapophysis.

a. The anapophysis; *a p*, its articular surface for the parapophysis; *a m*, its articular surface for the metapophysis.

z. The anterior zygapophyses, with articular surfaces looking upwards.

z'. The posterior zygapophyses, with articular surfaces looking downwards.

pl. Head and tubercle of pleurapophysis.

Fig. 22. Front and back views of the last (15th) dorsal vertebræ; which in addition to the parts specified above, shows—

mz. The articular surface of the metapophysis for the modified or newly developed posterior zygapophysis (*z'*).

zm. The articular surface of the posterior zygapophysis for the metapophysis.

The ordinary zygapophyses are almost suppressed.

Fig. 23. Front and back views of the third lumbar vertebra, which besides the parts specified above, shows—

da. The articular surface on the diapophysis for the anapophysis.

ad. The articular surface on the anapophysis for the diapophysis.

PLATE LI.

Fig. 24. Side view of a middle trunk-vertebra of the *Python Sebæ*.

Fig. 25. Front view of ditto.

Fig. 26. Back view of a middle trunk-vertebra of the *Python Sebæ*.
Fig. 27. Under view of ditto.
Fig. 27′. Vertebral ribs or 'pleurapophyses' of the *Python*.
Fig. 28. Front view of a middle tail-vertebra of ditto.
Fig. 29. Side view of a middle trunk-vertebra of the *Coluber elaphus*.
Fig. 30. Front view of ditto, with one of the ribs.
Fig. 31. Back view of ditto.
Fig. 32. Under view of ditto.
Fig. 33. Side view of a middle trunk-vertebra of the *Hydrus bicolor*.
Fig. 34. Side view of a middle trunk-vertebra of *Naja*.
Fig. 35. Back view of ditto.
Fig. 36. Side view of two middle trunk-vertebræ of the *Crotalus durissus*.
Fig. 37. Back view of ditto.
Fig. 38. Front view of ditto.
Fig. 39. Under view of ditto.
Fig. 40. Side view of a middle trunk-vertebra of the *Iguana*.
Fig. 41. Front view of ditto.
Fig. 42. Back view of ditto.
Fig. 43. Under view of ditto.
Fig. 44. Side view of a middle trunk-vertebra of the *Palæophis typhæus*.
Fig. 45. Front view of ditto.
Fig. 46. Back view of ditto.
Fig. 47. Under view of ditto.

The same letters indicate the same parts in each figure:—

c. The centrum.
d. The diapophysis, its articular tubercle.
d′. Its inferior production.
d″. Its superior production.
z. The anterior zygapophysis.
z′. The posterior zygapophysis.
ns. The neural spine.
n. The neurapophysis, produced into an angle in *Palæophis*.
pl. The pleurapophysis.
zs. The zygosphene.
za. The zygantrum.
hy. The hypapophysis.

PLATE LII.

Fig. 48. The five last cervical and five first dorsal vertebræ of the Great Penguin (*Aptenodytes patachonica*): an under view, showing the great development and metamorphoses of the hypapophyses (*h*).

Fig. 49. A back view of the first dorsal vertebra, showing the diverging hypapophysial plates (*hy*, *hy*).

Fig. 50. A back view of the second dorsal vertebra, showing the common base of the diverging plates (*hy*, *hy*).

Fig. 51. A front view of the fourth dorsal vertebra, showing the elongated base of the bifurcating hypapophysis (*hy*). In this vertebra the anterior surface of the centrum is simply convex, the posterior surface concave.

Fig. 52. A back view of the third cervical vertebra of a Crocodile, showing the diapophysis (*d*) and the parapophysis (*p*), or the upper and lower transverse processes of the right side; and the same, on the left side, with the co-articulated 'pleurapophysis' (*pl*) or 'hatchet-bone.' *c* is the ball on the back part of the centrum; *n* the neural arch; *ns* the neural spine; *z* is the prozygapophysis; *z'* the zygapophysis; *hy* the hypapophysis.

Fig. 53. Front view of part of the atlas of a Soft-tortoise (*Trionyx Dumerili*, O.). *c* the vacant space left by the absent centrum; *hy* the hypapophysis or 'wedge-bone'; *n* the neural canal; *n'* the anterior articular surface of the neurapophysis; *d* the diapophysis.

Fig. 54. Side view of the atlas of the same *Trionyx*. *c* the centrum; *hy* hypapophysis; *n* neurapophysis, its anterior articular process or prozygapophysis; *z'* zygapophysis.

Fig. 55. Anterior view of the centrum of the atlas, showing *c* its articular surface for the hypapophysis; *c n* its articular surface for its neurapophyses, and for the basioccipital or centrum of the occipital vertebra.

Fig. 56. Side view of the same; *x* the concave articular surface for the centrum of the dentata.

Fig. 57. Front view of the atlas of the Long-necked Tortoise (*Hydraspis longicollis*, Bell).

Fig. 58. Side view of the same vertebra. The centrum *c* ('odontoid process' of Anthropotomy) has coalesced with its neural arch *n*, and the typical character of the atlas, as it is manifested in the class of fishes and in enaliosaurians, is resumed; *hy*, the hypapophysis ('body of the atlas' in Anthropotomy).

Fig. 59. The eighth cervical vertebra of the *Hydraspis longicollis*, which, like the first caudal vertebra of the Crocodiles*, is biconvex; *c* the centrum; *z* the two prozygapophyses; *z'* the postzygapophysis; *d* the two diapophyses.

PLATE LIII.

Fig. 60. A front view of the first caudal vertebra of the Great Ant-eater (*Myrmecophaga jubata*).

c. The centrum.

n, *n*. The neurapophyses.

* Physiological Catalogue of the Museum of the Royal College of Surgeons, vol. i. (1833) p. 53, Prep. No. 250.

ns. The neural spine.
z. The prozygapophyses.
m. The metapophyses.
d. The diapophysial base of the 'transverse process.'
pl. The pleurapophysial apex of the 'transverse process.'
hy. The hypapophyses.
h. The hæmapophyses.

Fig. 61. A. An upper view of the seventh caudal vertebra of a Beaver (*Castor fiber*).
B. An under view of ditto.
C. A front view of ditto.

Fig. 62. A. An upper view of the ninth caudal vertebra of the Great Kangaroo, *Macropus major*.
B. An under view of ditto.
C. A front view of ditto, wanting the terminal epiphysis.

In both specimens *n n* are the neurapophyses, which here become exogenous processes.

m m. The metapophyses.
d. The anterior diapophyses.
d'. The posterior diapophyses.
hy. The hypapophyses.

Figs. 63—66. These figures of the seventh and eighth cervical vertebræ of the *Trionyx Dumerili* have been inserted because the latter (fig. 66) combines two remarkable modifications which are separately presented by the occipital vertebra of the skull in the Frog, and in Man. In the Frog, as in all other batrachians, the occipital vertebra articulates with the succeeding vertebra, called 'atlas,' by a transverse pair of convex condyles, similar to those shown at *c c*, figs. 65 and 66: in the Human subject, as in all mammalia, the centrum of the occipital vertebra (*processus basilaris ossis occipitis*) is flattened and united to that of the atlas (*processus odontoïdes*) by a ligament: the back part of the centrum of the eighth cervical in the *Trionyx* (fig. 66, *c*) is, in like manner, reduced to a depressed plate, and is joined by a ligament to the succeeding centrum, viz. that of the first dorsal vertebra. The last cervical vertebra of the *Trionyx* resembles the first cervical in Man by having no neural spine: it is further remarkable for the peculiar development and form of its anterior (*z*) and posterior (*z'*) zygapophyses. Fig. 64 shows the two concavities at the back part of the centrum of the seventh cervical vertebra, which receive the two convexities at the fore-part of the eighth.

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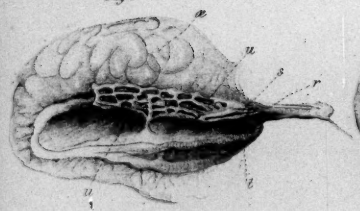
Fig. 5.



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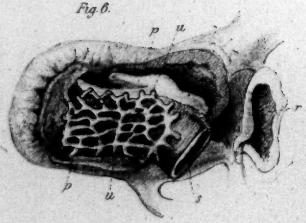

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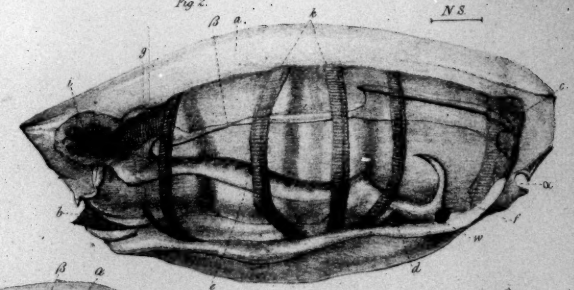

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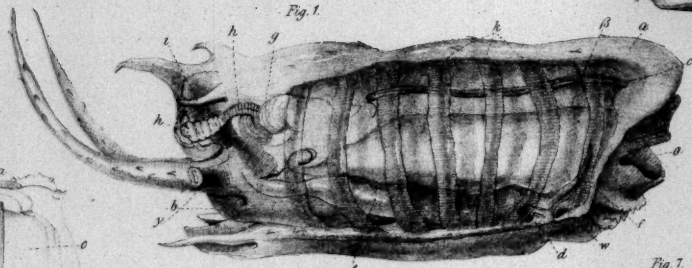

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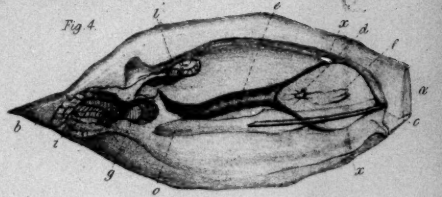

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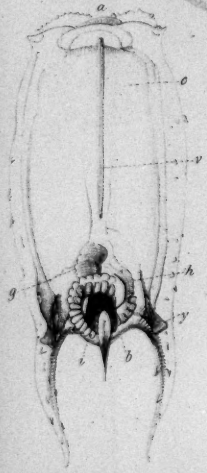

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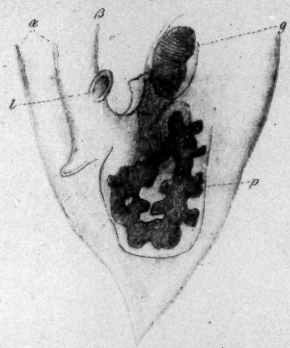

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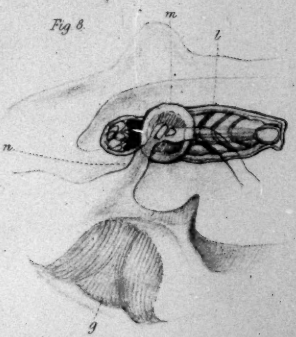

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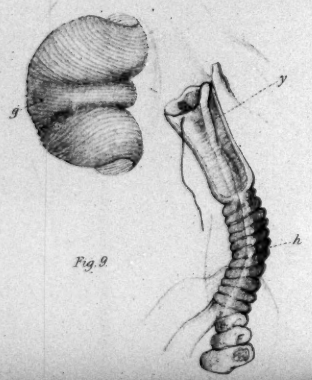

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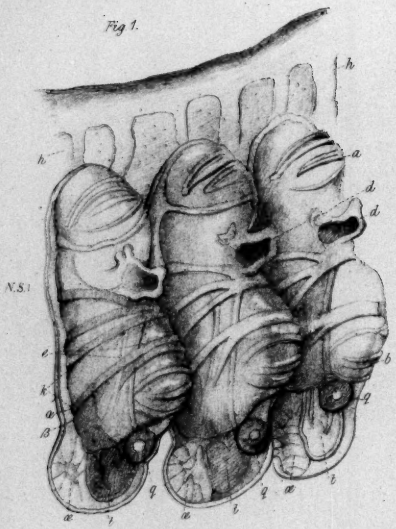

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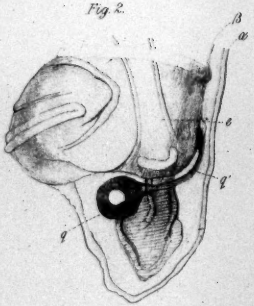

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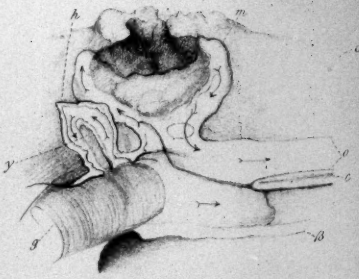

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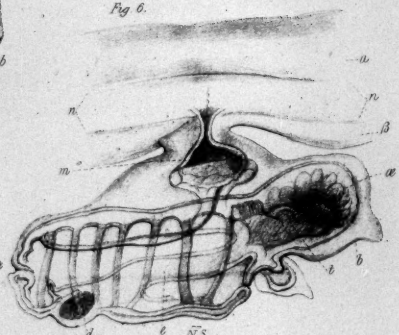
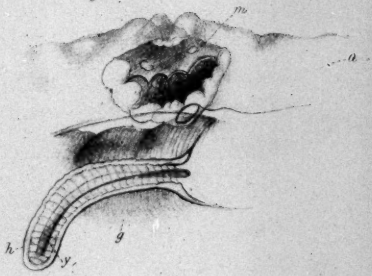


Fig. 3a.



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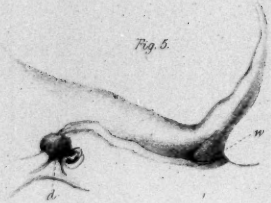

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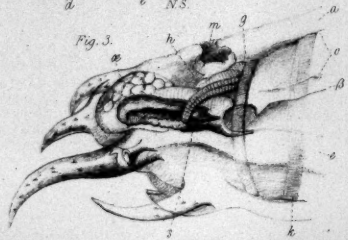
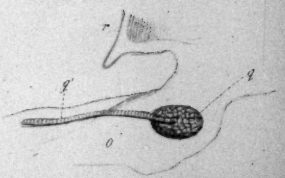


Fig. 7.

Fig. 8.



Ad. nat. delt. T.H.H.

J.Basire, Lith.

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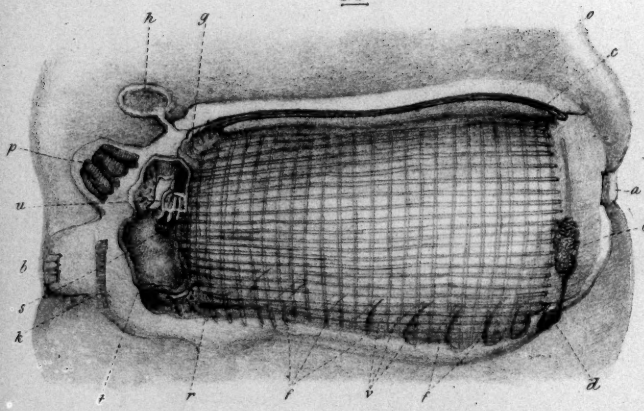

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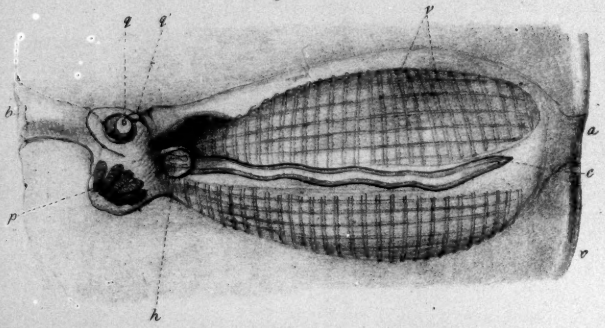

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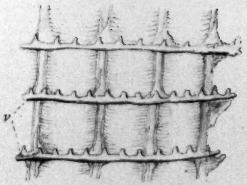

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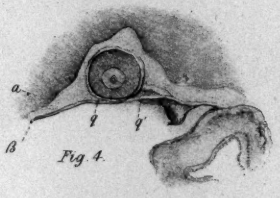

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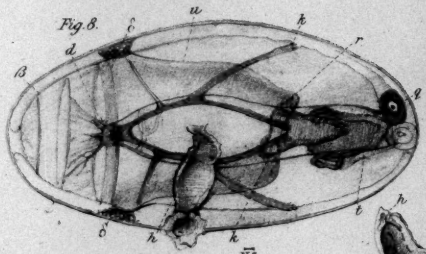

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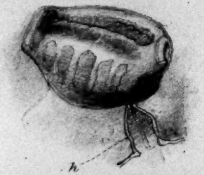

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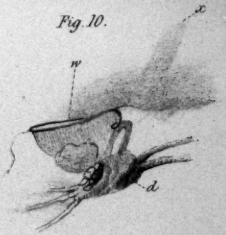

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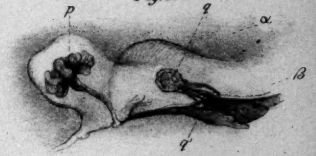
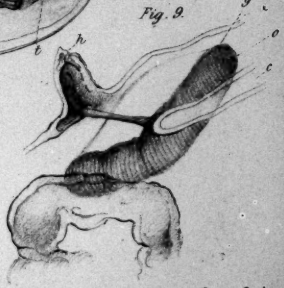


Fig. 6.

Fig. 9.



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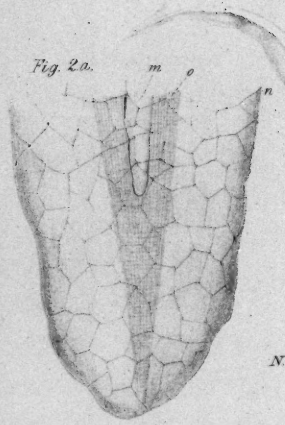
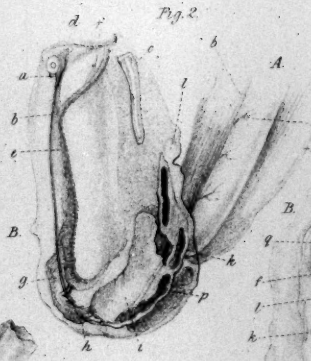
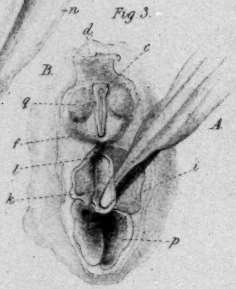
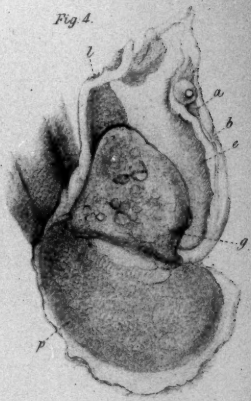
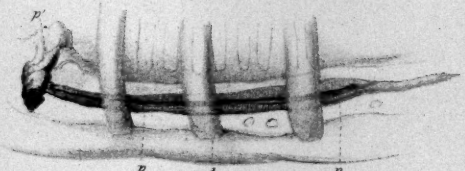
J.Basire, Lith.

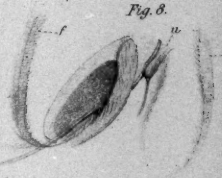


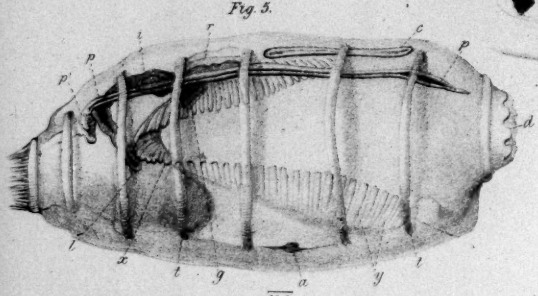


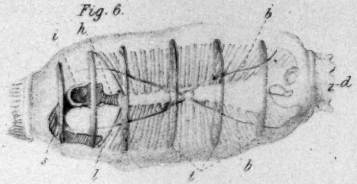












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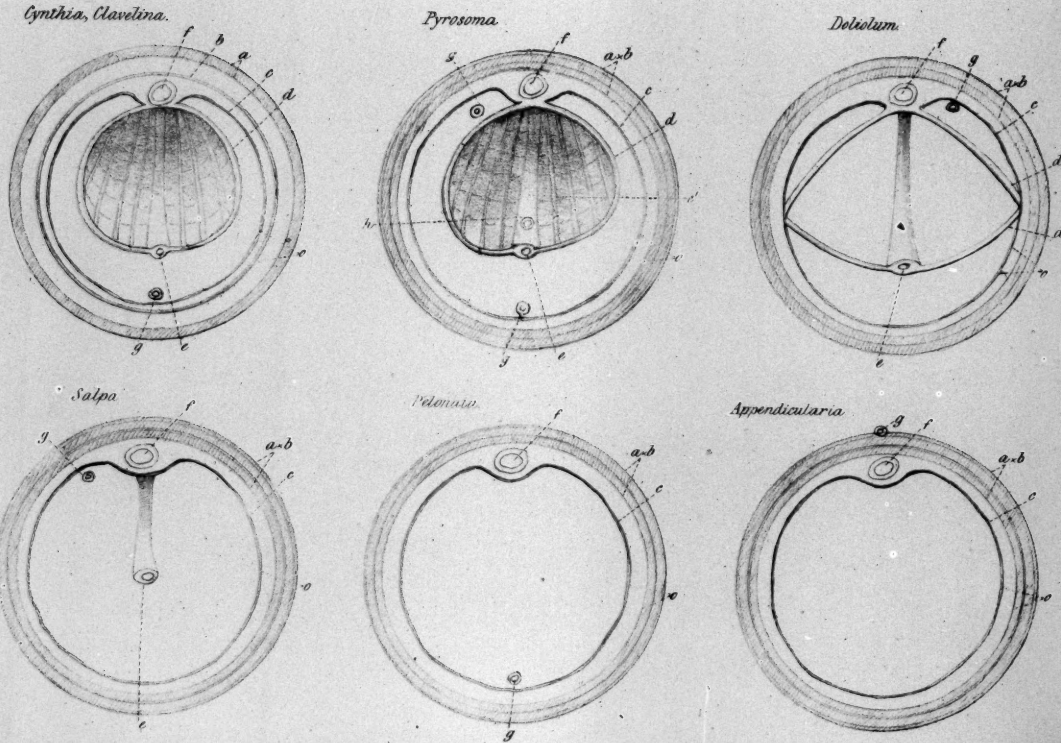

Cynthia, Clavelina.
Pyrosoma.
Doliolum.
Salpa.
Pelonaia.
Appendicularia.
a. Test. b. outer tunic. c. inner tunic. d. branchial sac.
e. epipharyngeal band. e. hypopharyngeal band. f. Thoracic vessel. f. alimentary canal. g. Anus. h. mouth. o. sinus system.

Sectional Diagrams of Tunicata.

The section supposed to be made perpendicularly to the axis of the branchial cavity.

J. Basire, Lith.

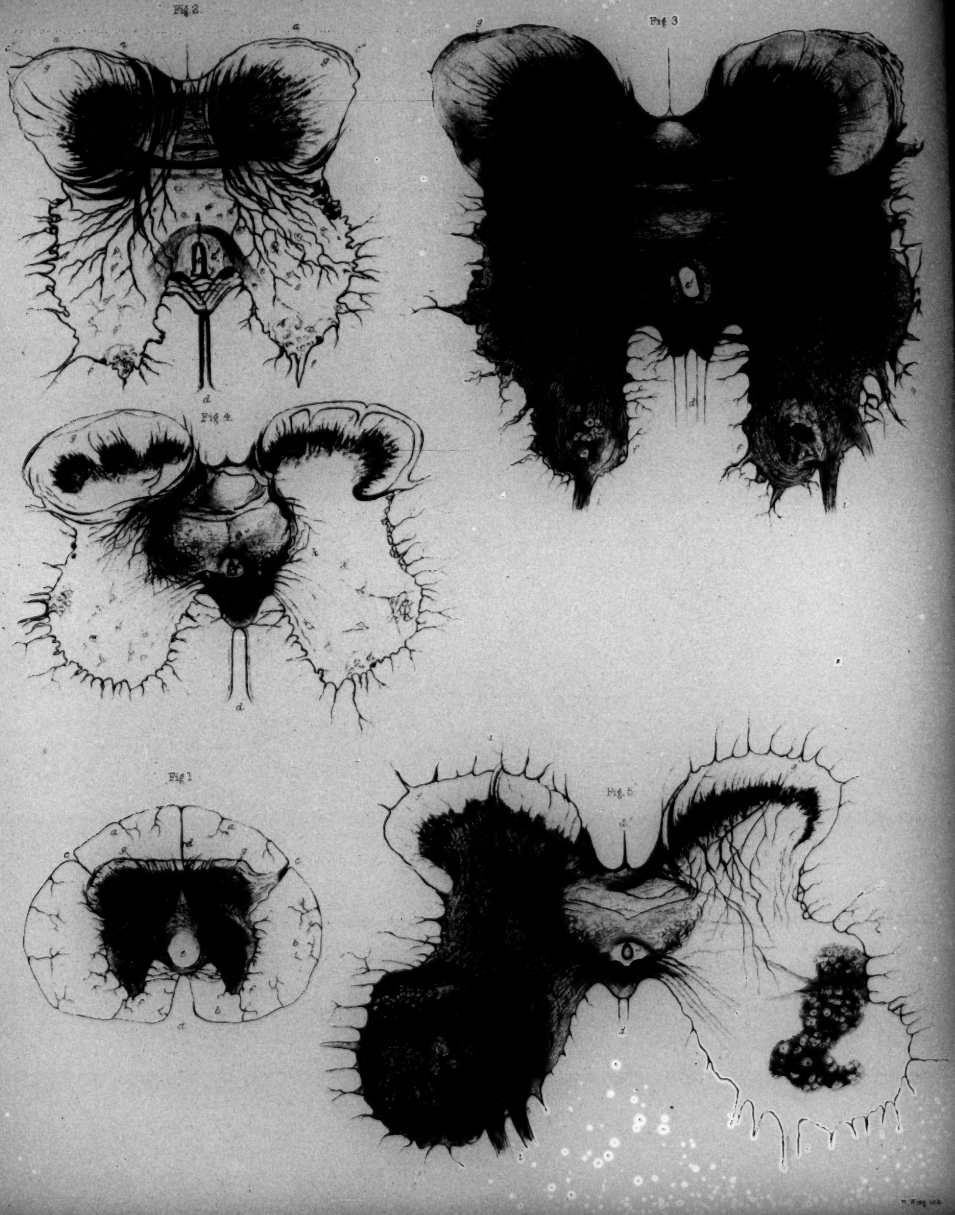
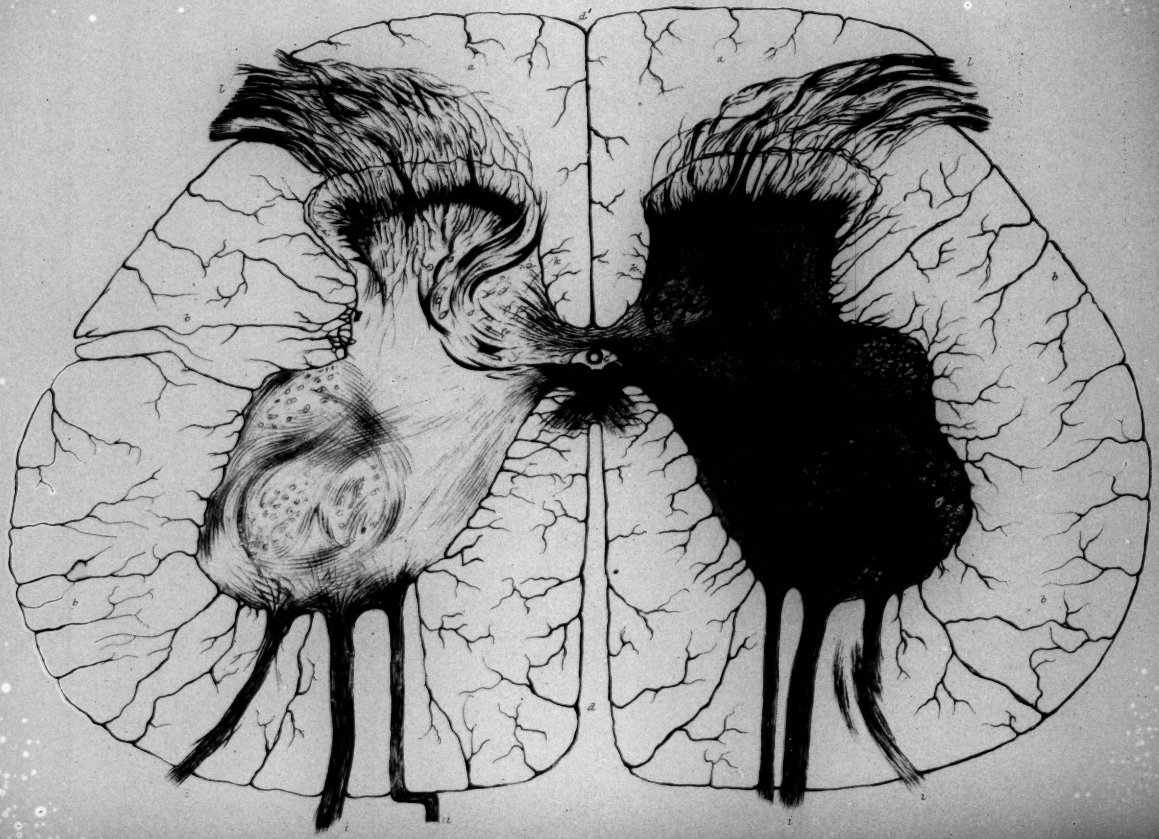


Fig 2.
Fig 3
Fig 4.
Fig 1
Fig 5.

Fig 6.



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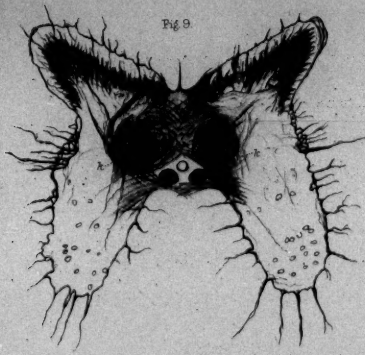

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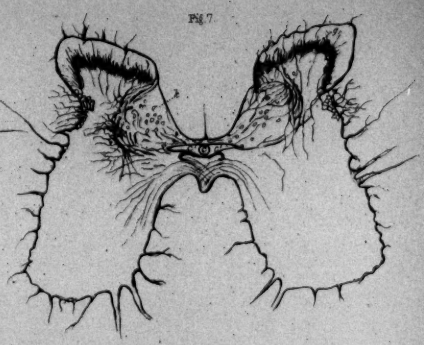

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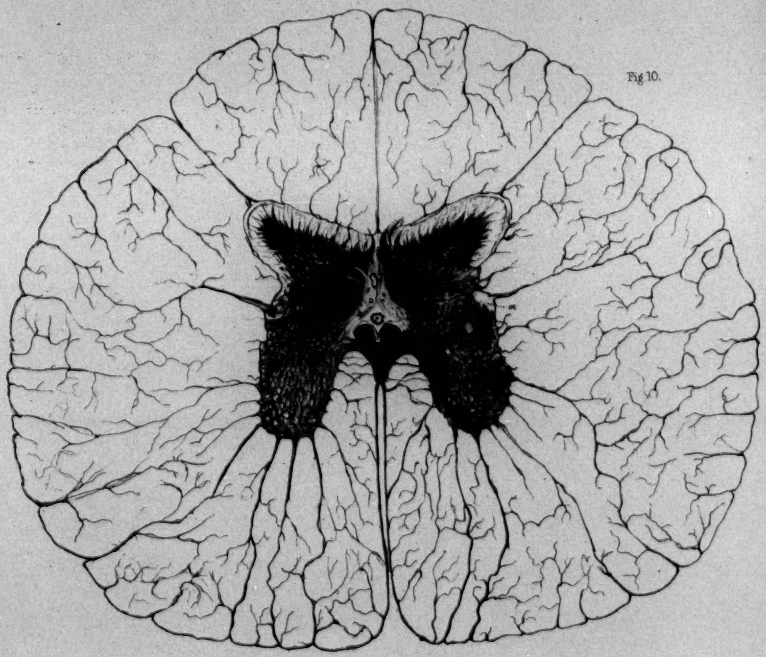

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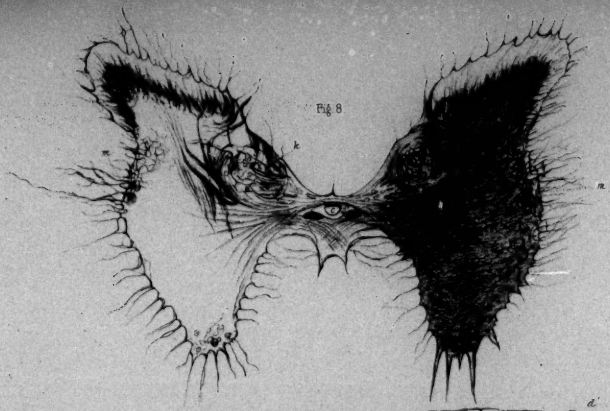

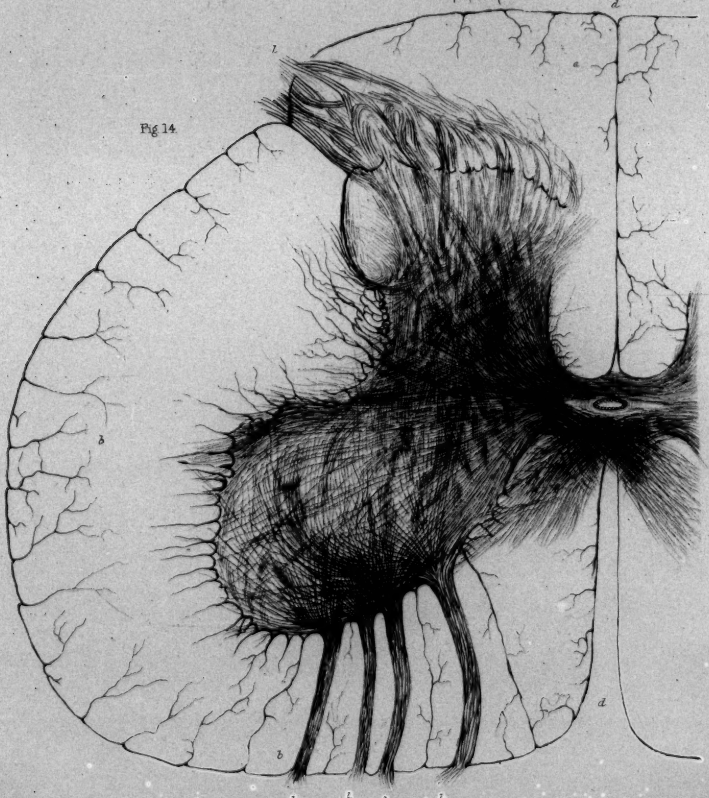

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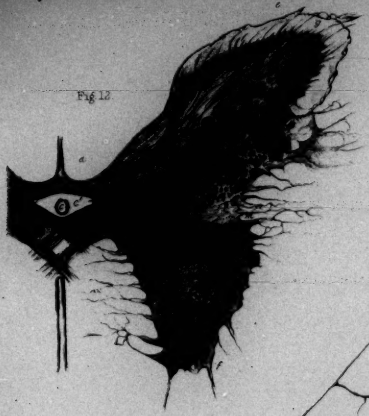
W. Wing, lith.
Ford & George, Lithographers, 5 & Hatton Garden.

Fig. 12.



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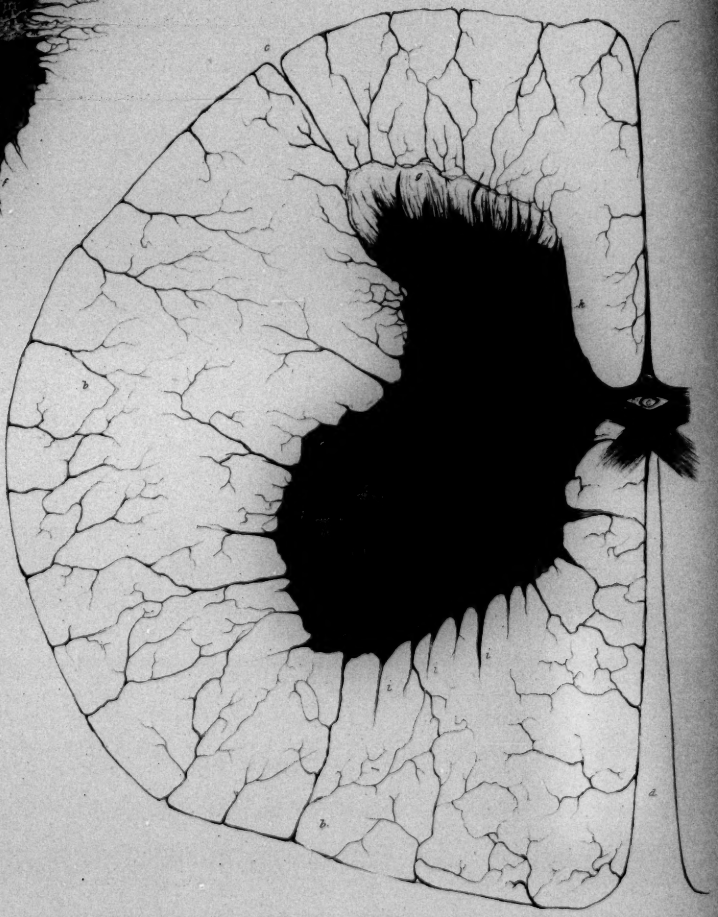
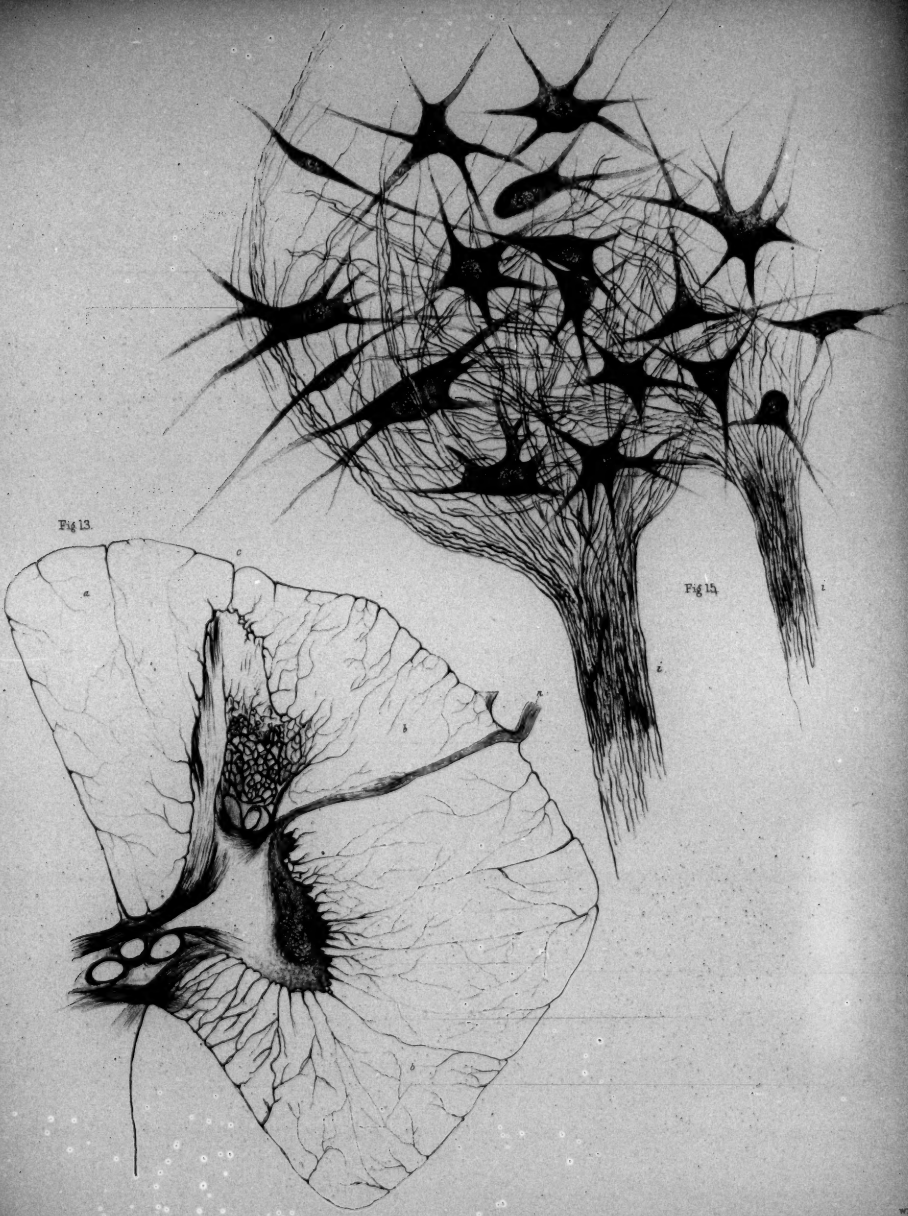


J. L. Clarke del. ad nat.

W. Wood, lith.

Printed by George Lithographers, 64 Hatton Garden.



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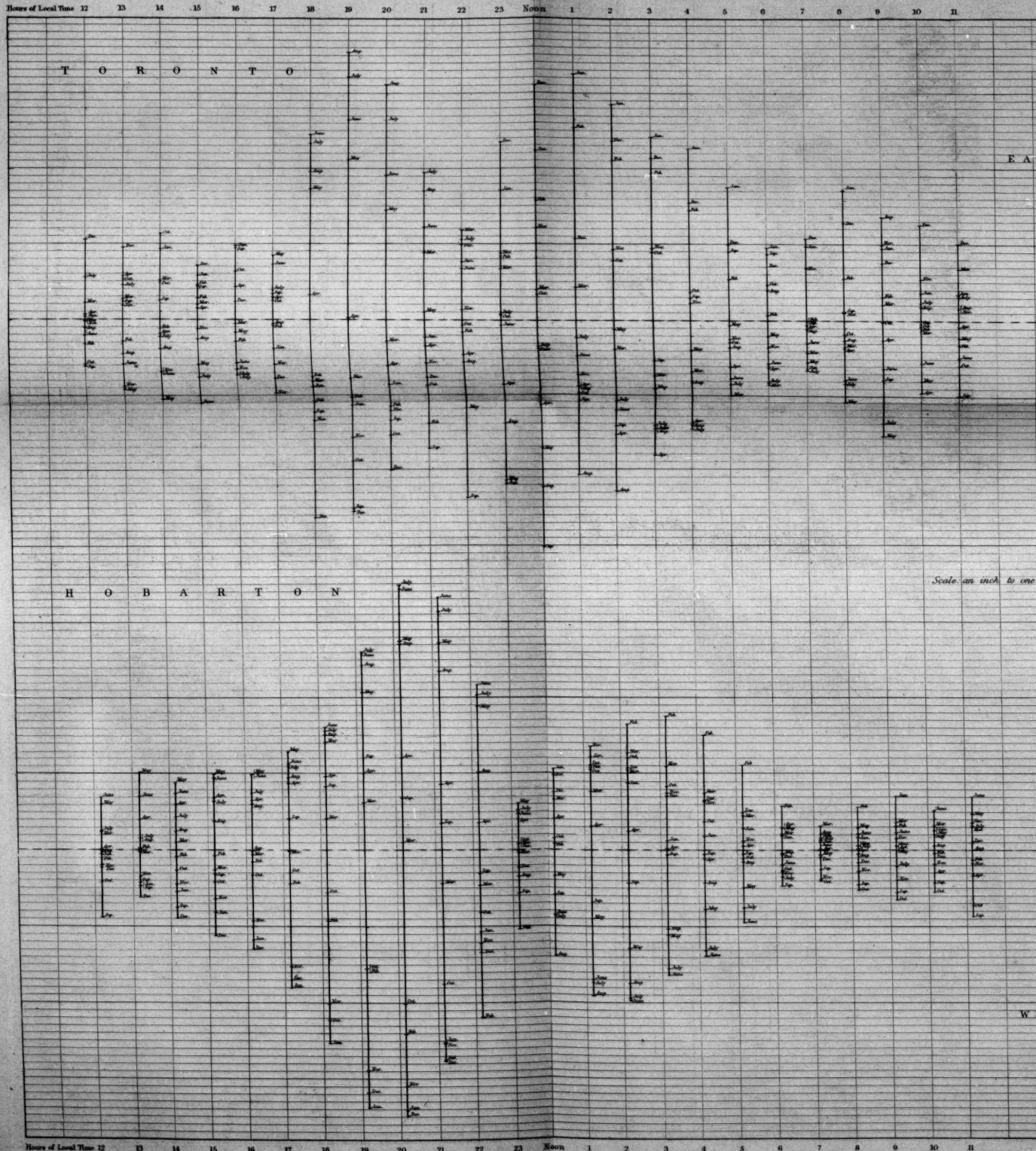
Ford & George, Lithographers, 56, Hatton Garden.
W.Wagstaff.

ANNUAL VARIATION of the MAGNETI


Hours of Local Time 12 13 14 15 16 17 18 19 20 21 22 23 Noon 1 2 3 4 5 6 7 8 9 10 11
TORONTO
E A
Scale: an inch to one
HOBARTON
W
Hours of Local Time 12 13 14 15 16 17 18 19 20 21 22 23 Noon 1 2 3 4 5 6 7 8 9 10 11
Drawn by Capt. W

TIC DECLINATION at the different hours.

12 13 14 15 16 17 18 19 20 21 22 23 Noon 1 2 3 4 5 6 7 8 9 10 11 Hours of Local Time.

ST HELENA

AST

one minute of Declination

CAPE of GOOD HOPE

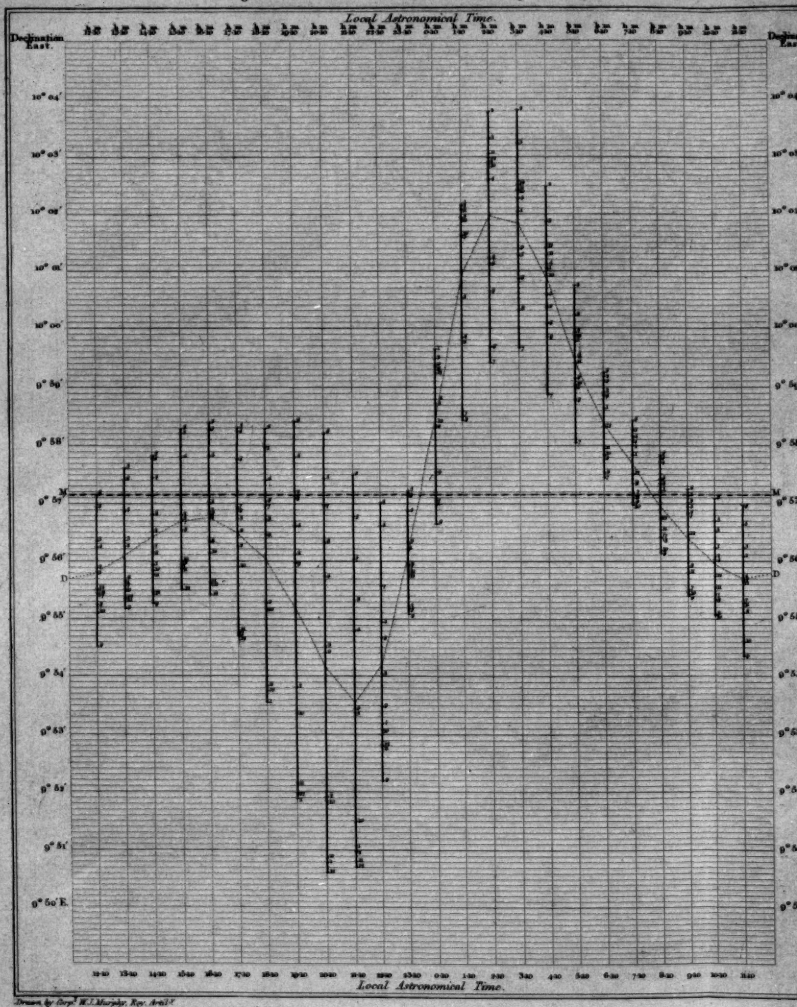
WEST

12 13 14 15 16 17 18 19 20 21 22 23 Noon 1 2 3 4 5 6 7 8 9 10 11 Hours of Local Time.

Lt. W.J. Murphy. *Royal Artillery.*

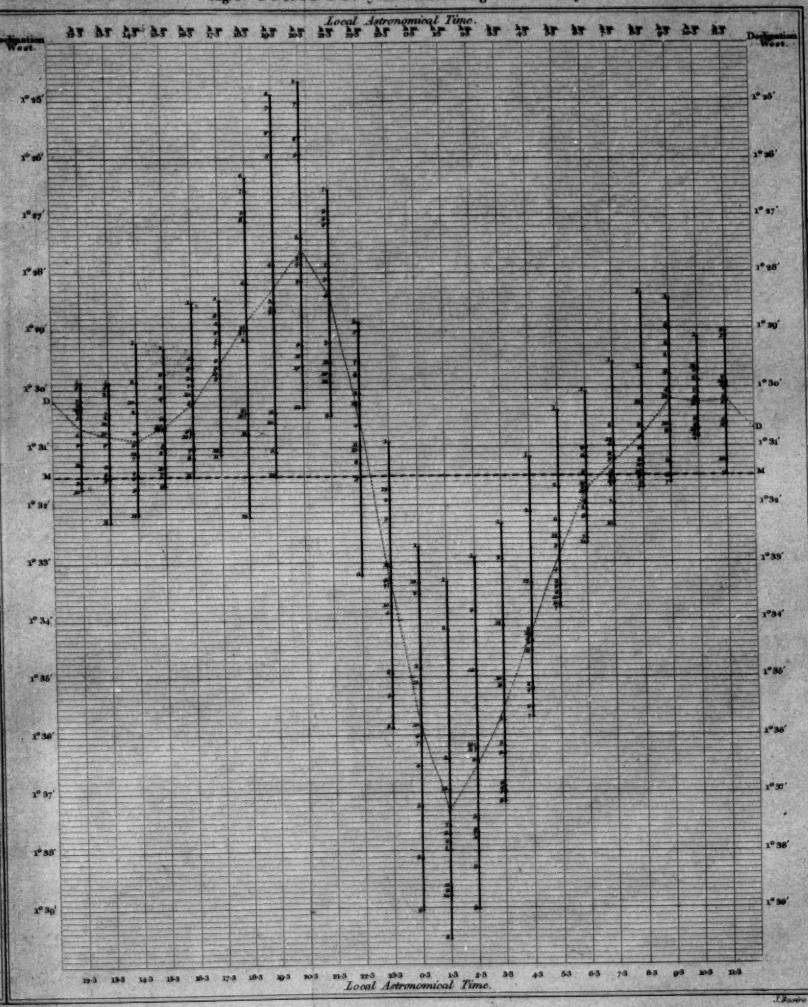
J. Basire *sc.*

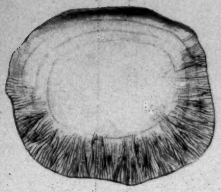
Fig. 1. HOBARTON: year commencing with July.

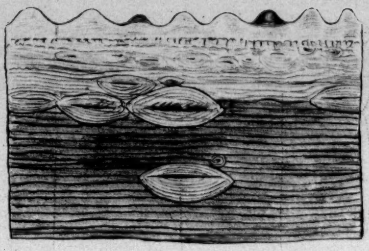


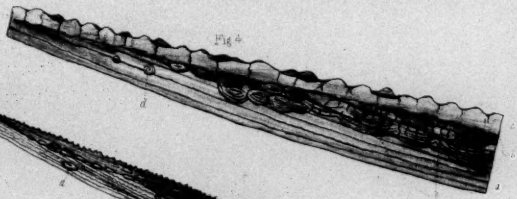
Drawn by Lieut. M. J. Murphy, Roy. Artill.

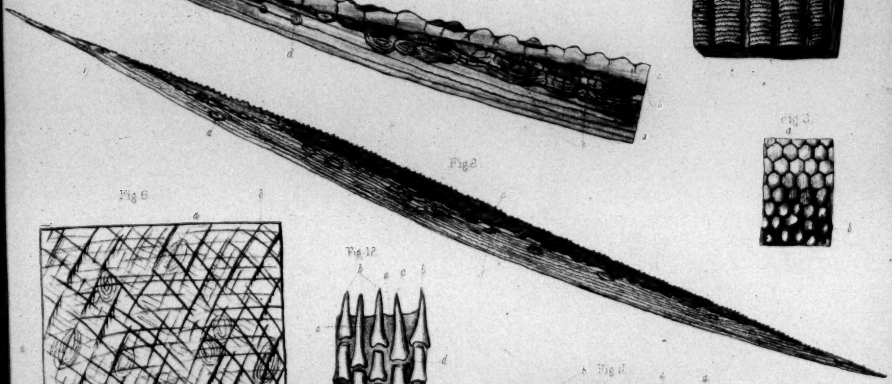
Fig. 2. TORONTO: year commencing with January.

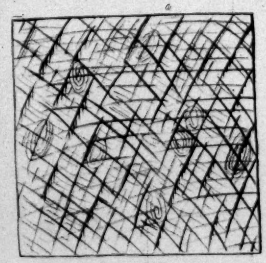


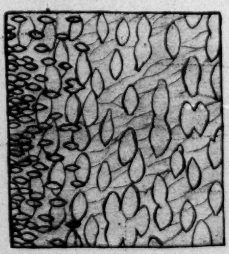


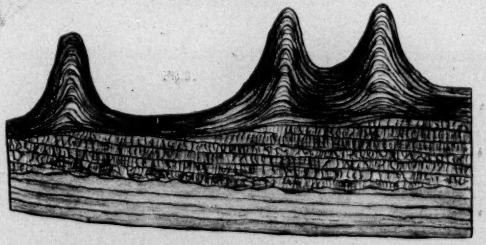












W. & W. West W. Wing Lith.

Ford & Greig, Lithographers, 14 Hatton Garden

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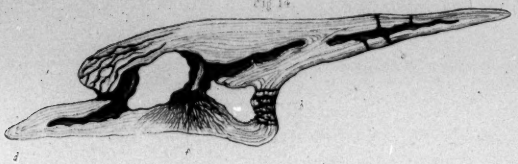

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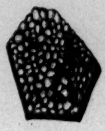
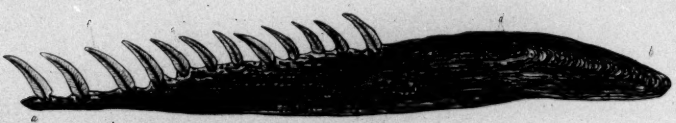


Fig 18.

Fig 15.



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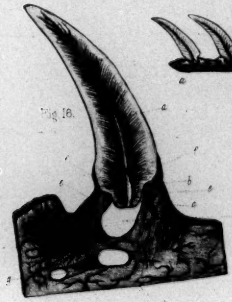
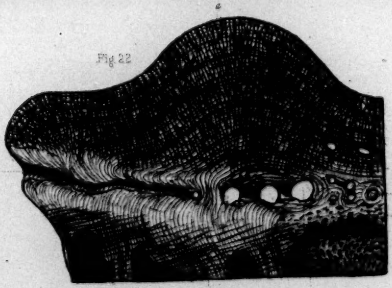


Fig 19.

Fig 22.



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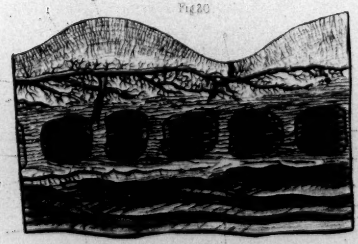

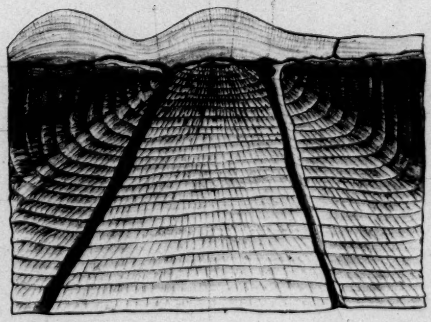

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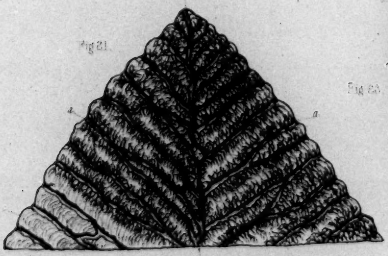


Fig 23.

W. D. W. del. W. Wing Lith.

Paul & Dangé, Lithographers 54 Holton Garden.

Fig. 25.

Fig. 24.

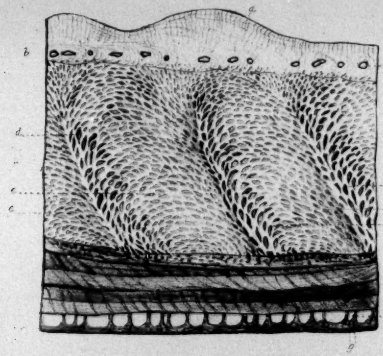
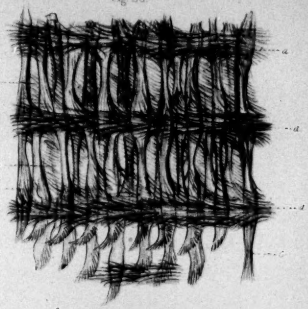


Fig. 26.



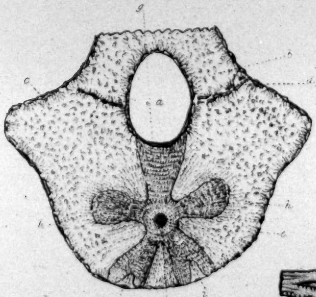


Fig. 28.

Fig. 29.

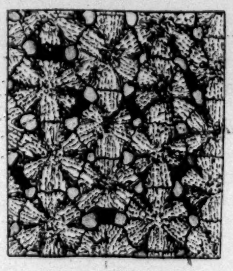


Fig. 27.

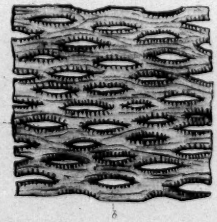


Fig. 32.

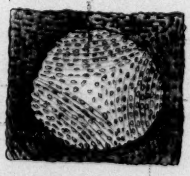
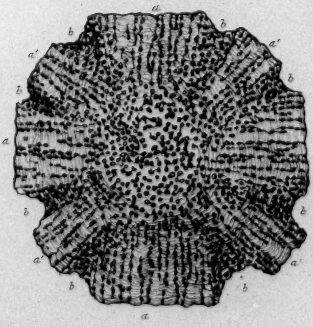


Fig. 30.



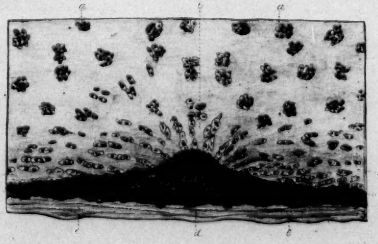


Fig. 31.

Fig. 33.

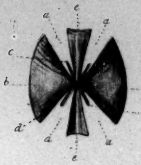


Fig. 37.

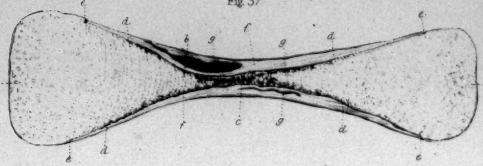


Fig. 38.

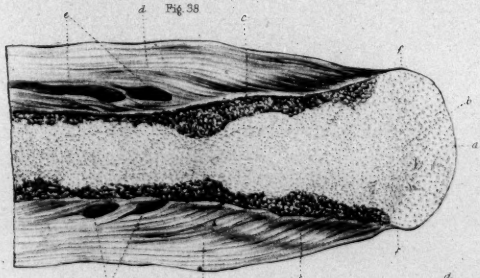


Fig. 34.

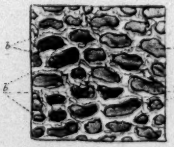


Fig. 39.

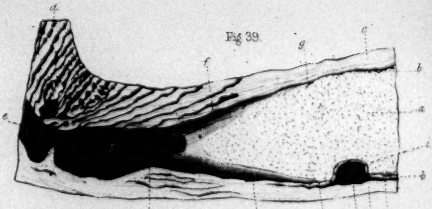
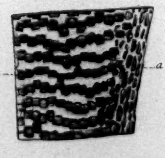


Fig. 35.



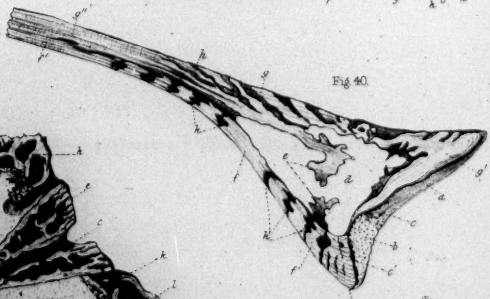


Fig. 40.

Fig. 41.

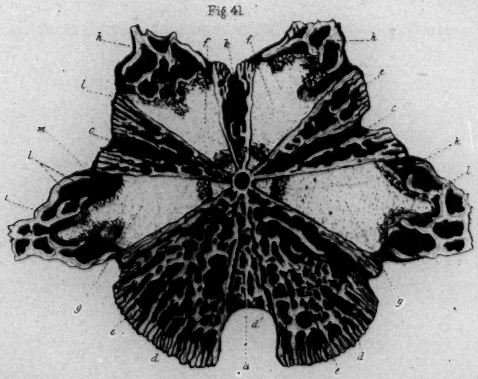


Fig. 36.

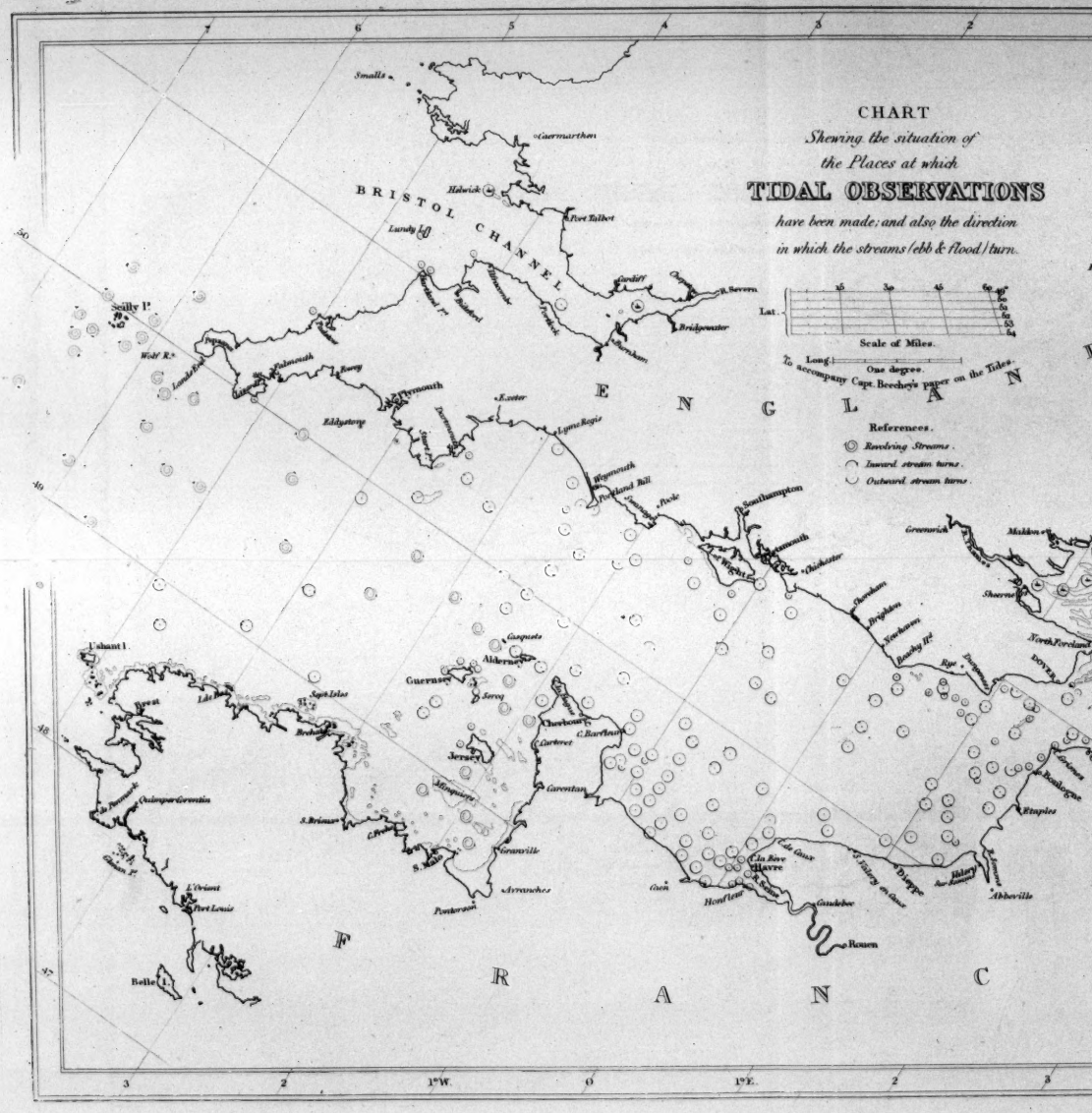
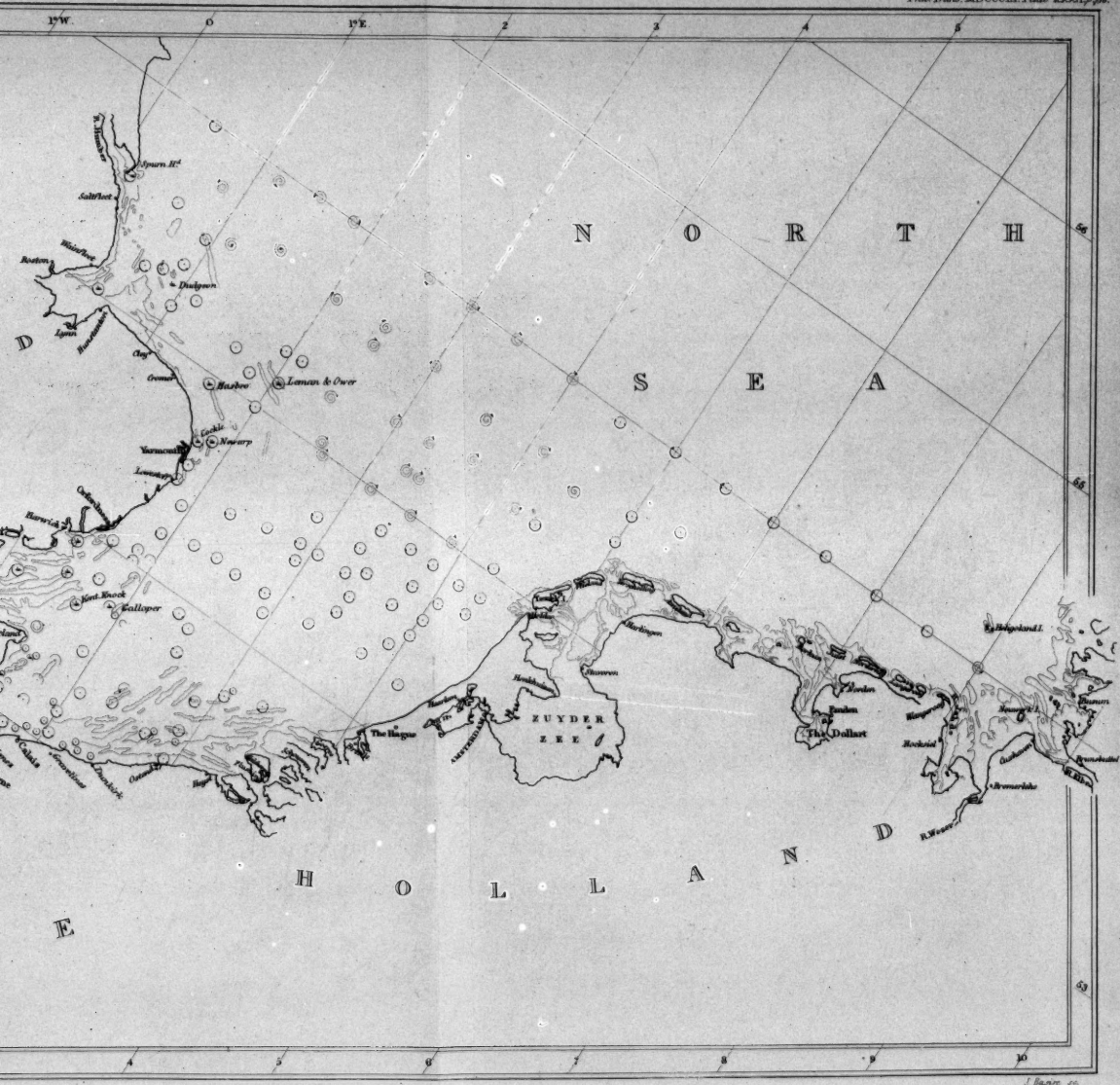


CHART
Shewing the situation of
the Places at which
TIDAL OBSERVATIONS
have been made; and also the direction
in which the streams ebb & flood turn.
Scale of Miles.
Long.
One degree.
To accompany Capt. Beechey's paper on the Tides.
References.
Revolving Streams.
Inward stream turns.
Outward stream turns.
BRISTOL CHANNEL
ENGLAND
FRANCE
Smalls
Caermarthen
Milford
Pembroke
Tenby
Lundy I.
Port Talbot
Cardiff
Newport
R. Severn
Bridgewater
Scilly I.
Wolf R.
Falmouth
Penzance
Plymouth
Eddystone
Exeter
Lyme Regis
Portland Bill
Weymouth
Poole
Southampton
Portsmouth
I. of Wight
Chichester
Shoreham
Brighton
Newhaven
Beachy Hd.
Rye
Dungeness
DOVER
Greenwich
Maldon
Sheerness
North Foreland
Margate
Ramsgate
Deal
Walmer
Sandwich
Ushant I.
Great Ldc Is.
Sept Isles
Brehat
Quiberon
Quiberon Greenlin
St Nazaire
Belle I.
L'Orient
Port Louis
Chats I.
St Brieux
Cancale
St Malo
Granville
Avranches
Pontorson
Jersey
Guernsey
Alderney
Gasquets
Hanois
Cherbourg
C. Barfleur
Carteret
Carentan
Hauteport
Trouville
Honfleur
Caen
Ecu River
Havre
St Adresse
Gonfleur
Rouen
Pont Audemer
Pont de l'Arche
Dieppe
St Valery en Caux
Le Crotoy
Etaples
Boulogne
Calais
Abbeville
50
49
48
47
6
5
4
3
2
1°W.
0
1°E.
2
3


1°W.
0
1°E.
2
3
4
5
NORTH
SEA
HOLLAND
ZUYDER ZEE
Lynn Rt.
Saltfleet
Wainfleet
Boston
Dudgeon
Lynn
Ransthorpe
Cley
Cromer
Haisbro
Lamon & Ower
Cockle
Newarp
Yarmouth
Lowestoft
Orfordness
Harwich
Kent Knock
Galloper
Ostend
Nieuport
Dunkirk
Calais
Gravelines
Boulogne
Ambleteuse
Hoy
The Hague
Amsterdam
Haerlem
Haerlingen
Vlieland
Texel
Helder
Helvoet
Sluys
Ameland
Schelling
Borkum
Emden
Borkum
Rottum
Rockzand
The Dollart
Oldeboorn
Bourtange
Delfzeil
Borkumbank
Bremerlake
R. Weser
Heligoland I.
55
56
57
4
5
6
7
8
9
10

J. Blaire sc.

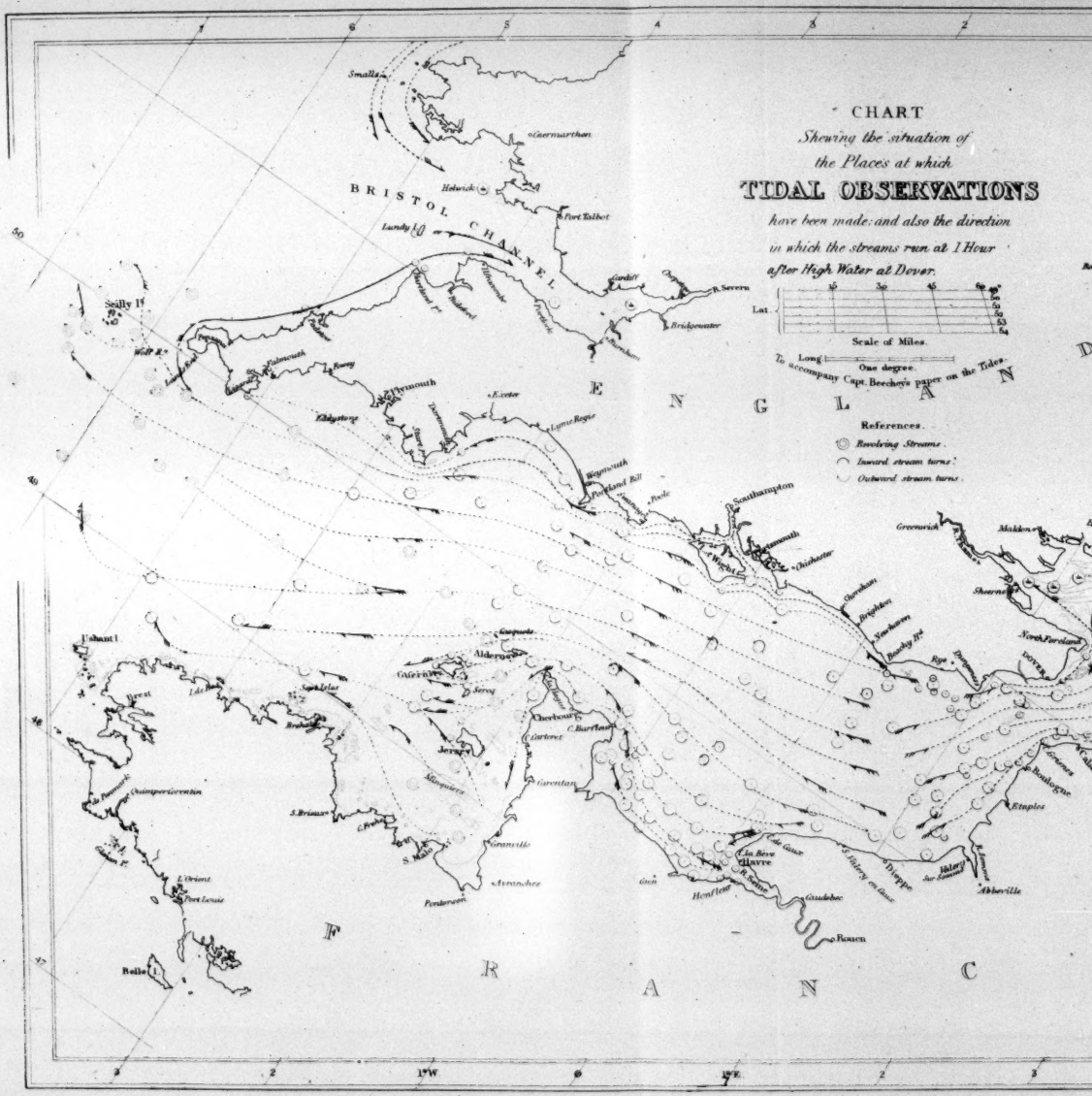
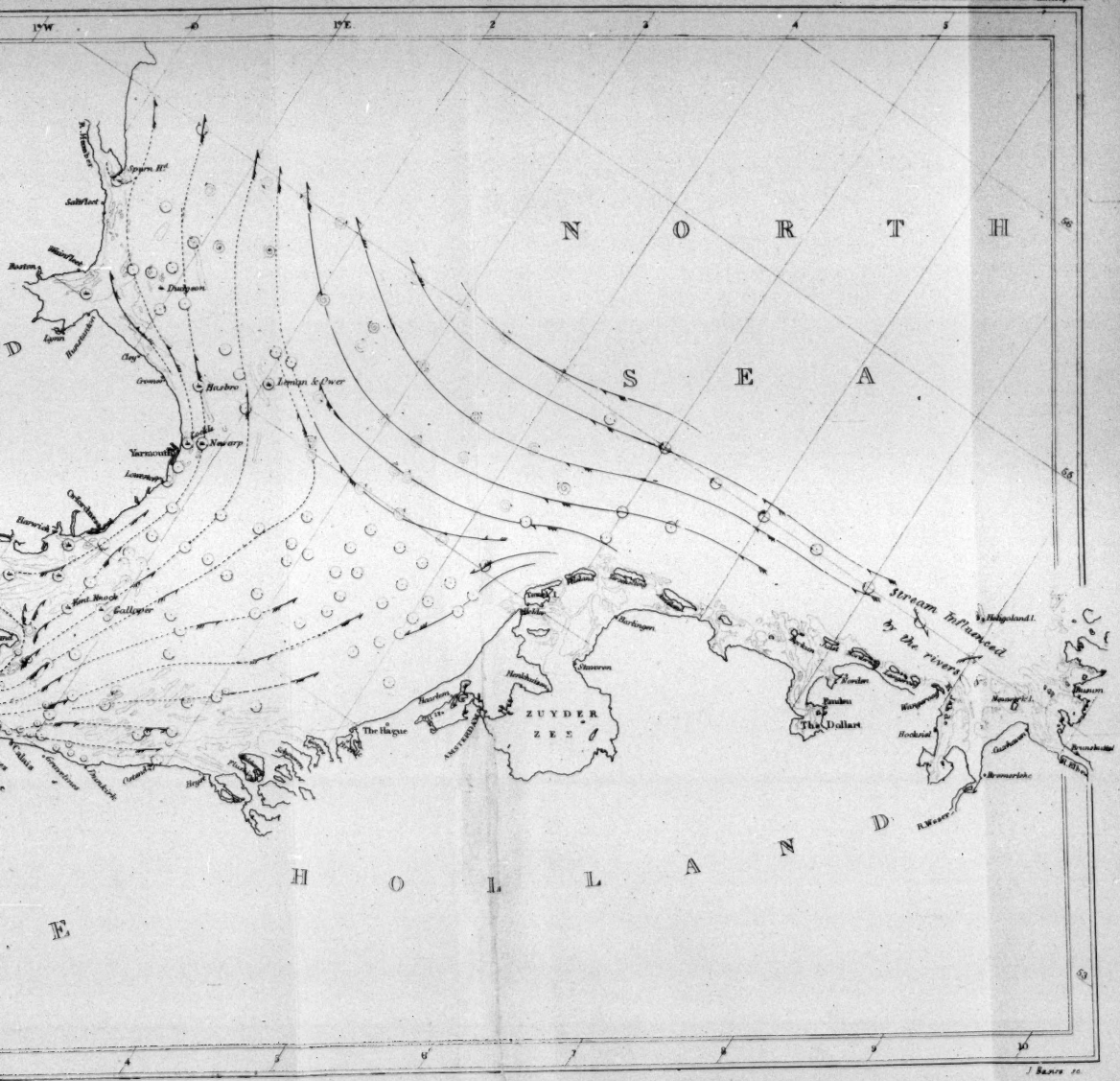


CHART
Shewing the situation of
the Places at which
TIDAL OBSERVATIONS
have been made: and also the direction
in which the streams run at 1 Hour
after High Water at Dover.
Lat.
Scale of Miles.
Long
One degree.
To accompany Capt. Beechey's paper on the Tides.
References.
Revolving Streams.
Inward stream turns.
Outward stream turns.
BRISTOL CHANNEL
ENGLAND
FRANCE
Smalls
Caermarthen
Helwick
Port Talbot
Lundy
Cardiff
Overton
R. Severn
Bridgewater
Scilly
Wolf R.
Falmouth
Torbay
Plymouth
Exeter
Ebbystone
Lyme Regis
Weymouth
Portland Bill
Poole
Southampton
Cowes
Wight
Chichester
Greenwich
Shoreham
Brighton
Newhaven
Beachy Hd.
North Foreland
Ushant
Brest
Audierne
Camaret
Alderney
Guernsey
Serq
Cherbourg
C. Barfleur
Carteret
Jersey
Granville
Mont St. Michel
St. Brieux
Pt. Pleurtuit
Quimperlé
Pt. Penmarch
L'Orient
Port Louis
Belle Isle
Pordenreen
Arranches
Ornes
Le Havre
Honfleur
Caudebec
Rouen
Dieppe
St. Valery en Caux
Abbeville
Boulogne
Etaples
Calais


NORTH
SEA
HOLLAND
ZUYDER
ZEE
Stream Influenced
by the rivers
Spurn Hd.
Saltfleet
Boston
Dunkeswell
Lynn
Cant
Ground
Haisbro
Denign & Owse
Yarmouth
Newarp
Lowestoft
Harwich
Kent Knock
Gallopper
Tessel I.
Helder
Burkengen
Helgoland I.
Heligoland
Texel
Haerlem
The Hague
AMSTERDAM
Flardin
Embden
The Dollart
Hoeksiel
Borkum
Brunsbuttel
Oostende
Brouwershe
R. Weser
Dunkirk
Calais
Gravelines
Ostend
Blyth
F.

J. Banks sc.

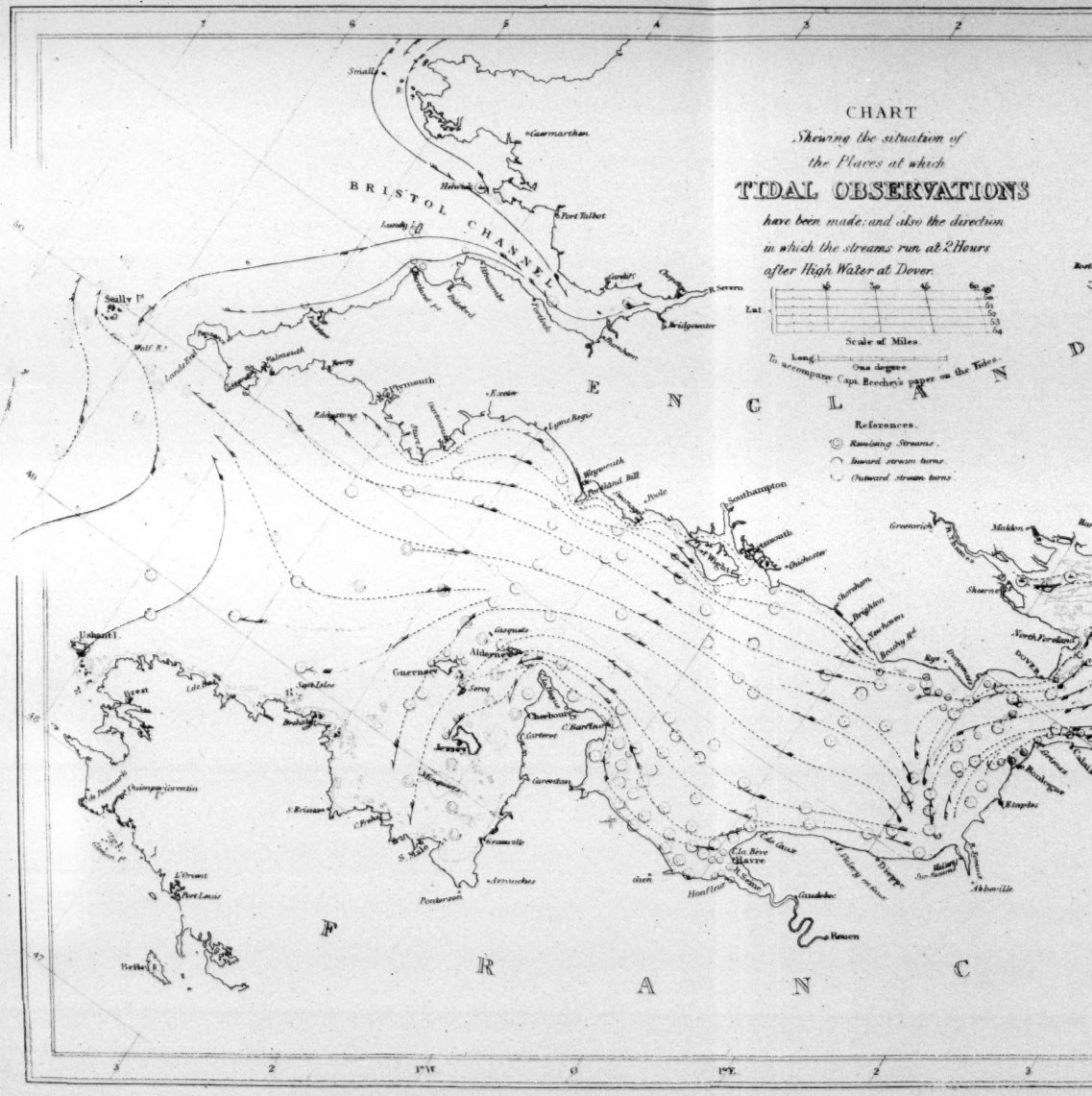
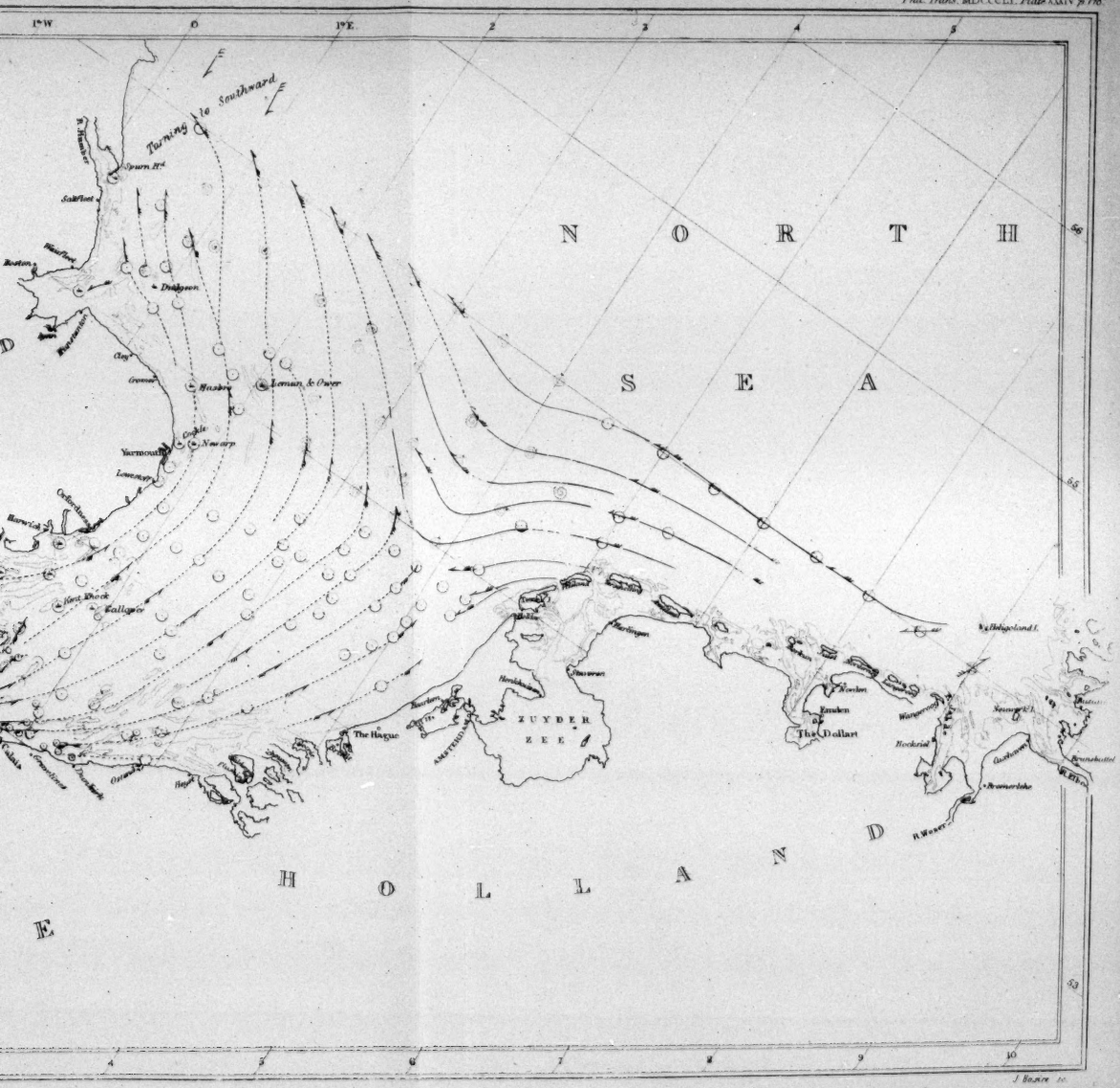


CHART
Shewing the situation of
the Places at which
TIDAL OBSERVATIONS
have been made; and also the direction
in which the streams run, at 2 Hours
after High Water at Dover.
Lat.
Scale of Miles.
Long.
One degree
To accompany Capt. Beeche's paper on the Tides.
References.
Rowling Streams.
Inward stream turns.
Outward stream turns.
BRISTOL CHANNEL
ENGLAND
FRANCE
Smalls
Carmarthen
Port Talbot
Cardiff
Bristol
Bridgwater
Scilly I.s
Wolf R.
Lizard Head
Falmouth
Plymouth
Exeter
Eddystone
Portland Bill
Poole
Southampton
Portsmouth
Chichester
Brighton
Beachy Hd
Rye
Greenwich
Sheerness
North Foreland
DOVER
Ramsgate
Margate
Deal
Walmer
Dungeness
Folkestone
Boulogne
Calais
Gris Nez
Ushant I.
Brest
Iles Sorlingues
Guernsey
Alderney
Sark
Jersey
Casquets
Cherbourg
C. Barfleur
Carteret
Granville
Pt. Jerbourg
Pt. Grouin
St. Brieux
St. Malo
Cape Frehel
L'Orient
Port Louis
Belle Isle
Penmarch Pt.
Audierne B.
Pt. Raz
Iles Glenan
Pt. Penmarch
Arromanches
Ouistreham
Iles Oeux
Cte. Bere
Havre
Honfleur
Caudebec
Rouen
Dieppe
Fecamp
St. Valery en Caux
Abbeville
Hastings
Rye
Beachy Hd
Eastbourne
Brighton
Shoreham
Worthing
Littlehampton
Bognor
Selsey Bill
Christchurch
Swanage
Portland Bill
Lyme Regis
Torbay
Start Pt.
Bolt Head
Salcombe
Dartmouth
Teignmouth
Exmouth
Padstow
Hartland Pt.
Ilfracombe
Barnstaple
Minehead
Watchet
Clevedon
Portishead
Newport
Chepstow
Aberystwith
Tenby
Milford
Haverfordwest
Swansea
Neath
Llanelly
Caermarthen
Aberavon
Cardiff
Newport
Chepstow
Bridgwater
Watchet
Minehead
Ilfracombe
Barnstaple
Padstow
Hartland Pt.
Milford
Haverfordwest
Tenby
Swansea
Neath
Llanelly
Aberavon
Aberystwith
Lundy I.


1°W
0
1°E
2
3
4
5
Turning to Southward
E
E
Spurn Hd.
Salt Fleet
Boston
Dudgeon
Cley
Greenc
Hasbro
Leman & Ower
Yarmouth
Cockle
Newcarp
Lowestoft
Orfordness
Harwich
Kent Knock
Gallopers
Goodwin Sands
Ramsgate
Margate
Dover
Calais
Boulogne
Ostend
Flushing
Helvoet
The Hague
Amsterdam
Harlingen
Terschelling
Vlieland
Texel
Helder
Hornbeken
ZUYDER ZEE
Enkhuysen
Medemblik
Stavoren
Urk
Kampen
Elburg
Harderwyk
Amersfort
Utrecht
Gorcum
Rotterdam
Delft
Leyden
Haerlem
Amsterdam
Embden
The Dollart
Borkum
Rockzand
Wangerooge
Norderney
Langeoog
Spiekeroog
Juist
Baltrum
Memmerdike
Oldenburg
Bremen
R. Weser
Heligoland I.
Neuwerk I.
Cuxhaven
Brunsbuttel
R. Elbe
NORTH SEA
HOLLAND
55
53
2
3
4
5
6
7
8
9
10

J. Basire sc.

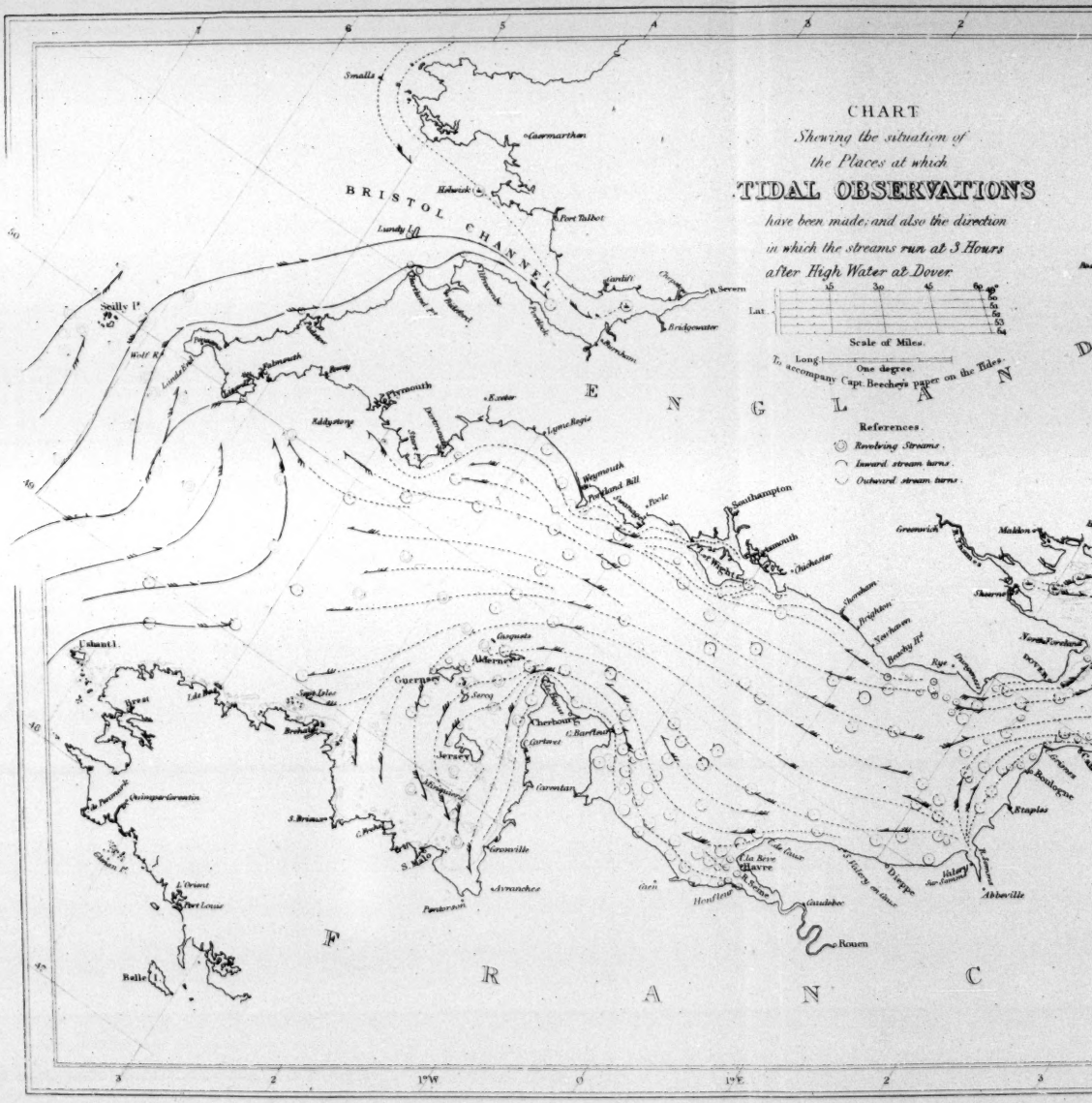
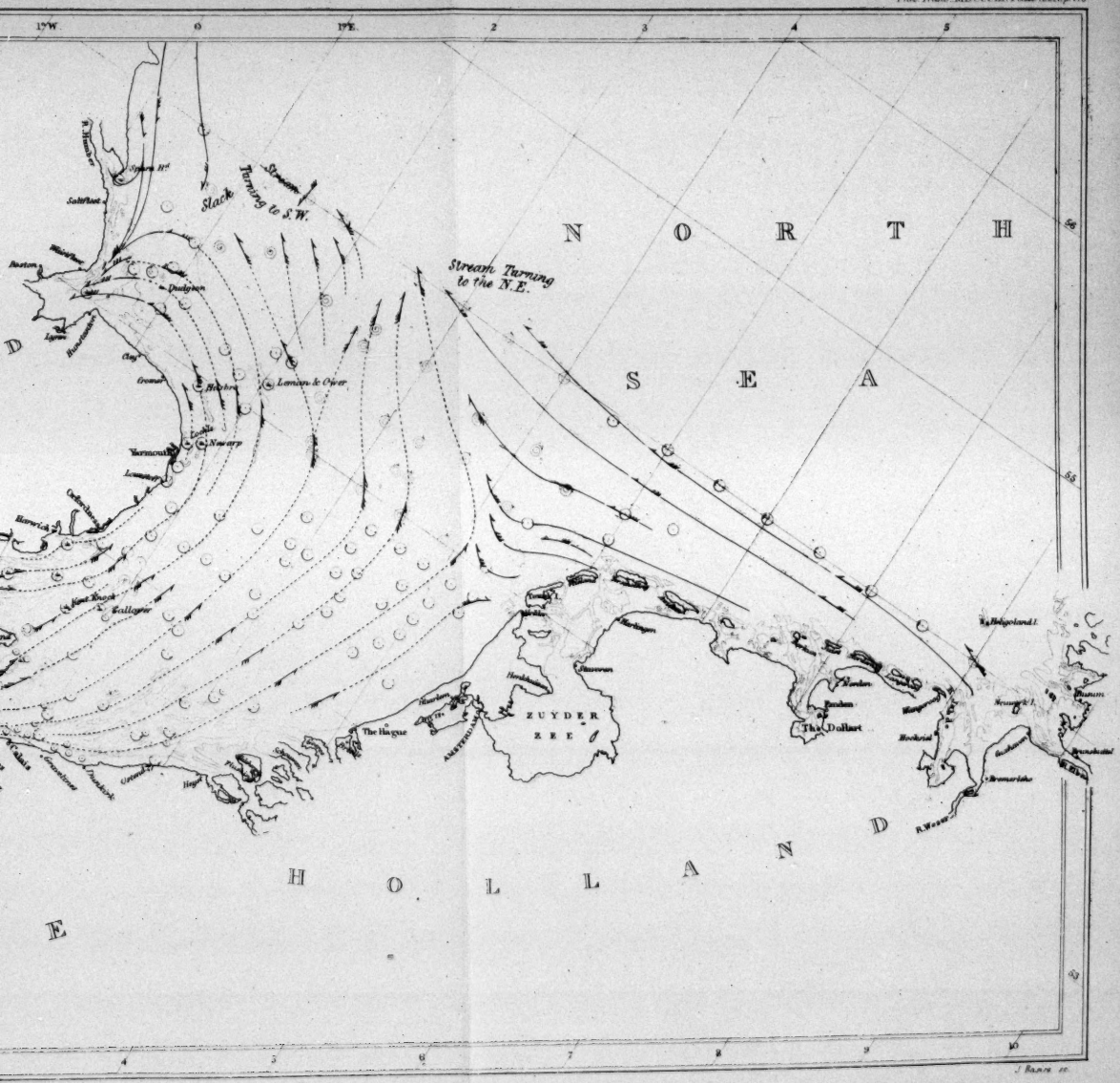


CHART
Shewing the situation of
the Places at which
TIDAL OBSERVATIONS
have been made; and also the direction
in which the streams run at 3 Hours
after High Water at Dover
Scale of Miles.
Long.
One degree.
To accompany Capt. Beechey's paper on the Tides.
References.
Revolving Streams.
Inward stream turns.
Outward stream turns.
BRISTOL CHANNEL
ENGLAND
FRANCE
Smalls
Caermarthen
Holyhead
Fort Talbot
Lundy Isd
Scilly I.
Falmouth
Plymouth
Exeter
Lyme Regis
Weymouth
Portland Bill
Poole
Southampton
Portsmouth
Chichester
Dungeness
Deal
Ramsgate
Margate
Sheerness
Maldon
Yarmouth
Lowestoft
Harwich
Orfordness
Aldborough
Brighton
Beachy Hd
Guernsey
Alderney
Casquets
Jersey
Cherbourg
C. Barfleur
C. Gurtaret
Carentan
Granville
Avranches
Pontorson
St Malo
Paimpol
Brehat
Ushant I.
Brest
Iles Glenan
L'Orient
Port Louis
Belle I.
Quimper Corentin
St Brieuc
Havre
La Heve
Honfleur
Caen
Cape la Hogue
Dieppe
Fecamp
Etretat
Abbeville
Rouen
Gravelines
Boulogne
Calais
Dover
Staples
Margate
Ramsgate
Sheerness
Harwich
Yarmouth
Lowestoft
Orfordness
Aldborough
Brighton
Beachy Hd
Dungeness
Deal
Maldon
Greenwich
Chatham
Sheerness
North Foreland
Dover
Ramsgate
Margate
Staples
Boulogne
Calais
Gravelines
Abbeville
Etretat
Fecamp
Dieppe
Cape la Hogue
Havre
La Heve
Honfleur
Caen
Gatteville
Rouen
Hastings
Rye
Winchelsea
Romney
Hythe
Folkestone
Dover
Deal
Ramsgate
Marg


1°W.
0
1°E.
2
3
4
5
N O R T H
S E A
Stream Turning to the N.E.
Slack
Stream Turning to S.W.
Lemon & Oyer
Newarp
Yarmouth
Lowestoft
Harwich
Gallopers
Ostend
Nieuport
Dunkirk
Calais
Boulogne
Hastings
Dungeness
Beachy Head
Dover
Ramsgate
Margate
Deal
Goodwin Sands
Sandwich
Canterbury
Whitstable
Herne Bay
Reculver
Sheerness
Chatham
Gravesend
Tilbury
Woolwich
Greenwich
London
Blackwall
Limehouse
Deptford
Lambeth
Westminster
Chelsea
Battersea
Putney
Kew
Richmond
Hounslow
Staines
Windsor
Egham
Chertsey
Weybridge
Hampton Court
Kingston
Wandsworth
Clapham
Streatham
Croydon
Dulwich
Peckham
Camberwell
Vauxhall
Lambeth
Chelsea
Hammersmith
Fulham
Kensington
Paddington
Hampstead
Highgate
Islington
Hackney
Stratford
Bow
Poplar
Blackwall
Greenwich
Deptford
Rotherhithe
Bermondsey
Southwark
Lambeth
Westminster
Pimlico
Chelsea
Brompton
Kensington
Hammersmith
Chiswick
Brentford
Isleworth
Twickenham
Hampton
Thames Ditton
Esher
Walton
Weybridge
Chertsey
Egham
Staines
Windsor
Eton
Slough
Uxbridge
Harrow
Edgware
Hendon
Finchley
Highgate
Hampstead
Kilburn
Paddington
Marylebone
Pentonville

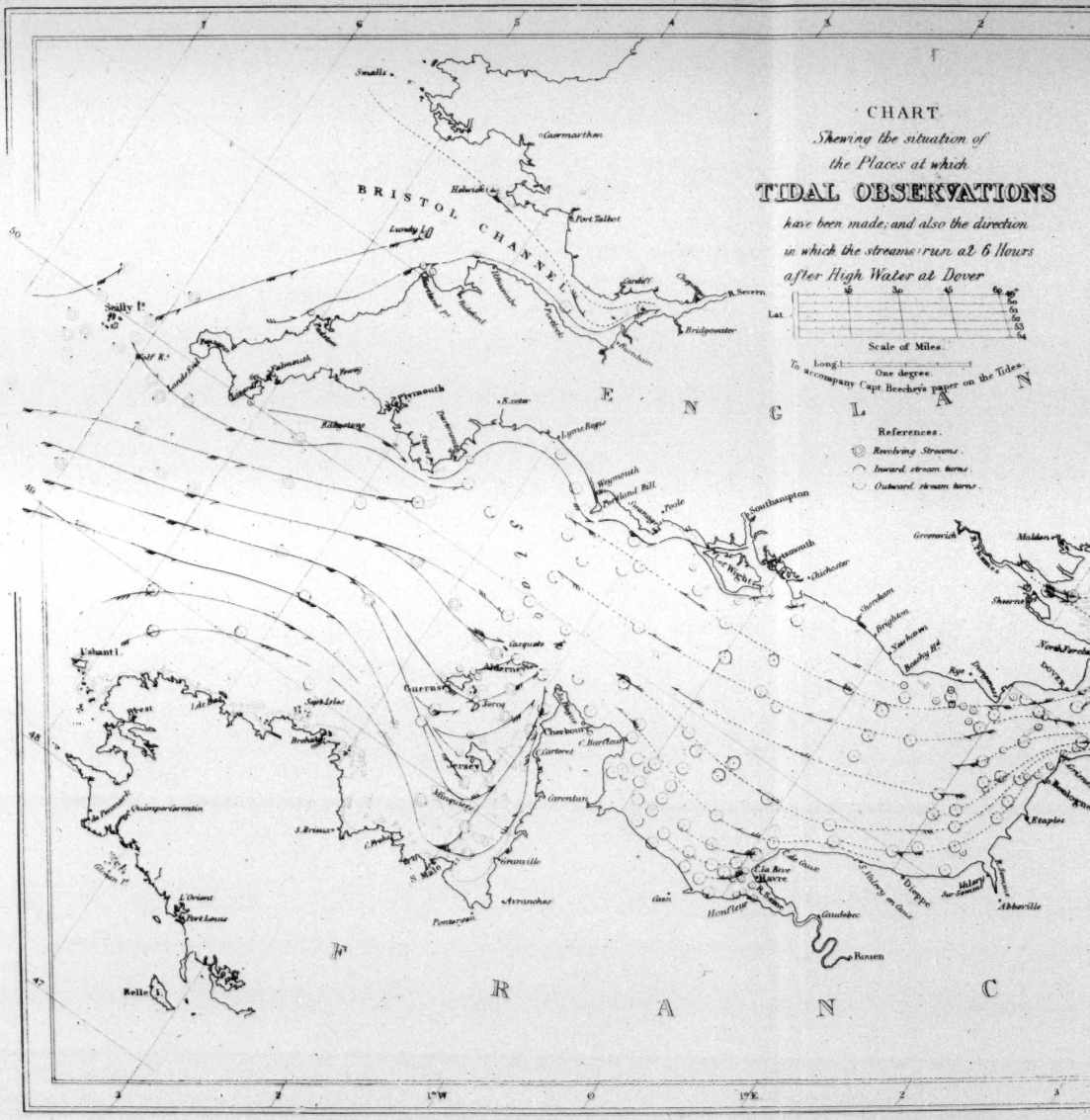
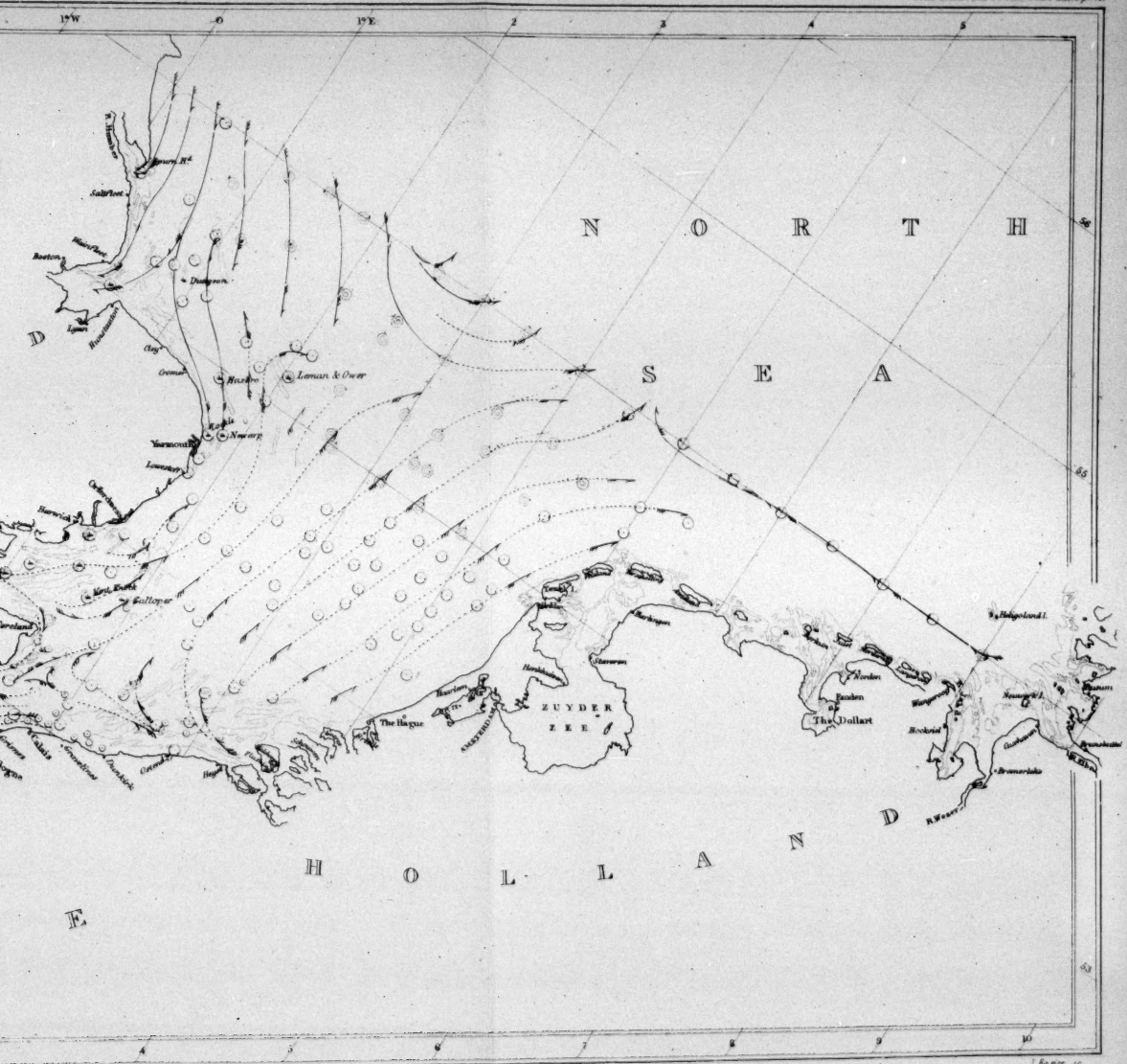


CHART
Shewing the situation of
the Places at which
TIDAL OBSERVATIONS
have been made; and also the direction
in which the streams run at 6 Hours
after High Water at Dover
Scale of Miles.
One degree.
To accompany Capt. Beechey's paper on the Tides.
References.
Revolving Streams.
Inward stream turns.
Outward stream turns.
BRISTOL CHANNEL
ENGLAND
FRANCE
Caermarthen
Swansea
Cardiff
Bridgewater
Scilly Is.
Falmouth
Plymouth
Weymouth
Portland Bill
Southampton
Portsmouth
Chichester
Brighton
Yarmouth
Rye
Dover
Greenwich
Sheerness
Guernsey
Alderney
Jersey
Cherbourg
C. Barfleur
Carteret
Granville
St. Malo
Arranches
Pontorson
L'Orient
Port Louis
Belle Isle
Ushant I.
St. Brieux
Le Havre
Honfleur
Caudebec
Rouen
Dieppe
Treport
Abbeville
Etaples
Boulogne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Cromer
Haisborough
Hollesley
Aldeburgh
Sunk
Galloper
Kentish Knock
Goodwin Sands
Margate
Ramsgate
Deal
Dover
Calais
Boulogne
Gravelines
Dunkirk
Ostend
Nieuport
Flushing
Helvoetsluys
The Hague
Amsterdam
Hoorn
Texel
Vlieland
Terschelling
Ameland
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Baltrum
Langeoog
Spiekeroog
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Helgoland I.
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Emden
The Dollart
Rockriff
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Neuwerk
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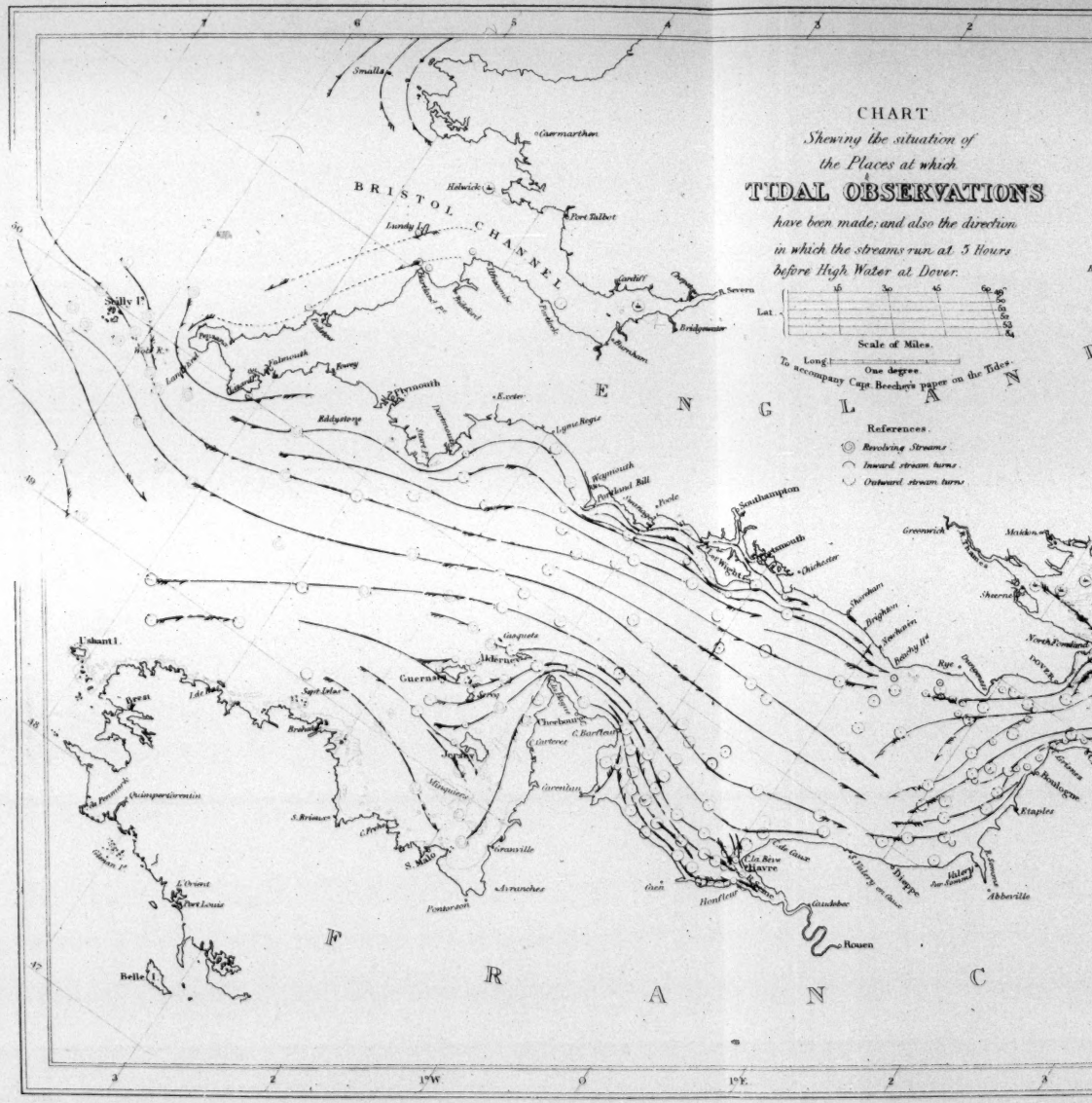
CHART
Shewing the situation of
the Places at which
TIDAL OBSERVATIONS
have been made; and also the direction
in which the streams run at 3 Hours
before High Water at Dover.
Scale of Miles.
One degree
To accompany Capt. Beecher's paper on the Tides
References.
Revolving Streams.
Inward stream turns.
Outward stream turns.
BRISTOL CHANNEL
ENGLAND
FRANCE
Smalls
Carmarthen
Holwick
Port Talbot
Lundy Id.
Cardiff
Newport
R. Severn
Bridgewater
Scilly I.
Falmouth
Looe
Plymouth
Exeter
Lyme Regis
Portland Bill
Weymouth
Poole
Southampton
Isle of Wight
Portsmouth
Chichester
Greenwich
Maidenhead
Sheerness
Thornham
Brighton
Rye
Dungeness
North Foreland
DOVER
Ushant I.
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Guernsey
Jersey
Cherbourg
C. Barfleur
Granville
Minquiers
S. Brieuc
S. Malo
Morlaix
Ostende
Ste. Cause
Havre
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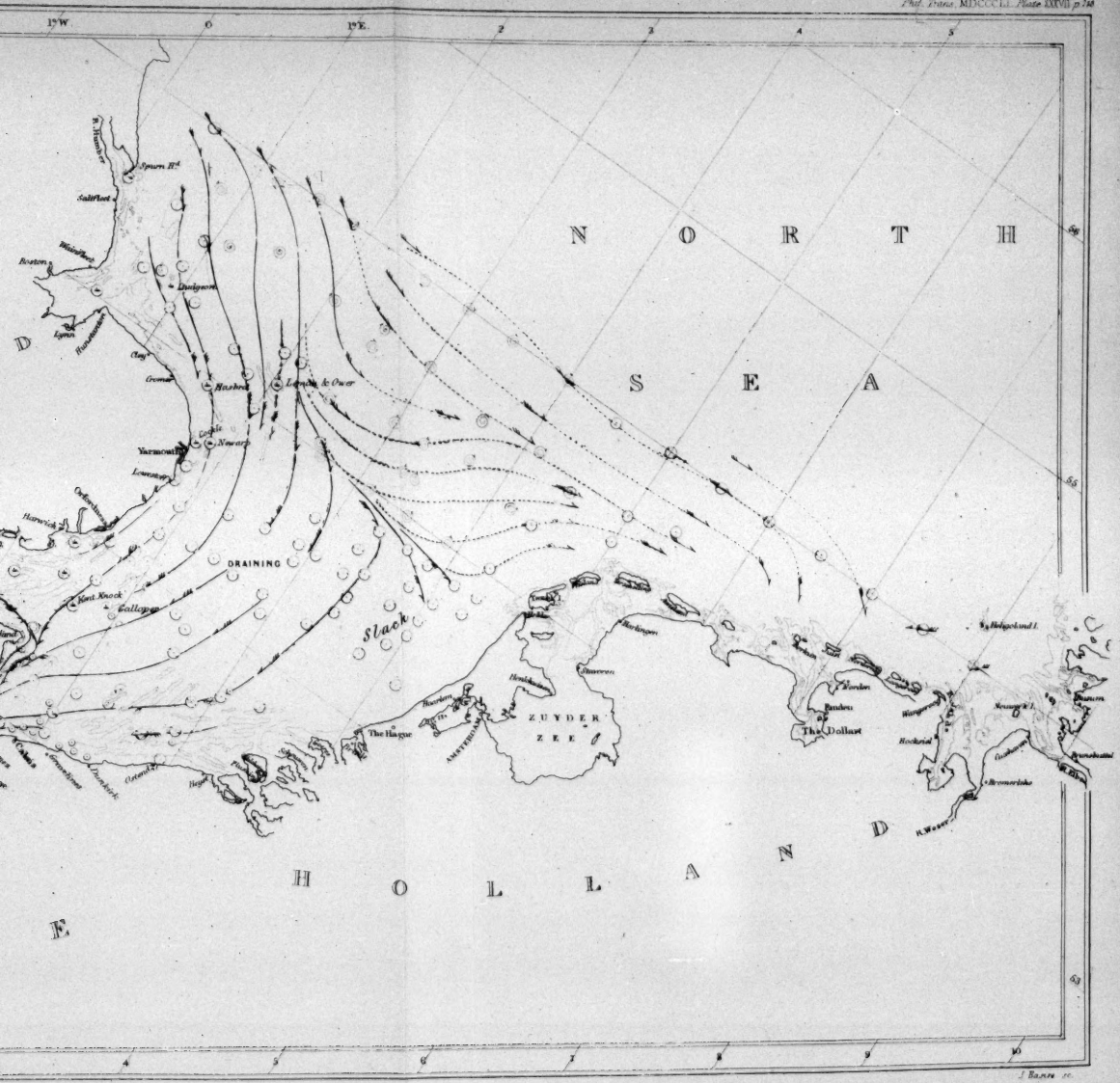
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Borkum
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Harlingen
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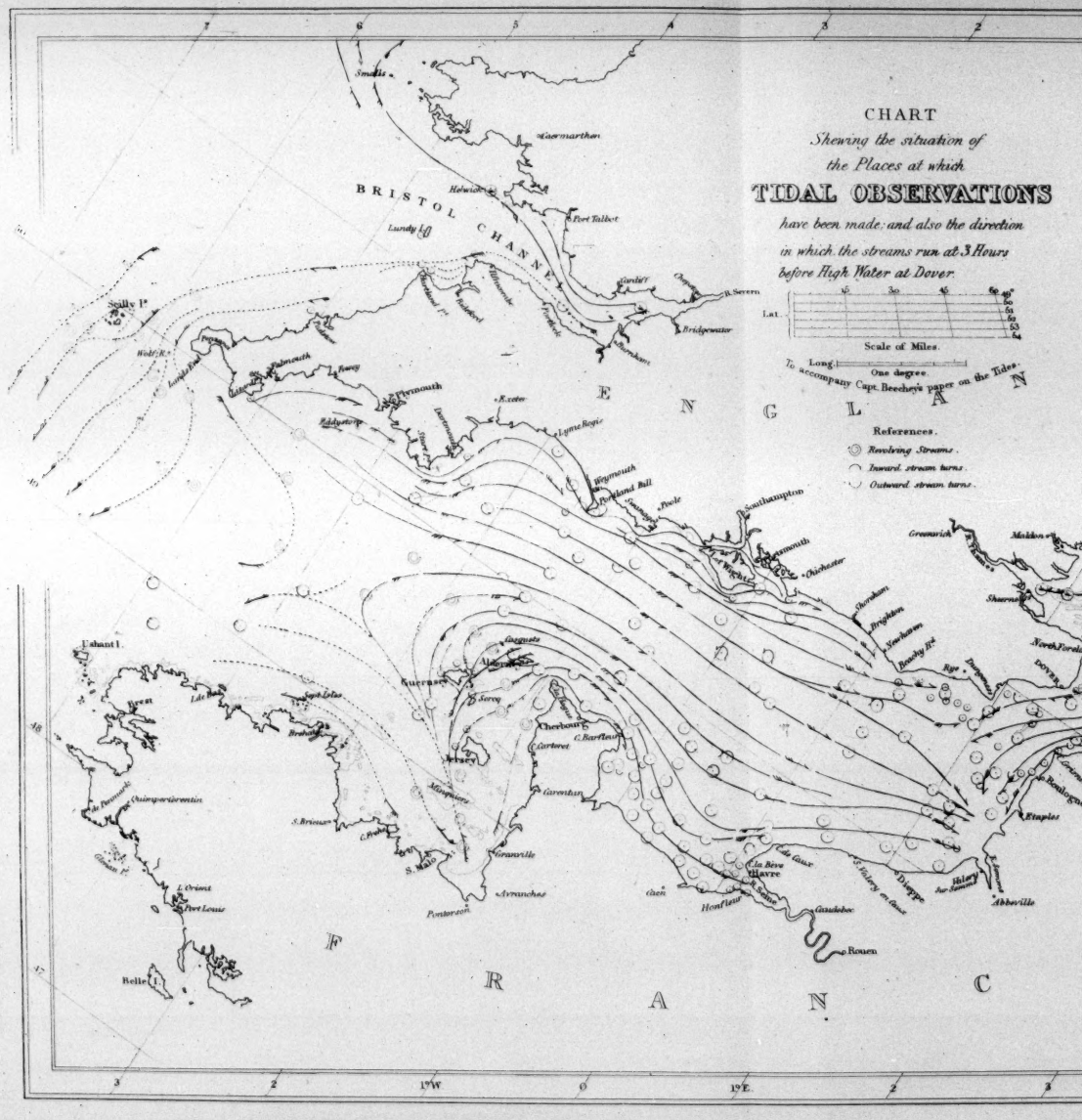
CHART
Shewing the situation of
the Places at which
TIDAL OBSERVATIONS
have been made: and also the direction
in which the streams run at 3 Hours
before High Water at Dover.
Lat.
Scale of Miles.
To accompany Capt. Beechey's paper on the Tides.
References.
Revolving Streams.
Inward stream turns.
Outward stream turns.
BRISTOL CHANNEL
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Guernsey
Sark
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C. Barfleur
Carteret
Arkney
Granville
S. Malo
Avranches
Pontorson
S. Brieuc
Quimperlé
Pte. Penmark
L'Orient
Port Louis
Belle I.
Pte. de Grave
Caen
Honfleur
Pte. Barfleur
Pte. Heve
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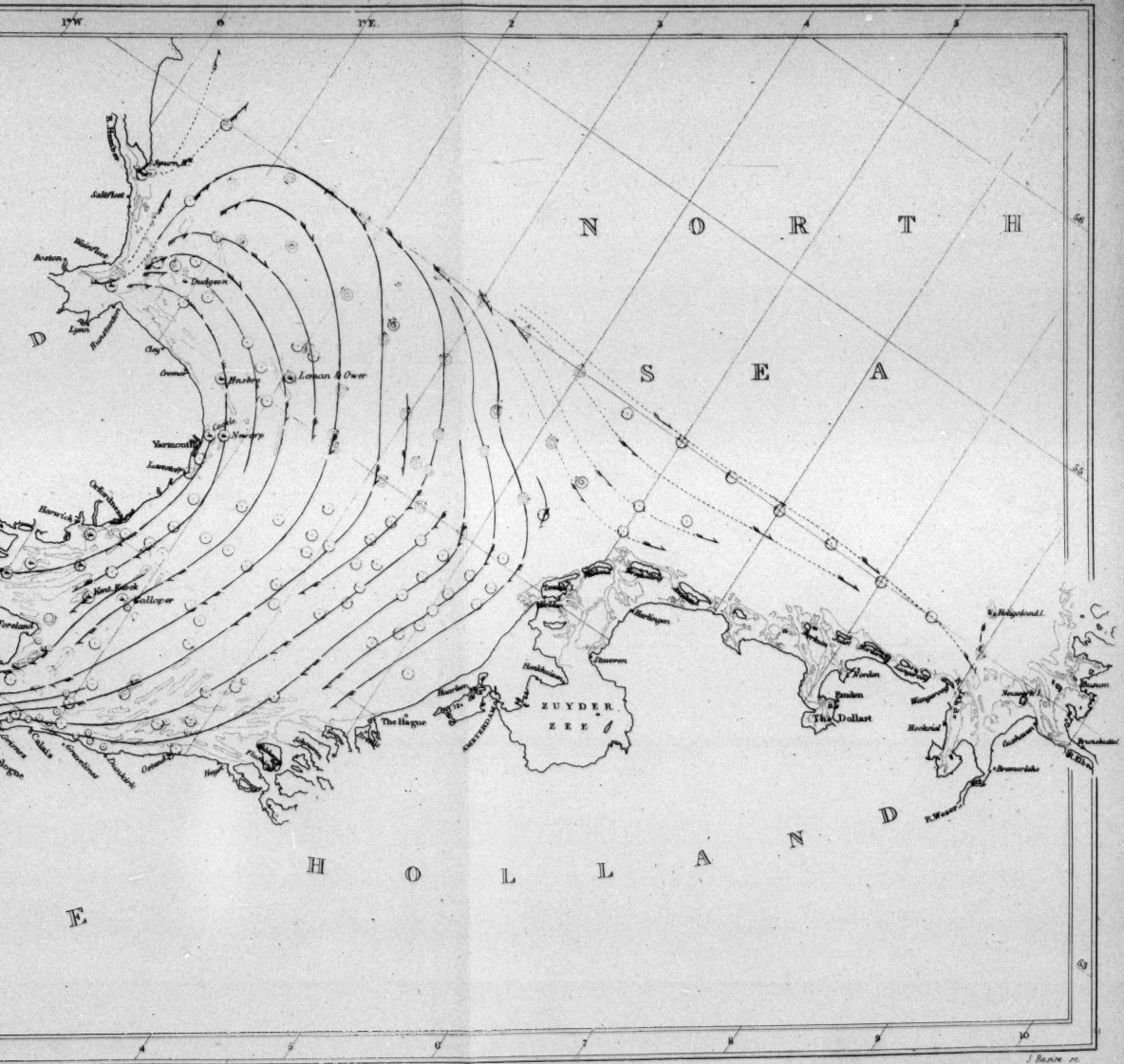
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Wangeroog
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Dunkirk
Ostend
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Utrecht
Gorcum
Dort
Helvoet
Brill
Goree
Zierickzee
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Vlissingen
Sluys
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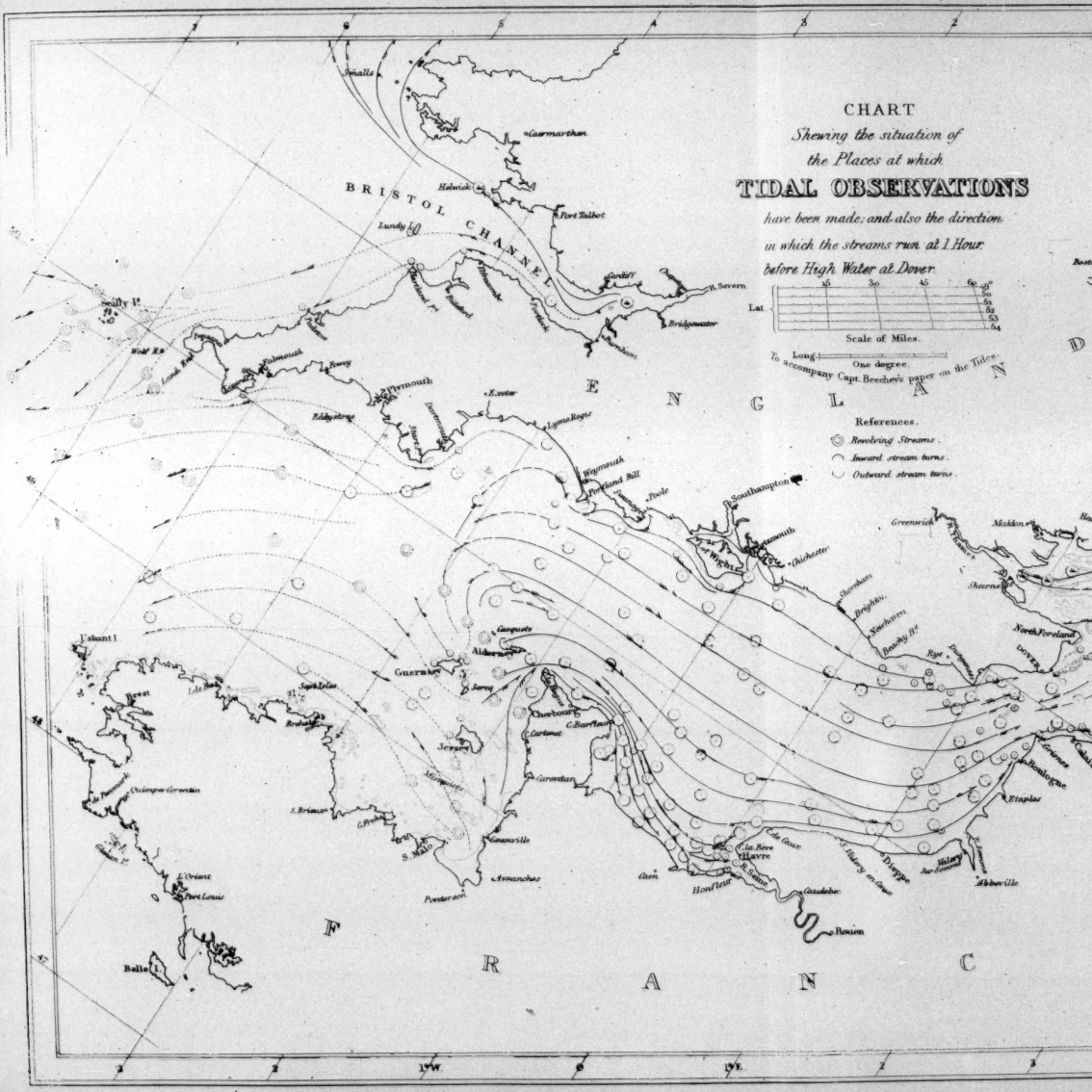
CHART
Shewing the situation of
the Places at which
TIDAL OBSERVATIONS
have been made; and also the direction
in which the streams run at 1 Hour
before High Water at Dover.
Lat.
Scale of Miles.
Long.
One degree.
To accompany Capt. Beechey's paper on the Tides.
References.
Revolving Streams.
Inward stream turns.
Outward stream turns.
BRISTOL CHANNEL
ENGLAND
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Cardiff
R. Severn
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Falmouth
Plymouth
Exeter
Lyme Regis
Weymouth
Portland Bill
Poole
Southampton
Isle of Wight
Chichester
Greenwich
Sheerness
North Foreland
DOVER
Brighton
Beachy Hd
Rye
Hastings
Ushant I
Brest
Guernsey
Alderney
Casquets
Jersey
C. Barfleur
Cherbourg
Granville
S. Malo
Arromanches
Port Louis
L'Orient
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Honfleur
Caudebec
Havre
R. Seine
Dieppe
St Valery en Caux
Abbeville
Etaples
Boulogne
Calais
Ostend
Gris Nez
Cape Grisnez
Pointe de Barfleur
Pte de Penmarch
Quiberon
Audierne
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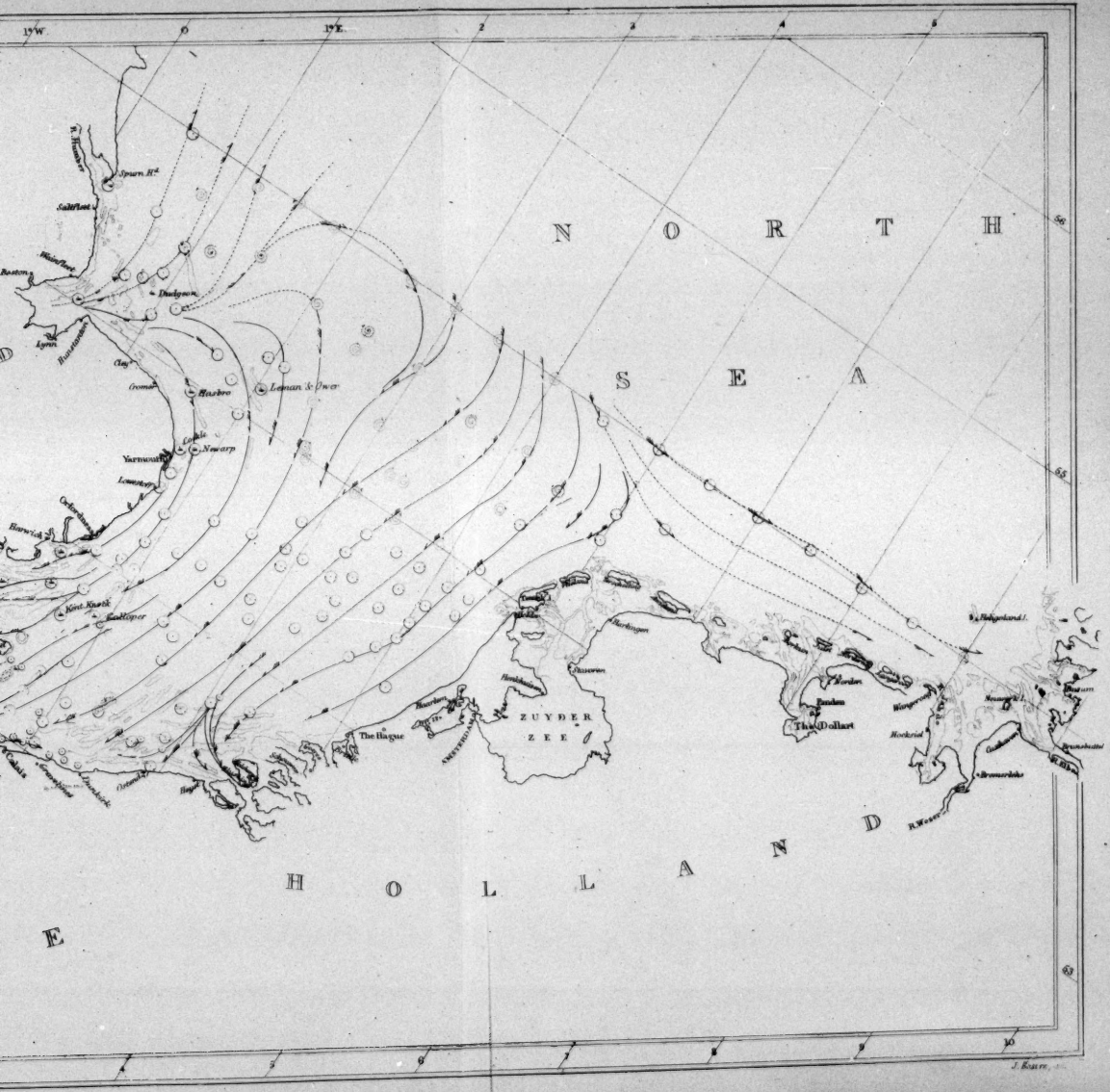
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Texel
Vlieland
Terschelling
Ameland
Schelling
Borkum
Huibergen
Oosterem
Harder
Enkhuysen
The Vollart
Hoorn
Medemblik
Stavoren
Workum
Hindeloopen
Schokland
Urk
Elburg
Bunschoten
Amersfort
Rhine
Vlie
Flie
Pollendam
Brouwershaven
R. Weser
J. Keare

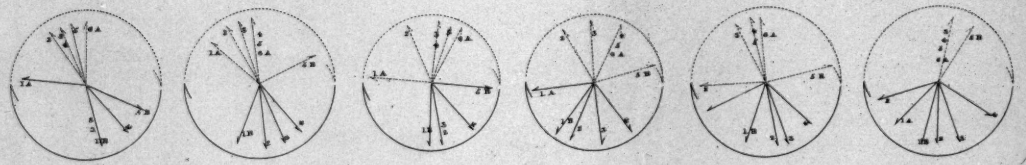
Phil. Trans. MDCCCXI. Plate XI. p. 316.

PLAN I.

Shewing the course of the stream both in the revolving and in the true Tides of the North Sea and English Channel.

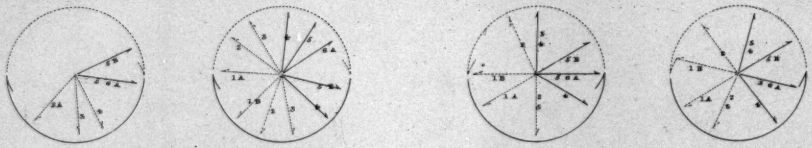
REVOLVING TIDES.

30 m. from the coast of Norfolk



27 mls. off the Texel H.d

25 mls. off the Start

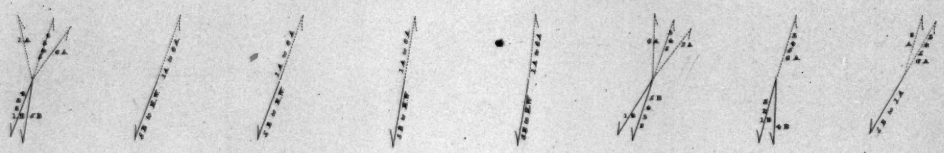


11 mls. off Guernsey

TIDES

observed in the true stream of the Channel.

11 mls. off Isboro'



11 mls. off Katwyk.

22 mls. off Portland

22 mls. off Beachey H.d

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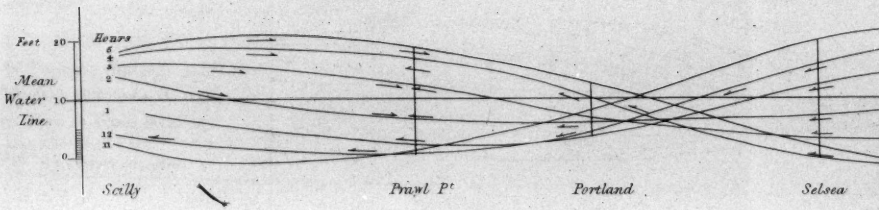
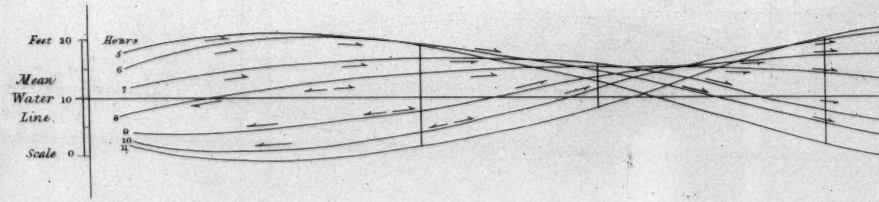
PL

Shewing the form of the surface of the water, and
at each

FALLING T



RISING T

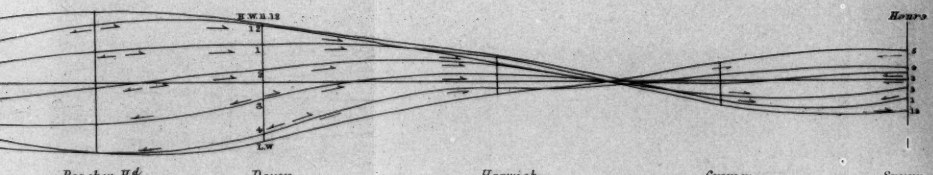


These sections are
as they occur upon

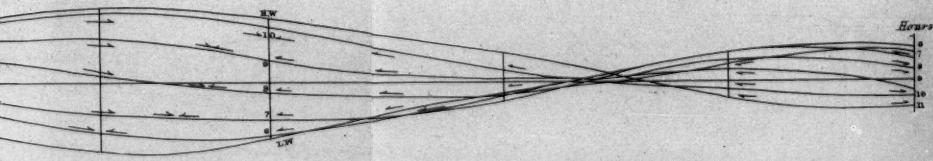
LAN 2.

l the direction of the stream between Scilly and Spurn Pt
h hour of the Tide.

TIDES AT DOVER.

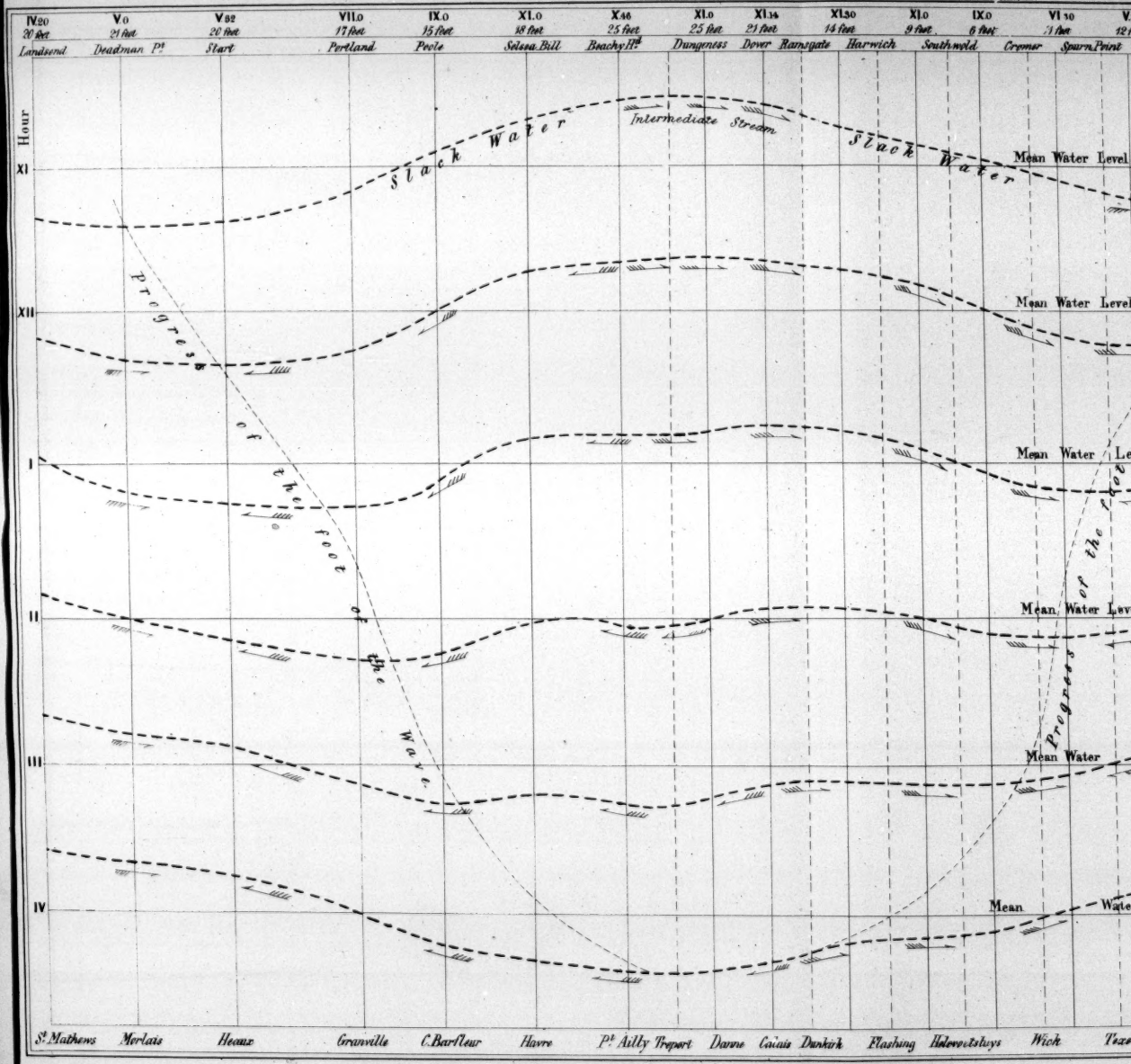


TIDES AT DOVER.



re THE MEAN of the Wave Lines
n the English and French Coasts.

J. Basire sc



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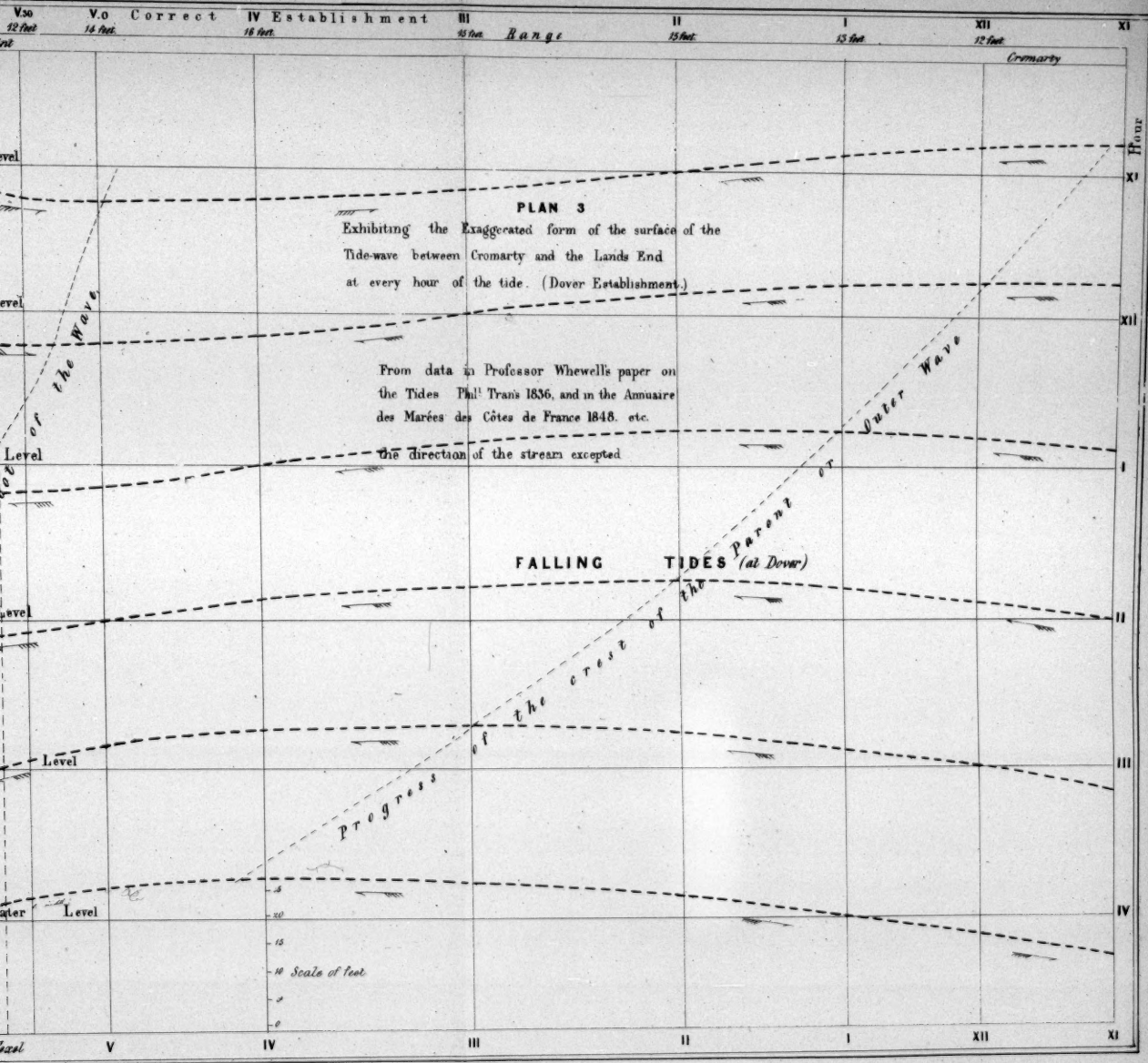
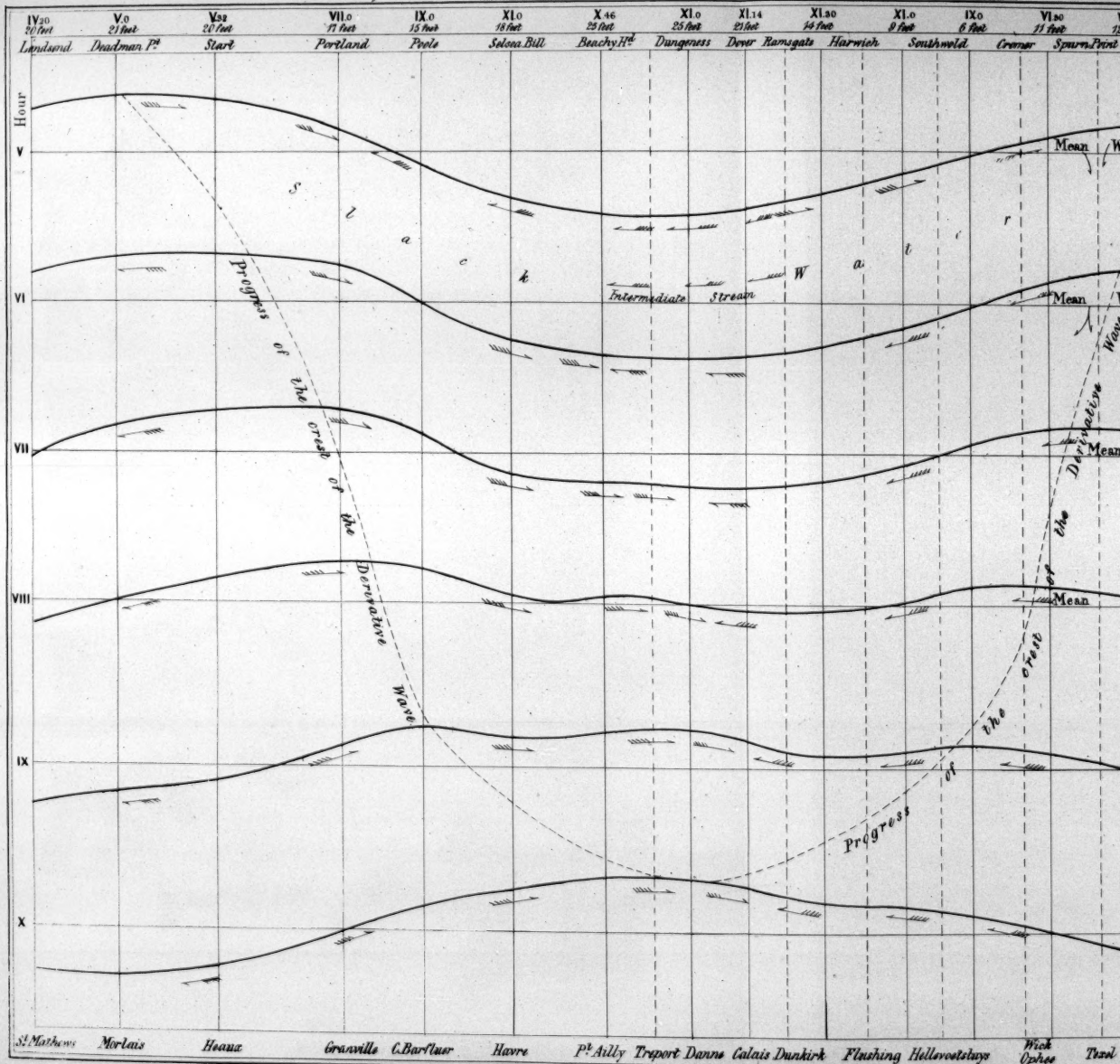
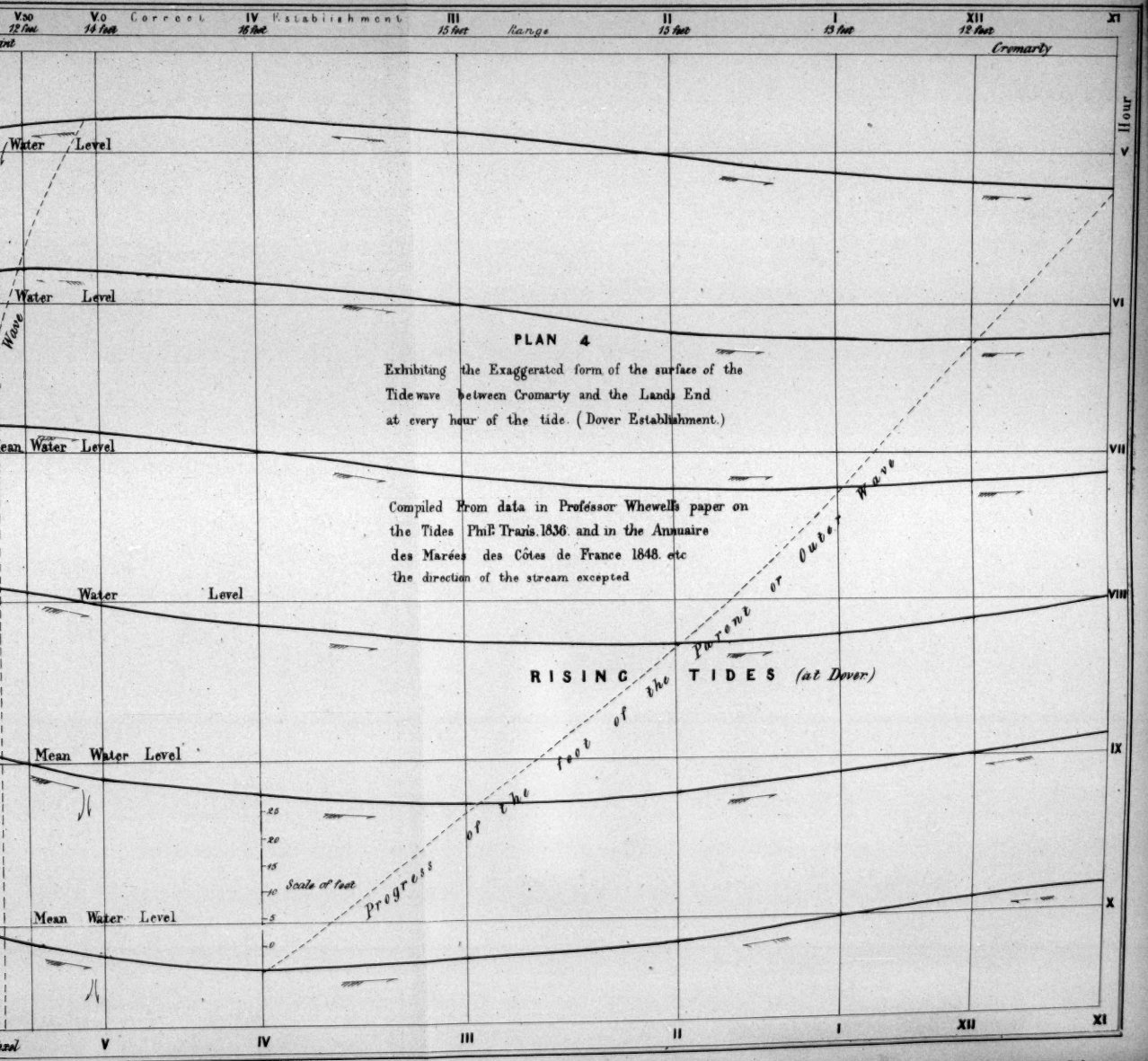


the Tides of the English Channel.

J. Basire, Lithe



To accompany Capt Beechey's report upon th



the Tides of the English Channel

Fig. 1.

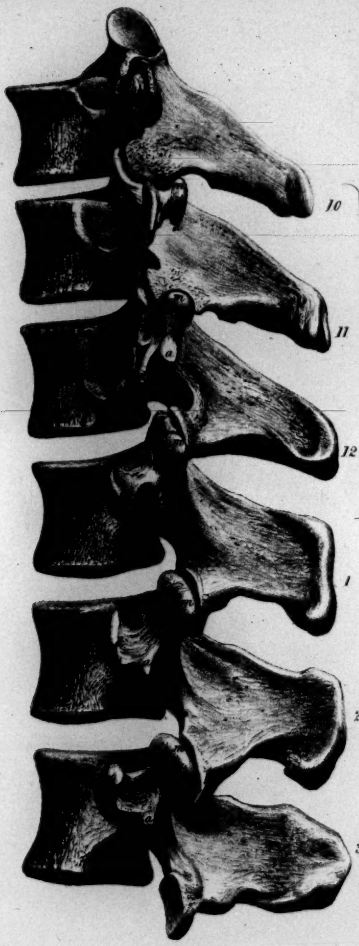
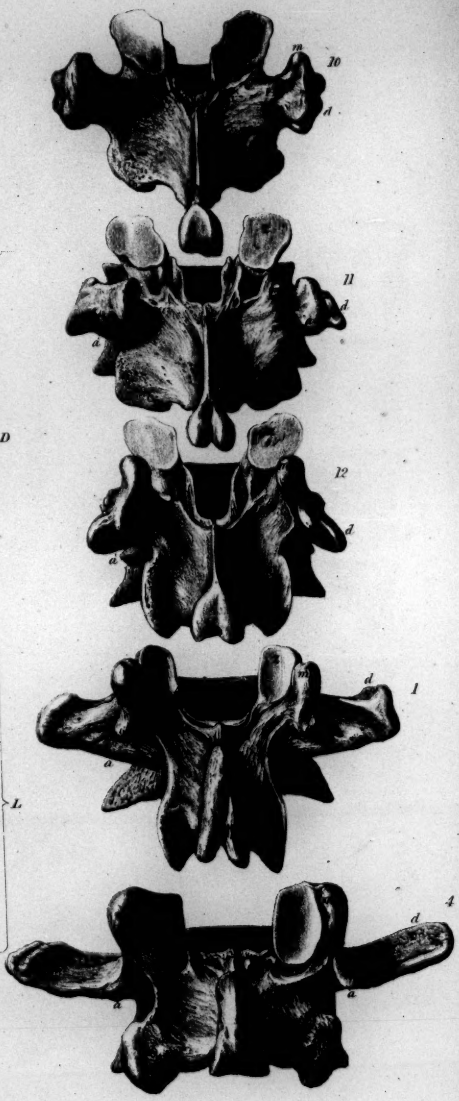


Fig. 2.



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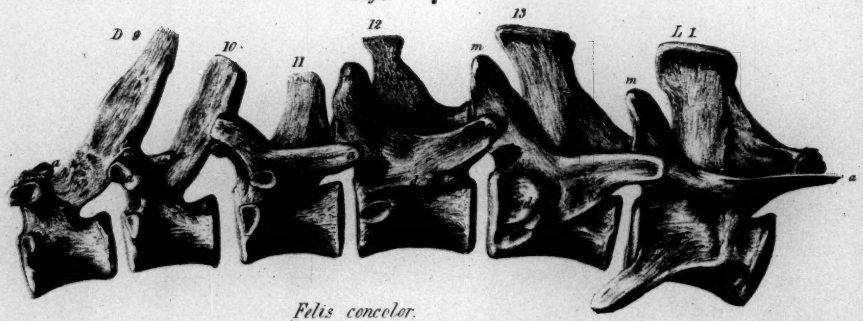
Printed by J. Squire.

Fig. 5.



Felis concolor.

Fig. 4.

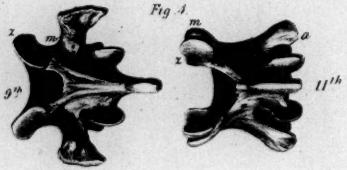
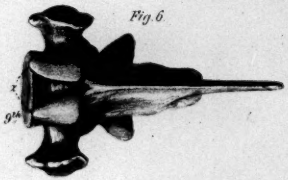


Fig. 6.



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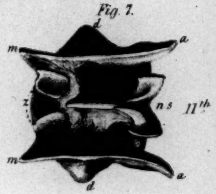
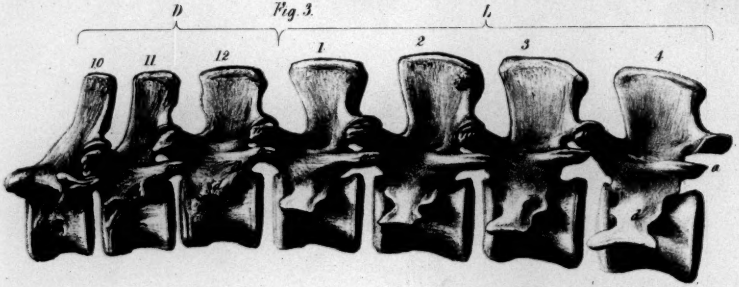


Fig. 3.



Macacus nemestrinus

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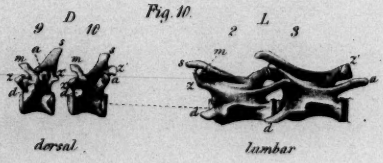
Fig. 8.

(Leptonyx.) Phoca serridens.

Fig. 9.

Helamys Capensis

Fig. 10.

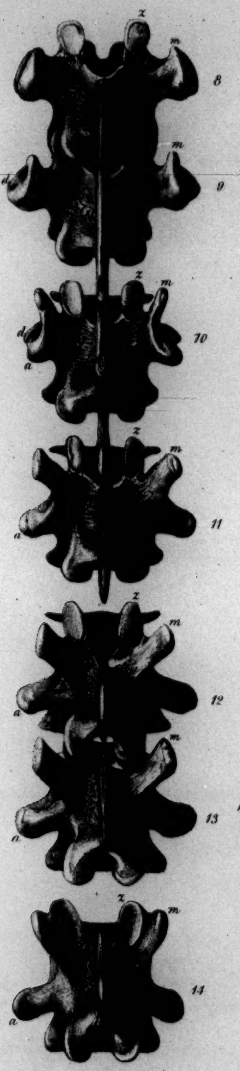


Lagotis

Dinkel. del. et lith.

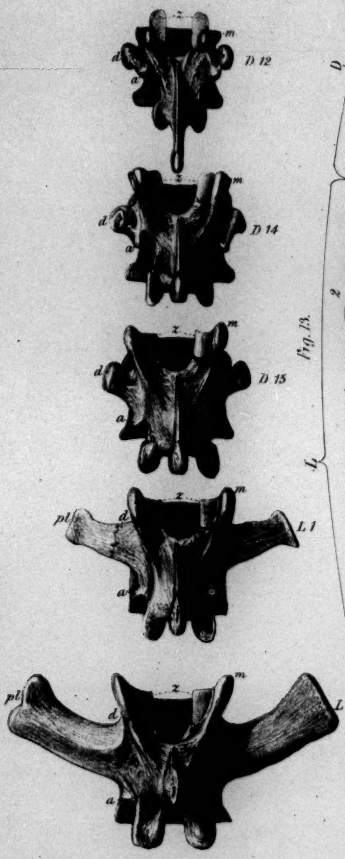
Printed by J. Basire.

Fig. 11.



Hydrochærus Capybara.

Fig. 12.



Phascolomys Wombat.

Fig. 13.

Macropus Parryi.

Dinkel. del. et lith.

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Delphinus Delphis.

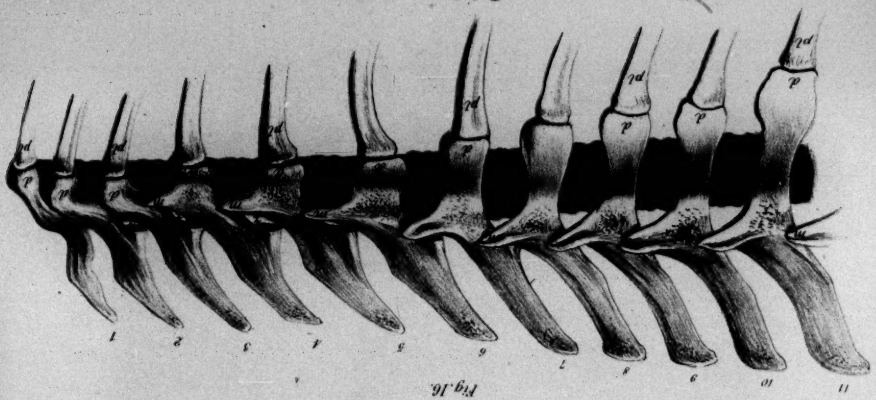

Fig. 16.

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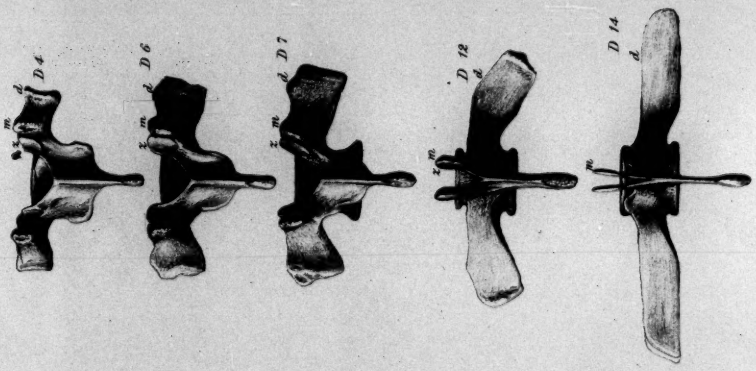
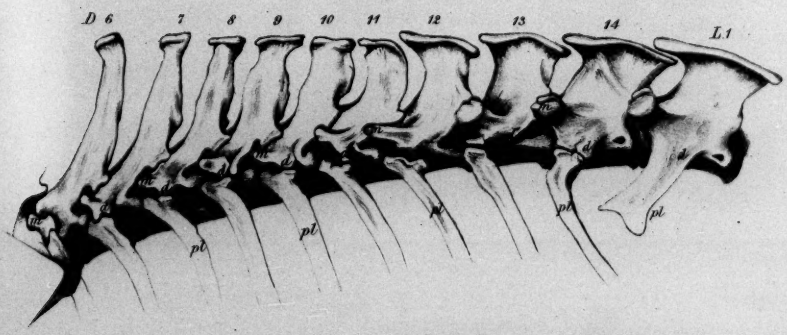
Fig. 15.



Delphinus Tursio.
½ nat. size.

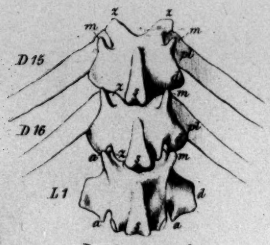
Fig. 14.



Moschus moschiferus.

J. Dinkel, del. et lith.

Fig. 17.



Bradypus tridactᵘˢ

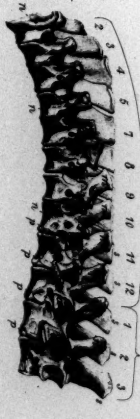
Fig. 18.

Dasypus longicaudus

Echidna

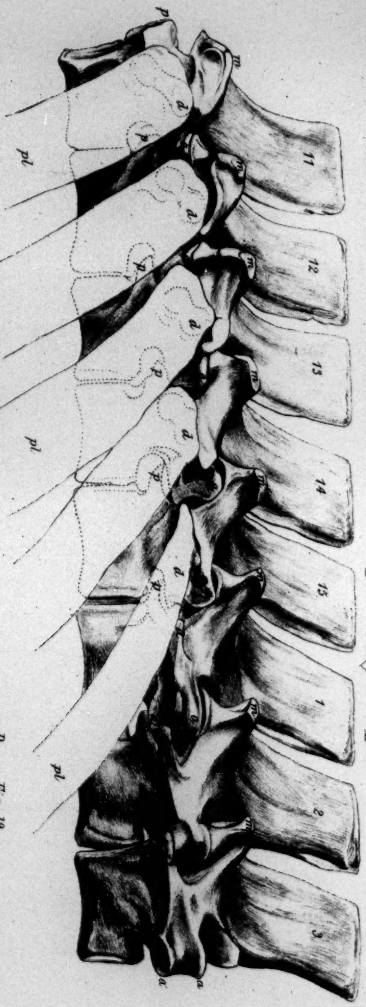
Myrmecophaga jubata.

Fig. 19.



Dasypus trinctus

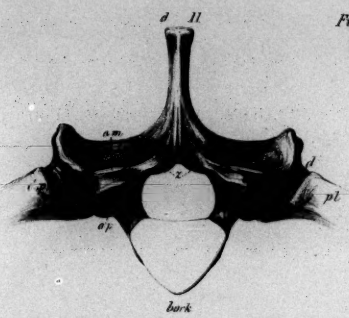
Fig. 20.



Dinkel del.

Printed by J. Basire.

Fig. 21.



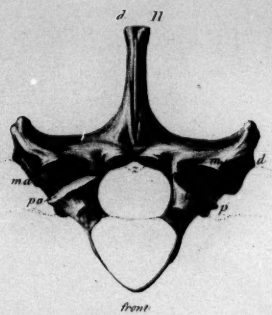
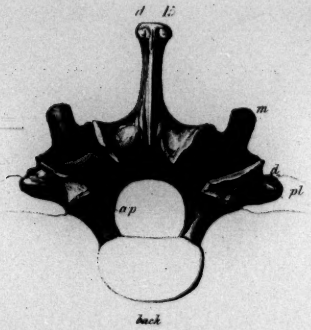


Fig. 22.



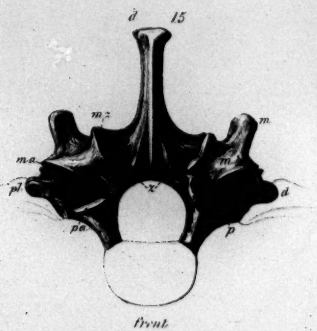
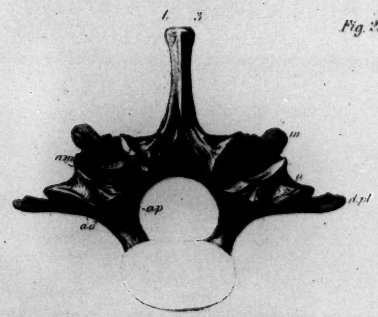
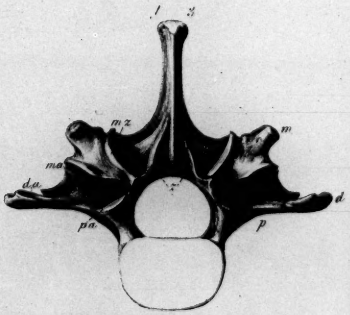


Fig. 23.





Myrmecophaga jubata

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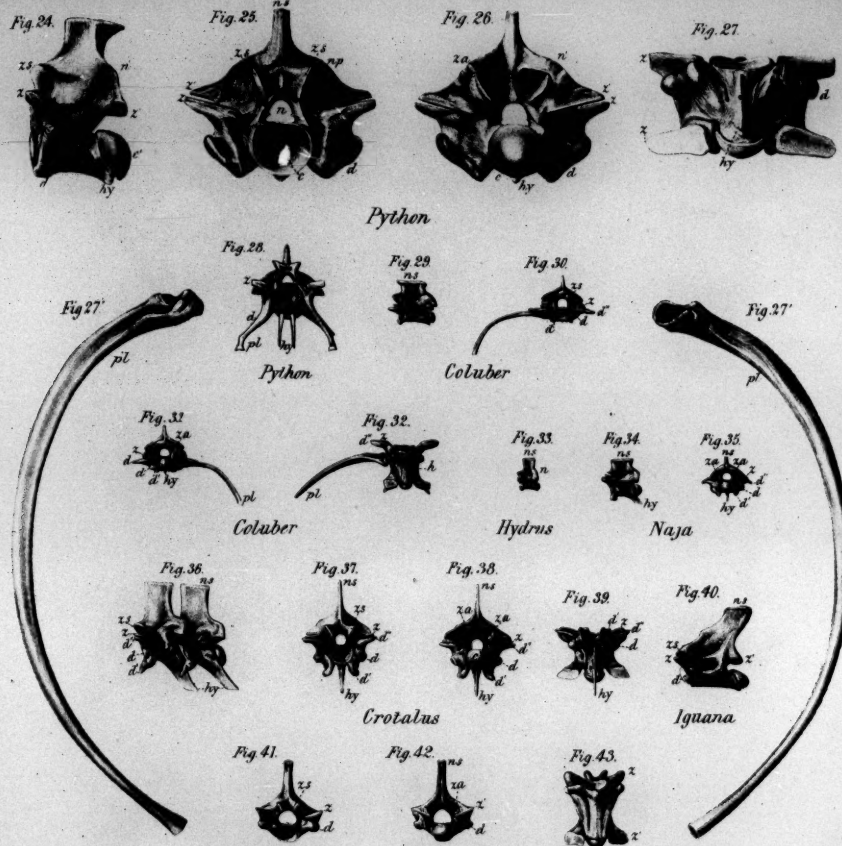
Fig. 24.
Fig. 25.
Fig. 26.
Fig. 27.
Python
Fig. 28.
Fig. 29.
Fig. 30.
Fig. 27'
Fig. 27'
Python
Coluber
Fig. 31.
Fig. 32.
Fig. 33.
Fig. 34.
Fig. 35.
Coluber
Hydrus
Naja
Fig. 36.
Fig. 37.
Fig. 38.
Fig. 39.
Fig. 40.
Crotalus
Iguana
Fig. 41.
Fig. 42.
Fig. 43.
Iguana
Fig. 44.
Fig. 45.
Fig. 46.
Fig. 47.
Palæophis

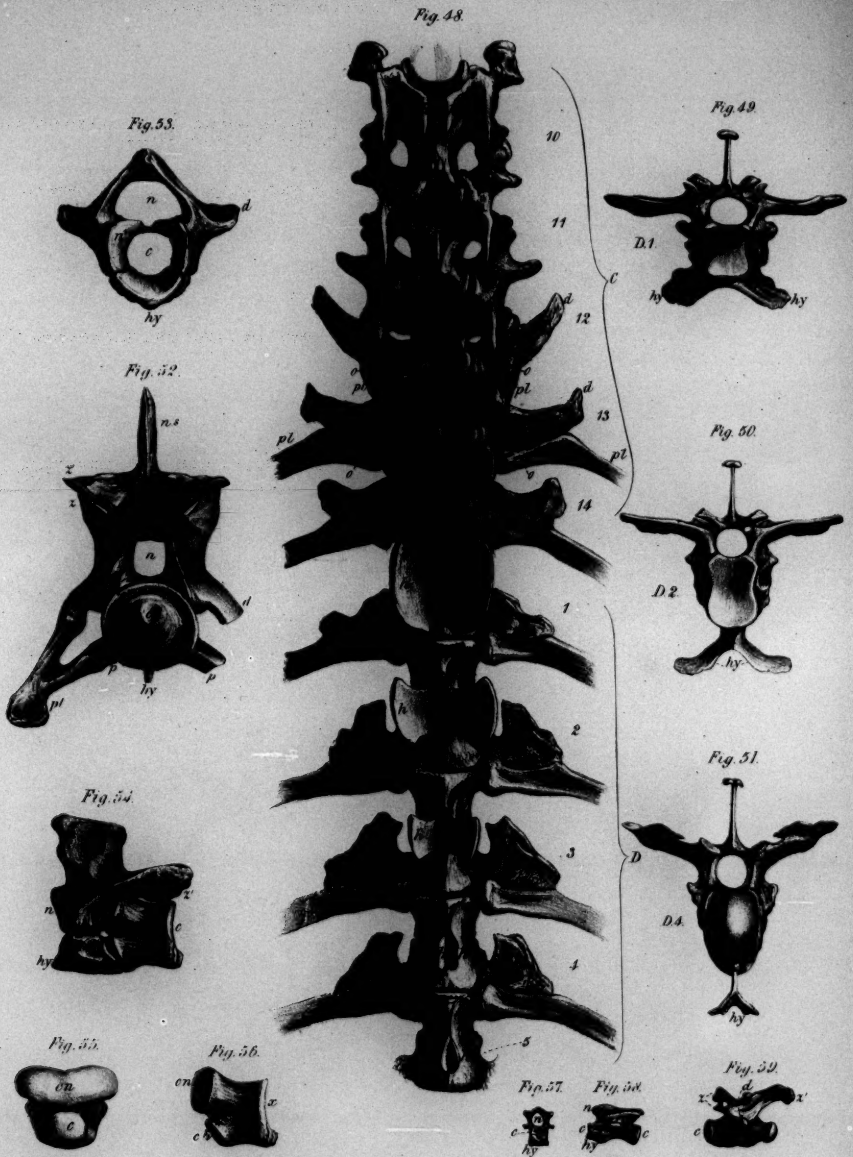

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Fig. 61.

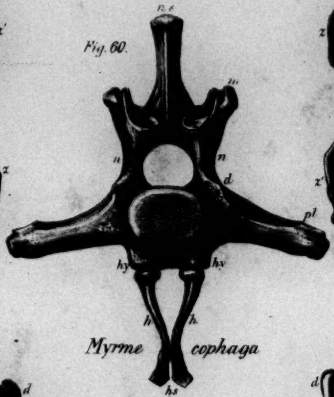
Castor

Fig. 63

Fig. 64.

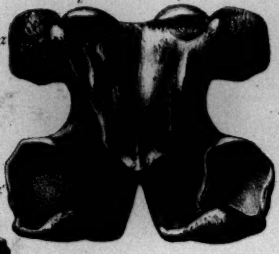
Trionyx

Fig. 60.

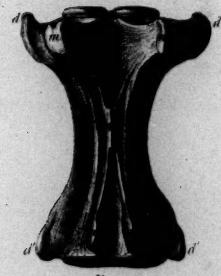


Myrmecophaga

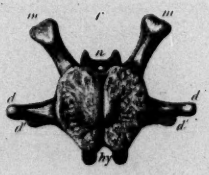
Fig. 65.



Trionyx

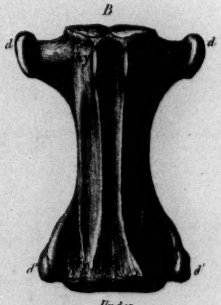


Upper



Macropus

9th Caudal



Under

J. Thwaites del. et lith.

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